Optical bistability induced by quantum coherence in a negative index atomic medium*

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(Received 26 December 2012; revised manuscript received 23 January 2013)

Bistability behaviors in an optical ring cavity filled with a dense V-type four-level atomic medium are theoretically investigated. It is found that the optical bistability can appear in the negative refraction frequency band, while both the bistability and multi-stability can occur in the positive refraction frequency bands. Therefore, optical bistability can be realized from conventional material to negative index material due to quantum coherence in our scheme.

Keywords: optical bistability, quantum coherence, negative refractive index, atomic medium

PACS: 42.65.Pc, 42.50.Gy **DOI:** 10.1088/1674-1056/22/10/104208

1. Introduction

Optical bistability (OB) is a phenomenon in which the system exhibits two output intensities for the same input intensity.[1,2] It has been an interesting topic because of its wide applications in optical transistors, memory elements, all optical switches, and so on.[3,4] Not only has it been realized in conventional schemes, such as a Fabry–Pérot resonator filled with a nonlinear material,[5] nonlinear couplers,[6,7] layered periodic structures,[8–10] an optical cavity filled with Bose–Einstein condensate (BEC),[11–14] and quantum well,[15] but also in schemes filled with negative index material (NIM).[16–19] NIM is a kind of material with a negative refractive index, in which, the wave vector is opposite to the direction of energy propagation.[20] In 2004, Feise *et al.*[16] demonstrated bistability for two types of band gaps of a layered structure composed of two materials with positive and negative refractive index. Afterwards, Litichinister *et al.*[17] proposed a nonlinear optical coupler with one channel filled with a negative index material, in which, optical bistability was realized because the negative index material channel could give rise to an effective feedback mechanism. Recently, Chen *et al.*[18] put forward a scheme composed of square plasmonic loops loaded by Kerr nonlinearities. In their scheme, a bistable and self-tunable negative index material can be realized at optical frequency.

Based on quantum coherence, OB in atomic systems has been studied theoretically and experimentally.[21–24] For example, as is well known, the OB and dynamic instability were experimentally observed and studied in a system consisting of three-level Λ-type rubidium atoms in an optical ring cavity.[21] Also, it was found that spontaneously generated coherence (SGC) could significantly affect OB behavior in a three-level atomic scheme.[22] A noticeable Doppler effect on OB has been investigated in an N-type active Raman gain atomic system.[23] It is shown that the Doppler effect can greatly enhance the dispersion and create OB or increase the bistable region. Furthermore, the phase fluctuation of the driven field can be used to control the OB response to a Λ-type atomic medium.[24] Up to now, although OB has been studied in various atomic systems, to our knowledge, little research work has been reported in a negative index atomic medium.

In this paper, we will discuss the OB behaviors in an atomic medium with a negative refractive index. We put forward a V-type level atomic scheme. Based the quantum coherence, the negative refractive index can be realized by adjusting the applied fields in this atomic system. It is shown that OB appears in the negative refraction frequency bands, however, both OB and optical multistability (OM) can occur in the positive refraction frequency bands. Changing the detunings and intensities of the applied laser fields, OB can be switched to OM from a negative index medium to a positive index medium.

2. Model and equations

Consider a V-type four-level atomic system as shown in Fig. 1. The two lower levels $|1\rangle$ and $|2\rangle$ have opposite parities

*Project supported by the Fundamental Research Funds for the Central University (Grant Nos. GK201002024 and GK201003003), the National Natural Science Foundation of China (Grant Nos. 11104176 and 11104185), the Natural Science Foundation of Shaanxi Province, China (Grant No. 2011JQ1008), the Special Fund of Shanghai Outstanding Young Teachers, China (Grant Nos. slg10054 and slg10023), and the Innovation Program of Shanghai Municipal Education Commission, China (Grant No. 11YZ118).

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with $d_{12} = \langle 1|\hat{\boldsymbol{d}}|2\rangle \neq 0$, where $\hat{\boldsymbol{d}}$ is the electric dipole operator. The two upper levels, $|3\rangle$ and $|4\rangle$, have the same parity and $\mu_{34} = \langle 3|\hat{\boldsymbol{\mu}}|4\rangle \neq 0$, where $\hat{\boldsymbol{\mu}}$ is the magnetic dipole operator. Levels $|1\rangle$ and $|2\rangle$ are coupled by a weak probe field $\Omega_{\rm p}$ with frequency $\omega_{\rm p}$, while levels $|1\rangle$ and $|3\rangle$ are coupled by a strong pump field $\Omega_{\rm s}$ with frequency $\omega_{\rm s}$. Levels $|2\rangle$ and $|4\rangle$ are coupled by a control field $\Omega_{\rm c}$ with frequency $\omega_{\rm c}$, which is a two-photon process. Here,

$$\Omega_{\rm p} = \frac{\boldsymbol{d}_{12}\cdot\boldsymbol{E}_{\rm p}}{2\hbar}, \quad \Omega_{\rm c} = \frac{\boldsymbol{d}_{24}\cdot\boldsymbol{E}_{\rm c}}{2\hbar},$$

and

$$\Omega_{\rm s} = \frac{\boldsymbol{d}_{13}\cdot\boldsymbol{E}_{\rm s}}{2\hbar}$$

denote the Rabi frequency for the corresponding transition, and $\boldsymbol{d}_{ij}$ are the corresponding electric dipole moment. The detunings of the probe, pump, and control fields are defined as $\Delta_{\rm p} = \omega_{21} - \omega_{\rm p}$, $\Delta_{\rm s} = \omega_{31} - \omega_{\rm s}$, and $\Delta_{\rm p} = \omega_{42} - \omega_{\rm c}$, respectively, in which $\omega_{ij} = \omega_i - \omega_j$ is the transition frequency of level $|i\rangle$ and $|j\rangle$ $(i, j = 1, 2, 3, 4)$.

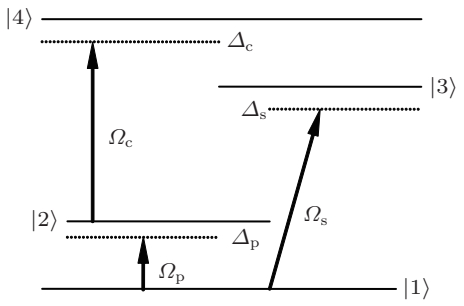


Fig. 1. A four-level V-type atomic scheme as described in the text.

Under the dipole approximation and the rotation wave approximation, the density matrix equations for such an atomic system are

$$\begin{aligned}
\dot{\rho}_{11} &= {\rm i}\Omega_{\rm p}(\rho_{21}-\rho_{12}) + {\rm i}\Omega_{\rm s}(\rho_{31}-\rho_{13}) \\
&\quad + \gamma_{31}\rho_{33} + \gamma_{21}\rho_{22}, && \text{(1a)}\\
\dot{\rho}_{22} &= {\rm i}\Omega_{\rm p}(\rho_{21}-\rho_{12}) + {\rm i}\Omega_{\rm c}(\rho_{42}-\rho_{24}) \\
&\quad + \gamma_{42}\rho_{44} + \gamma_{32}\rho_{33} - \gamma_{21}\rho_{22}, && \text{(1b)}\\
\dot{\rho}_{33} &= {\rm i}\Omega_{\rm s}(\rho_{13}-\rho_{31}) - (\gamma_{31}+\gamma_{32})\rho_{33} + \gamma_{43}\rho_{44}, && \text{(1c)}\\
\dot{\rho}_{12} &= -(\Gamma_{12} - {\rm i}\Delta_{\rm p})\rho_{12} + {\rm i}\Omega_{\rm p}(\rho_{22}-\rho_{11}) \\
&\quad + {\rm i}\Omega_{\rm s}\rho_{32} - {\rm i}\Omega_{\rm c}\rho_{14}, && \text{(1d)}\\
\dot{\rho}_{13} &= -(\Gamma_{13} - {\rm i}\Delta_{\rm s})\rho_{13} + {\rm i}\Omega_{\rm s}(\rho_{33}-\rho_{11}) + {\rm i}\Omega_{\rm p}\rho_{23}, && \text{(1e)}\\
\dot{\rho}_{14} &= -[\Gamma_{14} - {\rm i}(\Delta_{\rm p}+\Delta_{\rm c})]\rho_{14} + {\rm i}\Omega_{\rm p}\rho_{24} \\
&\quad + {\rm i}\Omega_{\rm s}\rho_{34} - {\rm i}\Omega_{\rm c}\rho_{12}, && \text{(1f)}\\
\dot{\rho}_{23} &= -[\Gamma_{23} - {\rm i}(\Delta_{\rm s}-\Delta_{\rm p})]\rho_{23} - {\rm i}\Omega_{\rm s}\rho_{21} \\
&\quad + {\rm i}\Omega_{\rm c}\rho_{43} + {\rm i}\Omega_{\rm p}\rho_{13}, && \text{(1g)}\\
\dot{\rho}_{24} &= -(\Gamma_{24} - {\rm i}\Delta_{\rm c})\rho_{24} + {\rm i}\Omega_{\rm p}\rho_{14} + {\rm i}\Omega_{\rm c}(\rho_{44}-\rho_{22}), && \text{(1h)}\\
\dot{\rho}_{43} &= -[\Gamma_{43} - {\rm i}(\Delta_{\rm p}+\Delta_{\rm c}-\Delta_{\rm s})]\rho_{43} - {\rm i}\Omega_{\rm s}\rho_{41} + {\rm i}\Omega_{\rm c}\rho_{23}, && \text{(1i)}
\end{aligned}$$

constrained by $\rho_{ij} = \rho_{ji}^*$ and $\rho_{11}+\rho_{22}+\rho_{33}+\rho_{44} = 1$. Here, Γ_{ij} denotes the coherence damping coefficient due to spontaneous emission, which is given by $\Gamma_{ij} = (\gamma_i + \gamma_j)/2$, where $\gamma_i = \sum_{k=1}^{4}\gamma_{ik}$ is the decay rate of level $|i\rangle$ and γ_{ik} $(i = 1,2,3,4)$ is the spontaneous emission decay rate from state $|i\rangle$ to state $|k\rangle$. We assume that the atoms are initially in state $|1\rangle$ and the strong pump approximation $\Omega_{\rm p} \ll \Omega_{\rm c} < \Omega_{\rm s}$.

Firstly, we need to search the negative refraction frequency band in this atomic system. As is well known, when the permittivity and permeability are negative simultaneously, the material is called a negative index material and the refractive index is defined as $n = -\sqrt{\varepsilon\mu}$ correspondingly.[20,25–27] So we will discuss the relative permittivity and permeability of the atomic medium to the probe field. Based on the quantum coherence, the induced complex electric dipole moment of the atom is given by $\boldsymbol{p}(\omega_{\rm p}) = {\rm Tr}\{\hat{\rho}\hat{\boldsymbol{d}}\} = \boldsymbol{d}_{12}\rho_{21}$. Moreover, according to the electromagnetic theory, the complex atomic polarizability tensor $\alpha_{\rm e}$ can be described as $\boldsymbol{p}(\omega_{\rm p}) = \varepsilon_0\alpha_{\rm e}(\omega_{\rm p})\boldsymbol{E}_{\rm p}(\omega_{\rm p})$. When we choose a probe electric field $\boldsymbol{E}_{\rm p}$ parallel to the atomic dipole $\boldsymbol{d}_{12}$, the electric polarizability $\alpha_{\rm e}$ can be expressed as a scalar

$$\alpha_{\rm e} = \frac{d_{12}\rho_{21}}{\varepsilon_0 E_{\rm p}}. \tag{2}$$

Similarly, the magnetic response of the medium to the probe field is relative to the coherent term ρ_{43} and the magnetizability can also be a scalar expressed as

$$\alpha_m = \frac{\mu_0 c\mu_{34}\rho_{43}}{\eta E_{\rm p}}, \tag{3}$$

where μ_0 is the permeability of vacuum, c is the speed of light in vacuum, and η is a unitary complex number depending on the polarization of the probe field. Considering the local field effects, we obtain the complex relative permittivity and permeability of the atomic medium with the density N as[27]

$$\varepsilon = 1 + N\alpha_{\rm e}\left(1 - \frac{N\alpha_{\rm e}}{3}\right)^{-1}, \tag{4}$$

$$\mu = \frac{1}{1 - N\alpha_m(1+\chi_{\rm e}/3)}, \tag{5}$$

where $\chi_{\rm e} = N\alpha_{\rm e}/(1 - N\alpha_{\rm e}/3)$ is the electric susceptibility of the medium relative to the probe field.

In order to study the bistability of the atomic system, we put the ensemble of N homogeneously broadened V-type atoms in a unidirectional ring cavity (see Fig. 2). Without loss of generality, we assume that the mirrors 3 and 4 each have 100% reflectivity, and the intensity reflection and transmission coefficient of mirrors 1 and 2 are R and T (with $R + T = 1$), respectively.

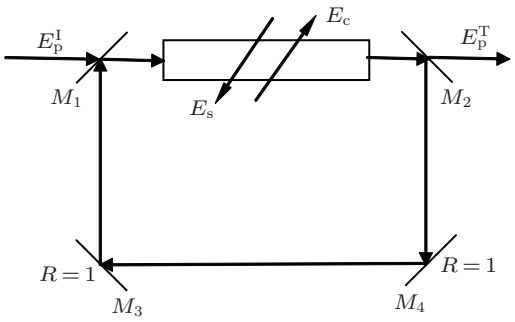


Fig. 2. Unidirectional ring cavity with an atomic sample of length L, $E_{\rm p}^{\rm I}$ and $E_{\rm p}^{\rm T}$ are the incident and transmitted probe fields, respectively, $E_{\rm s}$ and $E_{\rm c}$ are respectively the coherent pump and control fields which are noncirculating in the cavity.

The total electromagnetic field can be given as

$$\boldsymbol{E} = \boldsymbol{E}_{\rm p}\,{\rm e}^{-{\rm i}\omega_{\rm p}t} + \boldsymbol{E}_{\rm c}\,{\rm e}^{-{\rm i}\omega_{\rm c}t} + \boldsymbol{E}_{\rm s}\,{\rm e}^{-{\rm i}\omega_{\rm s}t} + {\rm c.c}, \tag{6}$$

where the probe field $\boldsymbol{E}_{\rm p}$ circulates in the ring cavity and the coupling field $\boldsymbol{E}_{\rm c}$ and the pump field $\boldsymbol{E}_{\rm s}$ do not circulate in the cavity. We apply the coupling field and pump field in the opposite directions so that the Doppler effects can be neglected. Under slowly varying envelope approximation,[28] the Maxwell's equation is

$$\frac{\partial E_{\rm p}}{\partial t} + c\frac{\partial E_{\rm p}}{\partial z} = {\rm i}\frac{\omega_{\rm p}}{2\varepsilon_0}P(\omega_{\rm p}), \tag{7}$$

where ε_0 is the permittivity of vacuum. $P(\omega_{\rm p})$ is the slowly oscillating term of the induced polarization in the transition $|1\rangle \leftrightarrow |2\rangle$ and it is given by

$$P(\omega_{\rm p}) = Nd_{12}\rho_{21}. \tag{8}$$

In the steady-state case, setting the time derivative in Eq. (7) to be zero, we can obtain the probe field amplitude as

$$\frac{\partial E_{\rm p}}{\partial z} = {\rm i}\frac{N\omega_{\rm p}d_{12}\rho_{21}}{2c\varepsilon_0}. \tag{9}$$

For a perfectly tuned cavity, in the steady state limit, the boundary conditions impose the following conditions between the incident field $E_{\rm p}^{\rm I}$ and the transmitted field $E_{\rm p}^{\rm T}$:

$$E_{\rm p}(L) = E_{\rm p}^{\rm T}/\sqrt{T}, \tag{10a}$$

$$E_{\rm p}(0) = \sqrt{T}E_{\rm p}^{\rm I} + RE_{\rm p}(L), \tag{10b}$$

in which, L is the length of the atomic sample. Note that the second term on the right-hand side of Eq. (10b) describes a feedback mechanism due to the mirrors, which is responsible for the bistibility. If $R = 0$, we do not expect any bistability. According to the mean-field limit,[29,30] using boundary conditions and normalizing the fields by setting $y = d_{12}E_{\rm p}^{\rm I}/(\hbar\sqrt{T})$ and $x = d_{12}E_{\rm p}^{\rm T}/(\hbar\sqrt{T})$, we can obtain the input–output relationship as

$$y = x + C\gamma_1\,{\rm Im}(\rho_{21}) - {\rm i}C\gamma_1\,{\rm Re}(\rho_{21}), \tag{11}$$

where $C = LN\omega_{\rm p}d_{12}^2/(2\hbar c\varepsilon_0 T\gamma_1)$ is the usual cooperation parameter in a ring cavity.

3. Results and discussion

The general solutions of Eqs. (1a)–(1i) for the steady state are tedious, and we only present the numerical analysis here. In our numerical calculation, we consider the case of atomic hydrogen to verify our scheme. In this case, the state $|1\rangle$ ($|2\rangle$) is the state $2{\rm P}_{1/2}$ ($2{\rm S}_{1/2}$), while the state $|3\rangle$ ($|4\rangle$) is the excited state $3{\rm D}_{3/2}$ ($3{\rm D}_{5/2}$). All the parameters are scaled by the spontaneous emission rate of level $|2\rangle$, which is $\gamma_{21} = \gamma = 100$ MHz. The wavelength of resonant probe field transition is 21 cm, which is in the radio wave range. The transition $|1\rangle \leftrightarrow |3\rangle$ is driven by the pump field $\Omega_{\rm s}$ which is resonant at 656.2 nm. The transition $|2\rangle \leftrightarrow |4\rangle$ is a two-photon process. Typical values for $\gamma_{42} = \gamma_{32} = \gamma_{31} = 0.5\gamma$ and $\gamma_{43} = \gamma_{41} = 0.01\gamma$ are used. The atomic density is $N = 5\times10^{25}/{\rm m}^3$. The Rabi frequency of the control field is $\Omega_{\rm c} = 0.1\gamma$.

The probe-detuning dependence of the real part of the relative permittivity ε and the real part of the relative permeability μ are shown in Fig. 3. It is obvious that the relative permittivity ε and the relative permeability μ are negative simultaneously in a frequency band about 3 MHz (shown as shade zone in Fig. 3). It is the result of quantum coherence. In our scheme, the level difference between level $|1\rangle$ and level $|2\rangle$ is nearly the same as that between level $|3\rangle$ and level $|4\rangle$. When the probe field is applied to the levels $|1\rangle$ and $|2\rangle$, large coherence ρ_{43} can be caused, which is related to magnetic transition. So it is possible to change the permeability of the medium greatly into being negative.[27] Then negative refractive index can be realized based on the quantum coherence in this scheme. In other words, we can obtain the negative index atomic medium by adjusting the detuning and intensity of the applied laser fields.

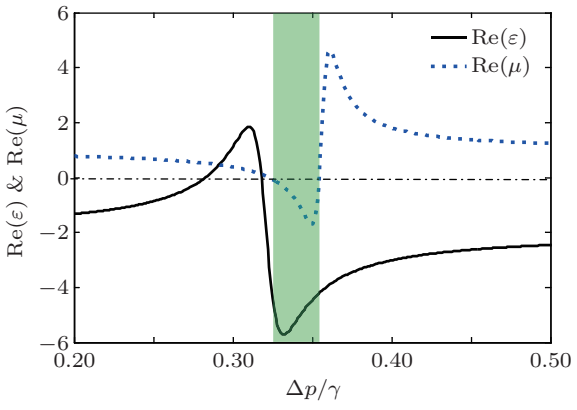


Fig. 3. (color online) Probe-detuning dependences on the real part of the relative permittivity ε and the real part of the relative permeability μ. The shade zone represents negative refraction band. The parameters are $N = 5\times10^{25}/{\rm m}^3$, $\gamma_{42} = \gamma_{32} = \gamma_{31} = 0.5\gamma$, $\gamma_{43} = \gamma_{41} = 0.01\gamma$, $\Omega_{\rm c} = 0.1\gamma$, $\Omega_{\rm s} = 5\gamma$, $\Delta_{\rm s} = \Delta_{\rm c} = 0.1\gamma$.

In what follows, we will focus on the optical bistability of this atomic medium in the negative refraction band. According to Fig. 3, we set the probe detuning $\Delta_{\rm p} = 0.34\gamma$ which is in the negative refraction band, and other parameters are the same as those in Fig. 3. The relationships between the input

probe intensity and output probe intensity with different cooperation parameters are plotted in Fig. 4. It is shown that the optical bistability appears with the gradual increase of the cooperation parameter C in the negative refraction band. When changing the detuning of the probe field in the negative refraction band, we find that the bistability still exists if the cooperation parameter is large enough. Quantum coherence plays an important role in realizing the bistability of the negative index atomic medium. In the V-scheme considered here, most of atoms are distributed not only in level $|1\rangle$, but also in level $|3\rangle$. This produces large coherence between these two levels, which makes it possible to achieve bistability in negative refraction band. Therefore, the bistability can be realized by quantum coherence in this negative index atomic medium.

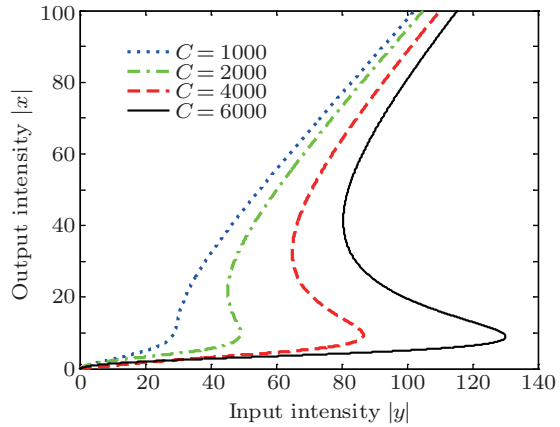


Fig. 4. (color online) Plots of output intensity $|x|$ versus input intensity $|y|$ in negative refraction band for different cooperation parameters. $\Delta_p = 0.34\gamma$. Other parameters are the same as those in Fig. 3.

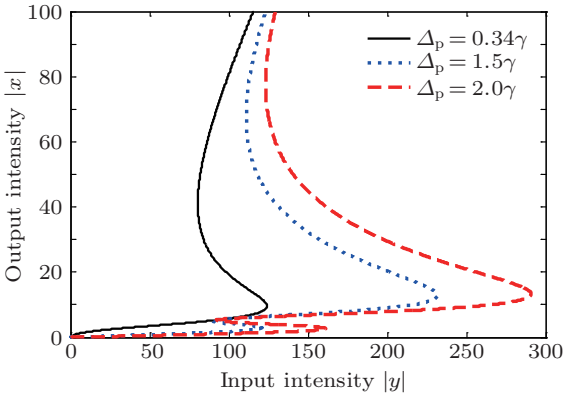


Fig. 5. (color online) Plots of output intensity $|x|$ versus input intensity $|y|$ in negative refraction band (solid line) and positive refraction band (dotted line and dashed line). $C = 6000$, other parameters are the same as those in Fig. 3.

In order to compare with the condition of positive refraction band, we plot the bistability behaviors of the probe field with different probe detunings in Fig. 5. It is shown that the multi-stability appears when the detuning reaches 1.5γ or 2γ, and it is in the positive refraction band. If we continue to increase the probe detuning in the positive refraction band, OM will be switched to OB. However, only in the negative refraction band does bistability appear. It is easy to understand because the negative refraction only occurs with the small probe detuning in which the quantum coherence is very large. According to Ref. [21], OB can appear near resonant region, but OM appears in the condition of large field detunings. Therefore, the bistability can be switched to multi-stability from the negative refraction band to the positive refraction band in this scheme by adjusting the detunings of the applied field.

4. Conclusions

In summary, we have illustrated the optical bistability and optical multi-stability behaviors in a V-type four-level atomic system. We find that optical bistability can appear in the negative refraction band. While, in the positive refraction band, optical multi-stability occurs in such an atomic medium within an appropriate parameter range. Our results provide a way to obtain desired bistability in a negative index medium or multi-stability in a positive index medium. We believe that this excellent performance offers the possibility of developing ultra-fast all-optical switches and quantum information processes.

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