The mobility of nonlocal solitons in fading optical lattices*

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We study the soliton mobility in nonlocal nonlinear media with an imprinted fading optical lattice. The results show that the transverse mobility of solitons varies with the lattice decay rate and the nonlocality degree, which provides an opportunity for all-optical control of light.

Keywords: optical lattice, nonlocal soliton

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1. Introduction

In recent years, the propagation of solitons in optical lattices has attracted a lot of attention for its unique properties for all-optical control of light.[1] Previous studies include diffraction-managed solitons in curved lattices,[2] soliton switching in quasi-continuous optical arrays,[3] and soliton control in chirped optical lattices.[4] Longitudinal modulation of lattices offers additional opportunities for soliton control. In particular, periodic longitudinal modulation of the coupling strength results in soliton oscillations and decay,[5] harmonic longitudinal modulation of the linear refractive index leads to parametric amplification of transverse soliton swinging,[6] while longitudinal modulation of the nonlinearity can be used to steer solitons.[7] Nowadays the technique in fabricating a desired optical lattice is mature. Even a three-dimensional optical lattice has been created by using the optical induction technique.[8,9]

The properties of solitons supported by media with a local nonlinear response are now well established. However, under appropriate conditions, the nonlinear response of materials can be highly nonlocal, that is a phenomenon which affects the properties of solitons supported by such media greatly.[10,11] Some unique properties are discovered in this context. For example, the nonlocality of the nonlinear response can greatly enhance the soliton mobility,[12,13] there exist thresholds of lattice parameters above which the nonlocal lattice solitons can exist,[14] and in the strongly nonlocal limit the soliton exhibits a pulsating propagation due to the effect of the nonlocality and the lattice.[15] Recently, the defect solitons in parity–time symmetric potentials with nonlocal nonlinearity were investigated. It was found that nonlocality can expand the stability region of defect solitons.[16]

In this paper, we study the soliton mobility in nonlocal nonlinear media with an optical lattice that fades exponentially along the propagation direction.

2. Soliton mobility

We consider the following dimensionless nonlocal nonlinear Schrödinger equation (NNLSE)[17,18] with a fading optical lattice[19]

$$\mathrm{i}\frac{\partial \widetilde{U}}{\partial z}+\frac{1}{2}\frac{\partial^2 \widetilde{U}}{\partial^2 x}+\widetilde{U}\int R(x-\xi)|\widetilde{U}(\xi,z)|^2\mathrm{d}\xi + \widetilde{U}pQ(x,z)=0, \tag{1}$$

where $\widetilde{U}(x,z)$ is the complex amplitude envelope of the light beam, x and z are transverse and longitudinal coordinates scaled to the beam width and diffraction length, respectively, $R(x)$ is the real symmetric nonlocal response function, and $pQ(x,z)=p\cos^2(\Omega x)\exp(-\delta z)$ describes the profile of the optical lattice (p is the lattice depth, Ω is the lattice frequency, and δ is the decay rate) (Fig. 1). In practice, such a kind of lattice can be induced optically. Specifically, the refractive index modulation in the transverse direction can be induced with two interfering plane waves.[20,21] Longitudinal modulation can

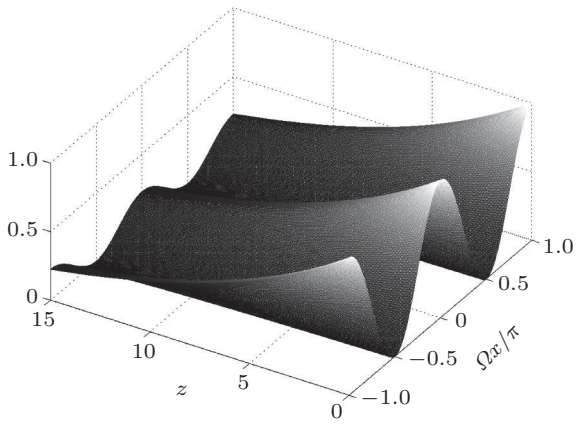


Fig. 1. Profile of a fading optical lattice with $\delta=0.1$ and $p=1$.

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be created by varying intensities,[22] intersection angles, or carrying wavelength[23] of the lattice-forming plane waves.

As previously indicated in Ref. [18], in the strongly nonlocal case if $R(x)$ is twice differentiable at $x=0$, equation (1) can be reduced to the following equation

$$\begin{aligned}&\mathrm{i}\frac{\partial \tilde{U}}{\partial z}+\frac{1}{2}\frac{\partial^2 \tilde{U}}{\partial^2 x}\\&+\tilde{U}\int\left[R_0+\frac{1}{2}R_0''(x-\xi)^2\right]|\tilde{U}(\xi,z)|^2\mathrm{d}\xi\\&+\tilde{U}p\cos^2(\Omega x)\exp(-\delta z)=0,\end{aligned}\tag{2}$$

where $R_0=R(0)$ and $R_0''=R''(0)$. For the NNLSE, the beam power is an invariant[24]

$$P=\int|\tilde{U}|^2\mathrm{d}x.\tag{3}$$

If we define the beam center as[25]

$$q(z)=\frac{\int\xi|\tilde{U}(\xi,z)|^2\mathrm{d}\xi}{\int|\tilde{U}(\xi,z)|^2\mathrm{d}\xi},\tag{4}$$

by making use of Eqs. (3) and (4), equation (2) is reduced to

$$\begin{aligned}&\mathrm{i}\frac{\partial \tilde{U}}{\partial z}+\frac{1}{2}\frac{\partial^2 \tilde{U}}{\partial^2 x}+\tilde{U}R_0P+\frac{1}{2}\tilde{U}R_0''P(x-q)^2\\&+\frac{1}{2}\tilde{U}R_0''\int(\xi-q)^2\mathrm{d}\xi\\&+\tilde{U}p\cos^2(\Omega x)\exp(-\delta z)=0.\end{aligned}\tag{5}$$

By the following transformation[25]

$$\begin{aligned}U=\tilde{U}\exp\Bigg\{-\mathrm{i}\Bigg[&R_0P\\&+\frac{R_0''}{2}\int_0^z\mathrm{d}z'\int(\xi-q)^2|\tilde{U}(\xi,z)|^2\mathrm{d}\xi\Bigg]z\Bigg\},\end{aligned}\tag{6}$$

equation (5) turns into

$$\begin{aligned}&\mathrm{i}\frac{\partial U}{\partial z}+\frac{1}{2}\frac{\partial^2 U}{\partial x^2}-\frac{U}{2}\gamma P(x-q)^2\\&\quad+Up\cos^2(\Omega x)\exp(-\delta z)=0,\end{aligned}\tag{7}$$

where $\gamma=-R_0''>0$.

Comparing Eq. (7) with the one-dimensional Schrödinger equation[26]

$$\mathrm{i}\hbar\frac{\partial}{\partial t}\psi(x,t)=\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}+V(x)\right]\psi(x,t),\tag{8}$$

in an equivalent quantum system with $\hbar=1$ and $m=1$, the equivalent potential energy of Eq. (7) is

$$V(x)=\frac{1}{2}\gamma P(x-q)^2-p\cos^2(\Omega x)\exp(-\delta z).\tag{9}$$

With the aid the of Ehrenfest theorem[26,27] and Eqs. (7)–(9), we get an equation of motion for the beam center as follows:

$$\frac{\mathrm{d}^2q}{\mathrm{d}z^2}=\frac{\int[-\gamma P(x-q)-\Omega p\exp(-\delta z)\sin(2\Omega x)]|U|^2\mathrm{d}x}{\int|U|^2\mathrm{d}x}.\tag{10}$$

In the context of strongly nonlocal cases, we employ the following Gaussian-type trial solution

$$U(x,z)=A\exp\left[-\frac{(x-q)^2}{2w^2}\right]\exp[\mathrm{i}\beta(x-q)+\mathrm{i}\phi],\tag{11}$$

where A and ϕ are the amplitude and phase of the beam respectively, w is the beam width, and β is the incident angle with respect to z axis. This choice of ansatz is justified since, in the limit $p\to0$, Eq. (11) describes solitons in uniform strongly nonlocal nonlinear media.[18] Substitution of Eq. (11) into Eq. (10) yields

$$\frac{\mathrm{d}^2q}{\mathrm{d}z^2}+\frac{\Omega_0^2}{2\Omega}\sin(2\Omega q)=0,\tag{12}$$

where $\Omega_0=[2p\Omega^2\exp(-\delta z)\exp(-\Omega^2w^2)]^{1/2}$ defines the frequency of small-amplitude oscillations of the soliton center.

Without loss of generality, we assume that a soliton is launched into the medium at the point $x=0$ with an angle β_0, so that $q|_{z=0}=0$ and $\mathrm{d}q/\mathrm{d}z|_{z=0}=\beta_0$. For $\delta=0$, it has been shown that the soliton's movement is separated by a critical angle $\beta_\mathrm{c}=\left[2p\exp(-\Omega^2w^2)\right]^{1/2}$. When the incident angle is smaller than the critical angle, the soliton is trapped within the waveguide induced by the optical lattices. Above the critical angle, the soliton leaves the waveguide and travels across the lattices.[13]

In the presence of longitudinal modulation, i.e., $\delta\neq0$, the critical angle

$$\beta_\mathrm{c}\sim\Omega_0/\Omega=\left[2p\exp(-\delta z)\exp(-\Omega^2w^2)\right]^{1/2},$$

indicating that β_c decreases with the propagation distance. As a result, even if the soliton is launched into the lattice with an incident angle $\beta_0<\beta_\mathrm{c}$, it will escape from the central waveguide once the condition $\beta_0>\beta_\mathrm{c}$ is satisfied. In the limit $\delta\to0$, the soliton is trapped by the first waveguide and never escapes from the central waveguide. On the contrary, it propagates along its original direction traveling across the lattices without oscillations. As the critical angle depends on the lattice decay rate, a modulation of the decay rate leads to a change of the soliton transverse mobility, a property that might be used for soliton steering.

3. Numerical simulation

To intuitively show the effect of the lattice and the nonlocality, we numerically simulate the propagation of a Gaussian-type beam in nonlocal nonlinear media with a fading optical lattice. We take Eq. (1) as the evolution equation and $U(x,z)|_{z=0}$ in Eq. (11) as the initial condition. The numerical arithmetic we use here is the split-step Fourier method.[28] The nonlocal response function is supposed to be a Gaussian function,[18] i.e., $R(x)=\exp(-x^2/2w_\mathrm{m}^2)/(\sqrt{2\pi}w_\mathrm{m})$, where we

introduce a nonlocal parameter α to define the degree of nonlocality (α is the ratio of the beam width w to the characteristic nonlocal response length $w_{\rm m}$ of the medium). The smaller the nonlocal parameter, the stronger the nonlocality degree.

The dependence of the soliton mobility on the lattice decay rate when $\alpha = 0.1$ is shown in Fig. 2, which indicates that the soliton is emitted at different transverse positions with the change of the lattice decay rate. Since the critical angle decreases more quickly for a larger decay rate, an increase of the rate leads to a decrease of the number of soliton oscillations. Depending on the lattice decay rate, solitons with the angle $\beta_0 < \beta_{\rm c}$ perform different numbers of oscillations and then start moving freely in diverse directions. When the rate increases to 0.5, the soliton does not oscillate but is always deflected in the same direction (Fig. 2(d)). To compare the theoretical prediction with the numerical simulation, we add dashed lines based on Eq. (12) to Fig. 2. It is found from Figs. 2(a), 2(b), and 2(c) that the theoretical prediction agrees well with the numerical simulation. However, we find a deviation from Fig. 2(d). The reason for the deviation is that the validity of the Ehrenfest theorem requires a slow change of the potential energy.[27] This requirement is not met in Fig. 2(d) since a large δ leads to a big change of the equivalent potential energy.

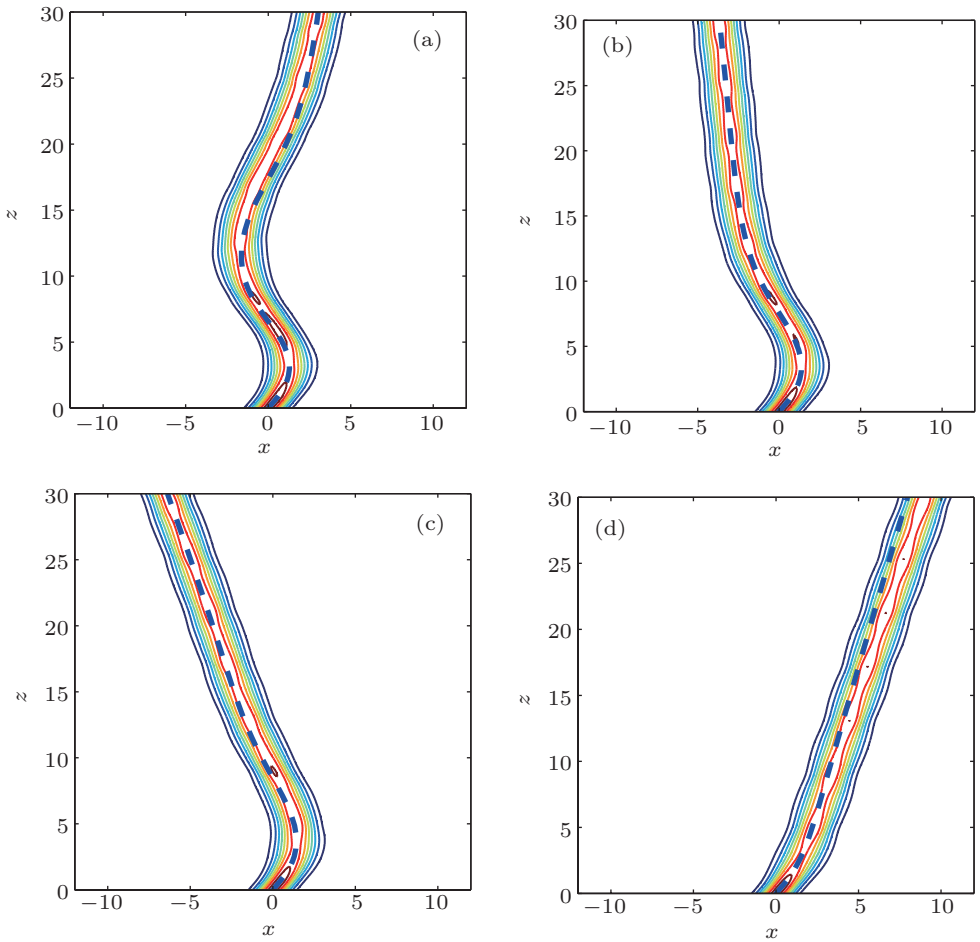


Fig. 2. (color online) Contour plots showing the dependence of the soliton mobility on the lattice decay rate. (a) $\delta = 0.1$; (b) $\delta = 0.15$; (c) $\delta = 0.2$; (d) $\delta = 0.5$. For all the cases, $\alpha = 0.1$, $p = 1$, $\Omega = 0.5$, and $\beta_0 = 0.7$. The dashed lines show the movement of the soliton center based on Eq. (12).

Figure 3 shows the dependence of the output position of solitons on the nonlocality degree. The curves in Fig. 3 indicate that the nonlocality produces an important effect on the soliton transverse mobility. The reason is that the Peierls–Nabarro potential barrier for solitons moving across the lattice is drastically reduced in the presence of nonlocality, which results in an obvious reduction of the radiation losses of solitons traveling across the lattices.[12] Since the nonlocality makes the radiative trapping less likely to happen, an increase of the nonlocality degree leads to an enhancement of the soliton mobility, as shown in Fig. 3. Similarly, to make a comparison between the theoretical result and the numerical simulation, we calculate the output position of the soliton center based on Eq. (12) and plot it in Fig. 3 as dashed lines. We see from Fig. 3 that the theoretical prediction agrees well with the numerical simulation when α is small. With the increase of α,

a deviation appears. The reason is that the theoretical result is obtained under the assumption of strong nonlocality, i.e., small α. For large α, energy radiation occurs and the theoretical result is no longer valid.

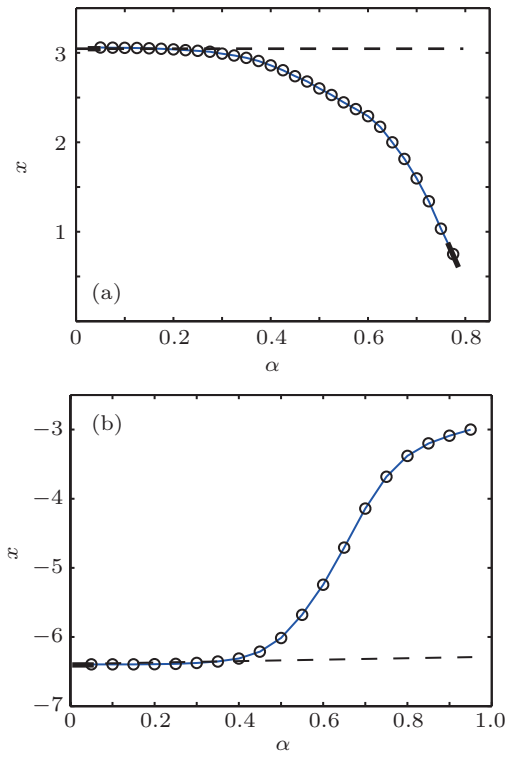


Fig. 3. (color online) Dependence of the output position of solitons on the nonlocality degree when the propagation distance is 30. (a) $\delta = 0.1$; (b) $\delta = 0.2$. For both cases, $p = 1$, $\Omega = 0.5$, and $\beta_0 = 0.7$. The dashed lines show the output position of the soliton center based on Eq. (12).

4. Conclusion

In conclusion, we have studied the mobility of solitons in nonlocal nonlinear media with a fading optical lattice. Based on the Ehrenfest theorem, we obtained an expression of the critical angle for the soliton's movement in the strongly nonlocal case, which decreases with the propagation distance. As a result, the soliton always escapes from the central waveguide as long as the critical angle is smaller than the incident angle. It is found that both the lattice decay rate and the nonlocality degree have important effects on the soliton mobility.

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