Phase grating in a doubly degenerate four-level system*

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In this paper, we suggest a doubly degenerate four-level system, in which the transition takes place between the hyperfine energy $5^2S_{1/2}$ $F = 1$ and $5^2P_{3/2}$ $F = 2$ in rubidium 87 D_2 line, for studying atomic phase grating based on the cross-Kerr and phase conjugation effects. The phase grating with high efficiency can be obtained by tuning phase shift Φ between the coupling and probe field, when the coupling intensity is much stronger than the strength of probe field. Under different coupling intensities, a high diffraction efficiency can be maintained. A new and simple way of implementing phase grating is presented. However, in such an atomic system, two main limitations must be taken into account. First, the independence between steady state probe susceptibility and the coupling intensity, when the population decay rate is larger than the Rabi frequency of the coupling field, cannot result in diffraction grating; second, the sample to be prepared should not be too long.

Keywords: phase grating, quantum interference, four-level system, standing-wave

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1. Introduction

Quantum interference induced by a strong laser field can control the optical response, and under appropriate conditions the absorption of a weak probe field vanishes due to destructive quantum interference. This phenomenon is called electromagnetically induced transparency (EIT).[1,2] The effect of EIT has disclosed new possibilities for nonlinear optics[3,4] and quantum information processing.[5] However, as atomic transition is coupled by an external standing-wave field in EIT schemes, the dynamic response of the probe field will exhibit periodical modulation. Electromagnetically induced grating (EIG)[6–10] is a direct result of the periodical modulation under the action of a standing-wave field. Recently, the study of EIG has received much attention due to its potential applications, including making a free background,[7] diffracting and switching a quantized probe field,[8] probing optical properties of a material.[10]

As is well known, the EIG based on EIT is a hybrid grating which consists of the absorption and phase modulation. The absorption grating tends to collect the energy at the center maximum, while phase grating will disperse much light into first-order and high-orders components. The diffraction efficiency of the grating increases dramatically due to phase modulation. Therefore, phase grating is interesting. In this letter, a doubly degenerate four-level system in ^{87}Rb is proposed to realize phase grating. As the coupling field is much stronger than the probe field, the probe absorption is completely compensated by the "cross-Kerr" effect in the medium, and the giant Kerr nonlinearity based on EIT without a dark state is created. Moreover, if there is an angle included between the coupling and probe field, the probe susceptibility will be phase-dependent with respect to the wave mixing process. Therefore, phase grating relies on both the giant Kerr nonlinearity and wave mixing process and can be manipulated effectively by tuning the phase shift between two coherent fields, which may be a new method to implement atomic grating. Through analysis, we find that the first-order diffraction efficiency of phase grating can reach up to 33%, the high-order efficiency can also be maintained by adjusting the phase shift between two coherent fields for different coupling intensities, lengths of atomic sample, and detunings.

2. Theoretical simulation and analysis

A doubly degenerate four-level system[11–16] in ^{87}Rb is proposed to accomplish atomic phase grating, as shown in Fig. 1(a). We analyze now the interaction of the system with mutually coherent ultrashort pulses. This situation occurs when the $S_{1/2}$–$P_{3/2}$ transition of atomic rubidium is excited with a σ-polarized pulse and a pair of π-polarized control pulses. The σ-polarized pulse propagating in atomic vapor can be considered as a pulse mixing of left circularly polarized σ^+ pulse and right circularly polarized σ^- pulse.

The π-polarized pulse strongly couples resonantly the parallel states ($|1\rangle \leftrightarrow |3\rangle$ and $|2\rangle \leftrightarrow |4\rangle$), while the weak σ^+ pulse couples resonantly the crossed states $|1\rangle \leftrightarrow |4\rangle$, and the weak σ^- pulse couples resonantly the crossed states $|2\rangle \leftrightarrow |3\rangle$. The electric fields propagate along the y axis in an optically dense medium. The σ-polarized probe field with Rabi frequency $\Omega_p (= \mu E_p/\hbar)$ and frequency ω is simultaneously ap-

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plied to transitions $|1\rangle \leftrightarrow |4\rangle$ and $|2\rangle \leftrightarrow |3\rangle$. Transitions $|1\rangle \leftrightarrow |3\rangle$ and $|2\rangle \leftrightarrow |4\rangle$ are simultaneously driven by a π-polarized coupling field with Rabi frequency $\Omega_c(=\mu E_c/\hbar)$ and frequency ω. The dipole matrix element μ is written as $\mu = \langle 3|\boldsymbol{\mu}\cdot\boldsymbol{e}_z|1\rangle$, and $\langle 1|\boldsymbol{\mu}_z|3\rangle = -\langle 2|\boldsymbol{\mu}_z|4\rangle$. The detuning $\Delta_0 = \omega_0 - \omega$, where ω_0 is the atomic transition frequency.

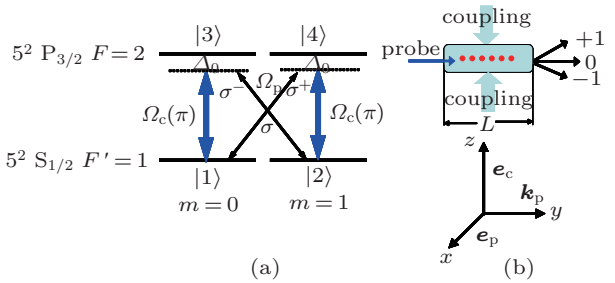


Fig. 1. (color online) (a) Schematic diagram of a doubly degenerate four-level of ^{87}Rb atoms; (b) field configuration with L being the length of the atomic sample.

In an interaction picture and rotating-wave approximation, the density matrix equations are given as

$$\begin{aligned}
\partial_t\sigma_{44} &= -\Gamma\sigma_{44} + i\Omega_c(\sigma_{42}-\sigma_{24}) \\
&\quad + i\Omega_p(\sigma_{14}\cdot e^{-i\Phi} - \sigma_{41}\cdot e^{i\Phi}), \\
\partial_t\sigma_{33} &= -\Gamma\sigma_{33} + i\Omega_c(\sigma_{13}-\sigma_{31}) \\
&\quad + i\Omega_p(\sigma_{23}\cdot e^{-i\Phi} - \sigma_{32}\cdot e^{i\Phi}), \\
\partial_t\sigma_{22} &= \Gamma''(\sigma_{44}+\sigma_{33}) + i\Omega_c(\sigma_{24}-\sigma_{42}) \\
&\quad - i\Omega_p(\sigma_{23}\cdot e^{-i\Phi} - \sigma_{32}\cdot e^{i\Phi}), \\
\partial_t\sigma_{41} &= -i(\Delta_0 - i\Gamma'')\sigma_{41} - i\Omega_c(\sigma_{21}+\sigma_{43}) \\
&\quad - i\Omega_p\cdot e^{-i\Phi}(\sigma_{44}-\sigma_{11}), \\
\partial_t\sigma_{42} &= -i(\Delta_0 - i\Gamma'')\sigma_{42} + i\Omega_c(\sigma_{44}-\sigma_{22}) \\
&\quad - i\Omega_p\cdot e^{-i\Phi}(\sigma_{43}-\sigma_{12}), \\
\partial_t\sigma_{43} &= -i\Gamma'\sigma_{43} - i\Omega_c(\sigma_{41}+\sigma_{23}) \\
&\quad + i\Omega_p(\sigma_{13}\cdot e^{-i\Phi} - \sigma_{42}\cdot e^{i\Phi}), \\
\partial_t\sigma_{31} &= -i(\Delta_0 - i\Gamma'')\sigma_{31} - i\Omega_c(\sigma_{33}-\sigma_{11}) \\
&\quad - i\Omega_p\cdot e^{-i\Phi}(\sigma_{34}-\sigma_{21}), \\
\partial_t\sigma_{32} &= -i(\Delta_0 - i\Gamma'')\sigma_{32} + i\Omega_c(\sigma_{12}+\sigma_{34}) \\
&\quad - i\Omega_p\cdot e^{-i\Phi}(\sigma_{33}-\sigma_{22}), \\
\partial_t\sigma_{21} &= -i\Omega_c(\sigma_{41}+\sigma_{23}) + i\Omega_p\cdot e^{i\Phi}\sigma_{31} \\
&\quad - i\Omega_p\cdot e^{-i\Phi}\sigma_{24}, \\
\sum_{i=1}^{i=4}\sigma_{ii} &= 1,
\end{aligned}$$

where Γ is the population decay rate; in the absence of radiative homogeneous dephasing processes, Γ' and Γ'' are the relaxation rates given as Γ and $\Gamma/2$, respectively; Φ is the phase shift between two coherent fields.

In the limit of a weak probe field and under the steady-state condition, we obtain the solution of $\sigma_{41}+\sigma_{32}$ from density matrix equation as follows:

$$\sigma_{41}+\sigma_{32} = \frac{[1+(\Omega_p/\Omega_c)^2 e^{-2i\Phi}](i\Gamma''+\Delta_0)\Omega_p e^{i\Phi}}{4\Gamma''/\Gamma\cdot|\Omega_c^2|\cdot\left|1+(\Omega_p/\Omega_c)^2 e^{-2i\Phi}\right|^2 + [1+(\Omega_p/\Omega_c)^2][(\Gamma'')^2+\Delta_0^2]}. \tag{1}$$

The induced polarization of the probe field is written as $P(\omega) = x\varepsilon_0 E_p = N\mu(\sigma_{41}+\sigma_{32})$. The real and imaginary parts of steady-state probe susceptibility are given by

$$\begin{aligned}
\mathrm{Re}(x) &= K\frac{[1+(\Omega_p/\Omega_c)^2\cos(2\Phi)][\Delta_0\cos(\Phi)-\Gamma''\sin(\Phi)]}{4\Gamma''/\Gamma\cdot|\Omega_c^2|\cdot\left|1+(\Omega_p/\Omega_c)^2 e^{-2i\Phi}\right|^2 + [1+(\Omega_p/\Omega_c)^2][(\Gamma'')^2+\Delta_0^2]} \\
&\quad + K\frac{(\Omega_p/\Omega_c)^2\sin(2\Phi)[\Gamma''\cos(\Phi)+\Delta_0\sin(\Phi)]}{4\Gamma''/\Gamma\cdot|\Omega_c^2|\cdot\left|1+(\Omega_p/\Omega_c)^2 e^{-2i\Phi}\right|^2 + [1+(\Omega_p/\Omega_c)^2][(\Gamma'')^2+\Delta_0^2]}, \qquad (2a) \\
\mathrm{Im}(x) &= K\frac{[1+(\Omega_p/\Omega_c)^2\cos(2\Phi)][\Delta_0\sin(\Phi)+\Gamma''\cos(\Phi)]}{4\Gamma''/\Gamma\cdot|\Omega_c^2|\cdot\left|1+(\Omega_p/\Omega_c)^2 e^{-2i\Phi}\right|^2 + [1+(\Omega_p/\Omega_c)^2][(\Gamma'')^2+\Delta_0^2]} \\
&\quad - K\frac{(\Omega_p/\Omega_c)^2\sin(2\Phi)(\Gamma''\sin(\Phi)+\Delta_0\cos(\Phi))}{4\Gamma''/\Gamma\cdot|\Omega_c^2|\cdot\left|1+(\Omega_p/\Omega_c)^2 e^{-2i\Phi}\right|^2 + [1+(\Omega_p/\Omega_c)^2][(\Gamma'')^2+\Delta_0^2]}. \qquad (2b)
\end{aligned}$$

Then in the case of $\Omega_c \gg \Omega_p$, Eq. (2) is reduced to

$$\mathrm{Re}(x) = K\frac{(\Delta_0/\Gamma'')\cos(\Phi)-\sin(\Phi)}{4\Gamma''\left\{\left|\Omega_c/\sqrt{\Gamma\Gamma''}\right|^2 + \frac{1}{4}[1+(\Delta_0/\Gamma'')^2]\right\}}, \tag{3a}$$

$$\mathrm{Im}(x) = K\frac{\cos(\Phi)+(\Delta_0/\Gamma'')\sin(\Phi)}{4\Gamma''\left\{\left|\Omega_c/\sqrt{\Gamma''\Gamma}\right|^2 + \frac{1}{4}[1+(\Delta_0/\Gamma'')^2]\right\}}. \tag{3b}$$

Here, $K = N\mu^2/\hbar\varepsilon_0$, N is the atomic density. In the medium, the probe field propagates along the y direction through an atomic sample of length L, which is described by Maxwell's equation. Here, under slowly varying envelope approximation and in the case of steady-state, Maxwell's equation is reduced to

$$\frac{\partial E_p}{\partial y} = i\frac{\pi}{\lambda\varepsilon_0}P(\omega)$$

or

$$\frac{\partial E_{\rm p}}{\partial y'} = \{-{\rm Im}[x] + {\rm i}\,{\rm Re}[x]\}E_{\rm p}, \tag{4}$$

where $y' = k(N\mu^2/2\hbar\varepsilon_0^2)y$, with $k(N\mu^2/2\hbar\varepsilon_0^2)$ being treated as the unit for y, and $k = 2\pi/\lambda$. When two overlapping coupling fields form a standing-wave in the z direction, the Rabi frequency of the standing-wave is expressed as $\Omega_{\rm c} = \Omega\sin(\pi z/\Lambda_{\rm cz})$. $\Lambda_{\rm cz}$ as the unit of z, it is spatial frequency of the standing-wave. Equation (4) shows that ${\rm Im}(x)$ and ${\rm Re}(x)$ each are a function of z, and $T(z)$ is the transmission function for sample of length L and expressed as $T(z) = {\rm e}^{-{\rm Im}[x]\cdot L}{\rm e}^{{\rm i}\,{\rm Re}[x]\cdot L}$. By Fourier transformation of $T(z)$, the Fraunhofer diffraction equation can be obtained as

$$I_{\rm p}(\theta) = \left|\int_0^1 T(z)\exp(-{\rm i}2\pi z\cdot\sin(\theta)R)\,{\rm d}z\right|^2 \cdot\frac{\sin^2(M\pi\sin(\theta)R)}{M^2\sin^2(\pi\sin(\theta)R)}, \tag{5}$$

where $R = \Lambda_{\rm cz}/\lambda$, θ is the diffraction angle of the probe field associated with the y direction, M is the constant factor and defined as the spatial width of the probe beam. Then the n-order diffraction intensity can be given by Eq. (5). The diffraction order n function is defined as $n = R\sin(\theta)$. The expression of first-order diffraction intensity is

$$I_{\rm p}(\theta_1) = \left|\int_0^1 T(z)\exp(-{\rm i}2\pi z)\,{\rm d}z\right|^2. \tag{6}$$

Phase grating is focused on in this article. The diffraction efficiency of phase grating depends on phase shift Φ, interaction length L, coupling intensity Ω, and detuning Δ_0. Next, we will discuss how these parameters affect the diffraction efficiency of phase grating.

In Fig. 2, diffraction patterns of phase grating each are considerd as a function of $\sin(\theta)$, and they are plotted for different values of phase shift Φ. The diffraction efficiency of phase grating changes with phase shift Φ. The probe field is absorbed strongly in medium at $\Phi = 0$, while $\Phi = \pi/2$ phase shift is present, the high efficiency transmission of the probe field is turned on, the 33% of the probe field energy is diffracted into the first-order component. A new phase grating with high efficiency is obtained. Tuning properly the phase shift Φ between two coherent fields opens up a new way for realizing atomic phase grating with high diffraction efficiency. The reason is that the realization of coherence $(\sigma_{41} + \sigma_{32})$ involves two quantum interactions: cross-Kerr and phase conjugate. The cross-Kerr interaction involves controlling the refractive index experienced by one electromagnetic field by the intensity of another. If a strong signal wave is incident on such a nonlinear material, it causes a change in teh refractive index or diffraction efficiency, and a weak probe wave therefore experiences a phase shift in propagating through the material which is proportional to the intensity of the signal wave. The change in refractive index or the diffraction efficiency is called the optical Kerr effect.

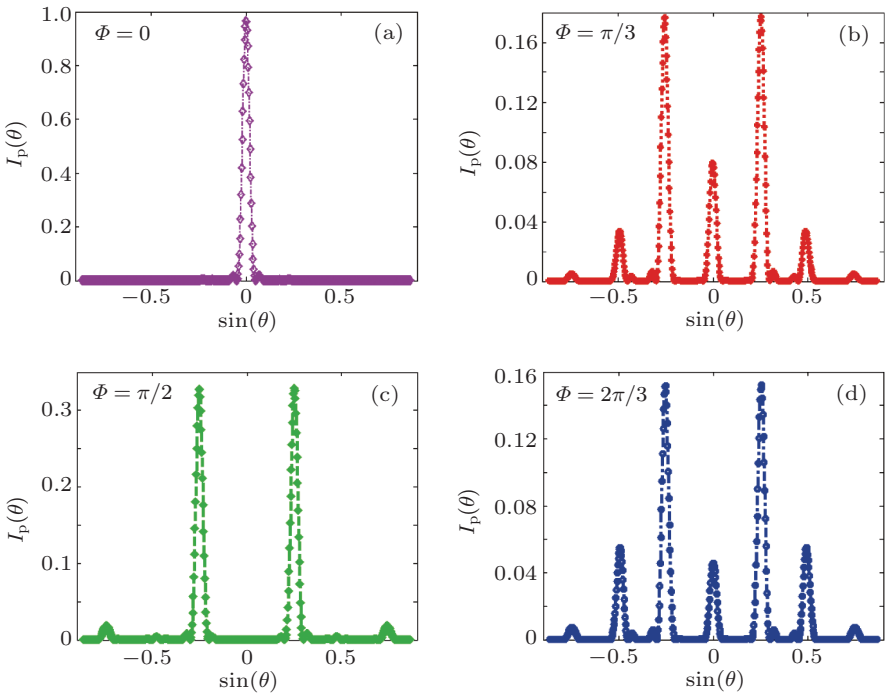


Fig. 2. (color online) Diffraction patterns of phase grating each as a function of $\sin(\theta)$ at various values of phase Φ: $\Phi = 0$ (a), $\pi/3$ (b), $\pi/2$ (c), and $2\pi/3$ (d). Other parameters are $\Gamma'' = \pi\times 5.9$ MHZ, $\Gamma = 2\Gamma''$, $\Omega = 5$ MHZ, $\Delta_0 = 2$ MHZ, $R = 4$, $M = 5$, and $L = 600$.

The phase conjugation relies on a wave mixing process. Degenerate four-wave mixing (DFWM)[17] is a nonlinear process in which two planar counter-propagating pump waves interact in a nonlinear medium with a probe field entering at an arbitrary angle to the pump waves and yield a fourth (output) wave. The fourth wave is a phase conjugate wave that is proportional to the spatial complex conjugation of the probe wave. The frequency of a phase conjugate beam is strictly equal to that of the incident beam in the DFWM mechanism. So the phase conjugation leads to high diffraction efficiency.

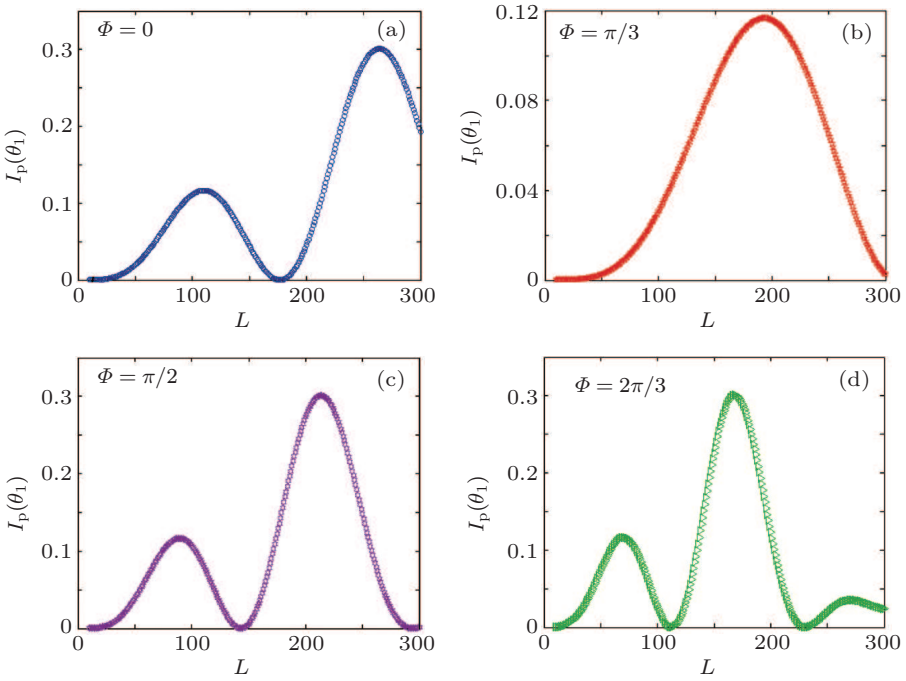


Fig. 3. (color online) Plots of first-order diffraction patterns of phase grating versus interaction length L at different values of phase shift Φ between two coherent fields. Other parameters are $\Gamma'' = \pi \times 5.9$ MHZ, $\Gamma = 2\Gamma''$, $\Delta_0 = 15$ MHZ, and $\Omega = 20$ MHZ ($\Omega > \Gamma''$).

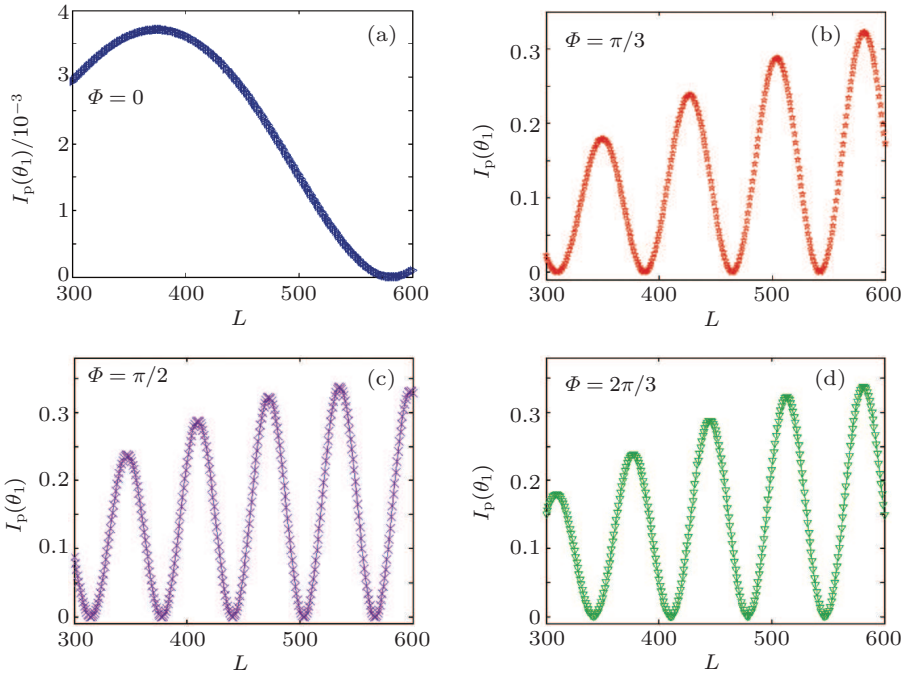


Fig. 4. (color online) Plots of first-order diffraction patterns of phase grating versus atomic sample length L at different values of phase shift Φ between two coherent fields. Other parameters are $\Gamma'' = \pi \times 5.9$ MHZ, $\Gamma = 2\Gamma''$, $\Delta_0 = 2$ MHZ, and $\Omega = 5$ MHZ ($\Omega < \Gamma''$).

Figures 3 and 4 show the plots of the first-order diffraction intensity $I_p(\theta_1)$ versus interaction length L at various values of Φ. We find that the high efficiency of phase grating is obtained by varying phase shift Φ at different values of L. In order to keep the high efficiency, the coupling intensity Ω needs to be considered: when $\Omega > \Gamma''$, short interaction length L should be chosen; if $\Omega < \Gamma''$, long interaction length L would be applied, because the dynamic response of the probe field to transitions $|1\rangle \leftrightarrow |4\rangle$ and $|2\rangle \leftrightarrow |3\rangle$ is modified due to the application of different coupling intensities to the transitions $|1\rangle \leftrightarrow |3\rangle$ and $|2\rangle \leftrightarrow |4\rangle$, which leads to different diffraction efficiencies. Although interaction length L can be controlled precisely, it is still a manipulated limitation to the fulfillment of phase grating. The probe susceptibility is independent of the coupling intensity because of $\Omega \ll \sqrt{\Gamma''\Gamma}$, therefore, phase grating could not be created.

The curves of the first-order diffraction intensity $I_p(\theta_1)$ versus detuning Δ_0 with different values of phase shift Φ are depicted in Figs. 5 and 6. The results show that under a strong-field standing wave and a fixed short interaction length, the efficiency of approximately 30% is attained by making a large detuning at $\Phi = \pi/2$; under a weak field standing-wave and a fixed long interaction length, the diffraction efficiency may reach up to 33% by making a slight detuning at $\Phi = \pi/2$; moreover, in these two cases, the diffraction efficiency is always lowest as $\Phi = 0$. For the phase shift Φ dependent on probe susceptibility to wave mixing process that causes the interference effect inducing the phenomenon between gain and dispersion, the medium turns from an absorber at $\Phi = 0$ to an amplifier at $\Phi = \pi/2$.

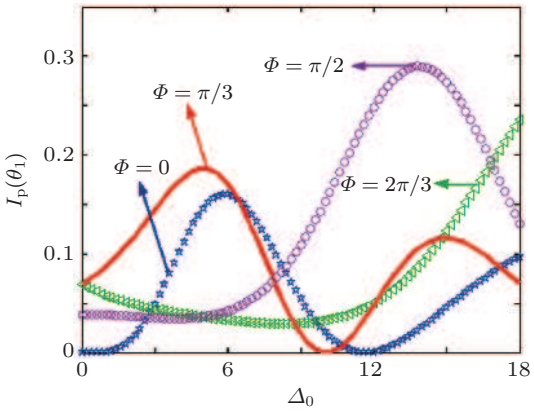


Fig. 5. (color online) First-order diffraction patterns of phase grating versus detuning Δ_0 at different values of phase shift Φ between two coherent fields. Other parameters are $\Gamma'' = \pi \times 5.9$ MHZ, $\Gamma = 2\Gamma''$, $L = 200$, and $\Omega = 20$ MHZ ($\Omega > \Gamma''$).

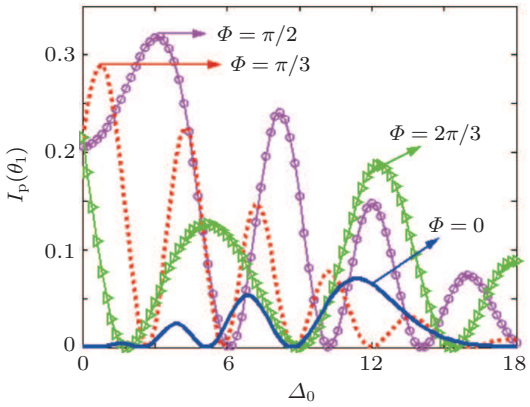


Fig. 6. (color online) Curves of the first-order diffraction patterns of phase grating versus detuning Δ_0 at different values of phase shift Φ between two coherent fields. Other parameters are $\Gamma'' = \pi \times 5.9$ MHz, $\Gamma = 2\Gamma''$, $L = 480$, and $\Omega = 5$ MHz ($\Omega < \Gamma''$).

3. Conclusions

In this paper, atomic phase grating is studied in a doubly degenerate four-level system in ^{87}Rb. The simulation results show that atomic phase grating can be fulfilled and a diffraction efficiency of approximately 33% is obtained by tuning phase shift Φ between two coherent fields, as the strength of coupling field is much stronger than probe field ($\Omega_c \gg \Omega_p$). In order to keep the high efficiency, the short interaction length and the large detuning are required for $\Omega > \Gamma''$; in the opposite case, the interaction length and slight detuning should be considered. However, in such an atomic system, we must pay attention to two limitations: (i) in the case of $\Omega \ll \sqrt{\Gamma''\Gamma}$, the probe susceptibility is independent of the coupling intensity, phase grating cannot be accomplished; (ii) the interaction length of sample cannot be too long.

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