Chaos synchronization of a chain network based on a sliding mode control*

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A sliding mode control approach is proposed to implement the synchronization of the chain tree network. The double-scroll circuit chaos systems are treated as nodes and the network is constructed with the state variable coupling. By selecting a switching sliding surface, the chaos synchronization of the network is achieved with one control input only. The stability analysis and the numerical simulations demonstrate that the complete synchronization in a chain network can be realized for all nodes.

Keywords: chaos synchronization, sliding mode control, Lyapunov stability, double-scroll circuit

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1. Introduction

The subject of complex networks has attracted considerable attention and been extensively studied, owing to their theoretical significance and potential applications, such as in the areas of cell biology, electric power grids, the Internet, etc.[1] The investigations on complex networks reveal that the synchronization serves as an important way to understand the structures and the functions of complex systems in the nature and the human society.[2] Actually, synchronization, as an emerging phenomenon, has fascinated researchers in various fields of nonlinear science.[3]

Recently, a great many valuable papers have been published, which focus on the synchronization processes.[4] These meaningful papers have made a significant contribution to our further studying, and some excellent approaches to the synchronization of complex networks have been developed.[5] Sorrentino and Ott[6] studied the synchronizing of general complex networks by means of an adaptive strategy. Chai *et al.*[7] proposed a backstepping approach for chain networks between multi-spatiotemporal chaotic systems. Yu and Zhang[8] implemented the new generic criteria of global chaos synchronization according to the way of coupling. The analysis of synchronization is of benefit to the understanding of not only the chaos synchronization but also the topology of complex networks.[9] Much work on synchronization complex networks has been derived from the research of two-system synchronization processes.[10]

Since the pioneer work on variable structure systems with sliding modes,[11] significant progress has been made.[12] An adaptive fuzzy sliding mode controller was designed for a class of nonlinear uncertain systems involving uncertainties and disturbances.[13] The sliding mode control was applied to the generalized projective synchronization of the uncertain chaotic systems with external disturbance.[14] The sliding mode control approaches were widely used to study the synchronization between two systems, owing to its characteristic including insensitivity to parameter variations and complete rejection of disturbances. These advantages of sliding mode approaches should be applied to networks to understand the synchronization of networks. However, there are only a few sliding mode approaches that have been proposed in synchronization of networks. In Ref. [15], Lü *et al.* implemented the synchronization of dynamical networks based on the sliding mode approaches.

Furthermore, according to the reports on the sliding mode approaches, two phenomena are worth noting.[16] One is that too many controllers are needed for the high-dimensional systems,[17] and the other is that some approaches require the system to have the special structure in Ref. [14]. However, there are many nodes in networks and their structures may not meet special structures in Ref. [14]. Consequently too many controllers or special structures of nodes may be unrealistic in natural and man-made systems. Roopaei *et al.*[18] proposed a robust adaptive sliding mode control strategy with only one controller for a class of uncertain chaotic systems.[18] There is an interesting problem whether the sliding mode control approaches could be developed to implement the synchronization of general networks with only one control input and their nodes without special requirements. The present paper will deal with the problem.

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So far, a variety of actual networks having different topological structures have been widely used.[19–22] The topological structures in each community are essentially of irregularity according to their functions in practice.[23–27] In this work, the chain network is treated as an example for the synchronization in circuit networks. The double-scroll circuit chaos systems are chosen as nodes and the network is constructed with the state variable coupling. The main questions addressed here are to drive any node of complex networks to be synchronized even though they have different initial conditions. A sliding mode control approach is applied to implementing the synchronization of complex networks. Based on this approach, a switching surface and only one control input are designed. The simulation results demonstrate the effectiveness of the synchronization principle.

2. Synchronization principle and control design methodology

Consider a dynamical network consisting of m nodes, then the state equations of the nodes will be identical chaotic systems, where $\boldsymbol{x}_i$ is the n-dimensional variable of node i, $\boldsymbol{x}_i(t) = [\boldsymbol{x}_{i1}(t), \boldsymbol{x}_{i2}(t), \ldots, \boldsymbol{x}_{in}(t)]^{\mathrm{T}} \in \boldsymbol{R}^n$. When the coupling action is considered, the equation of state of node i can be described by the following equation of state:[28,29]

$$\dot{\boldsymbol{x}}_i(t) = \boldsymbol{F}(\boldsymbol{x}_i(t)) + \xi \sum_{l=1}^{m} G_{il} \boldsymbol{H}(\boldsymbol{x}_i(t)), \quad (i = 1, 2, \ldots, m), \tag{1}$$

where $\boldsymbol{F} : \boldsymbol{R}^n \to \boldsymbol{R}^n$ and $\boldsymbol{H} : \boldsymbol{R}^n \to \boldsymbol{R}^n$ denote the coupling functions of the connected nodes, the parameter ξ is the relative weight of the coupling strength among the nodes of the network, and the parameter G_{il} is the matrix element of the coupling matrix whose specific form depends on the connection type of the network.

In order to enhance the applicability and to reduce the cost of synchronization in the network, only one control input is applied to the network, for example, the last node. Then the last node of the controlled network can be rewritten as

$$\begin{cases} \dot{x}_{m,1} = F_1(x_{m,1}, x_{m,2}, \ldots, x_{m,n}) + h_1(x_{m,1}) + u(t), \\ \dot{x}_{m,2} = F_2(x_{m,1}, x_{m,2}, \ldots, x_{m,n}) + h_2(x_{m,2}), \\ \cdots, \\ \dot{x}_{m,j} = F_j(x_{m,1}, x_{m,2}, \ldots, x_{m,n}) + h_j(x_{m,j}), \\ \cdots, \\ \dot{x}_{m,n} = F_n(x_{m,1}, x_{m,2}, \ldots, x_{m,n}) + h_n(x_{m,n}), \end{cases} \quad (j = 1, 2, \ldots, n), \tag{2}$$

where $u(t) \in R$ represents the controller and $h_j(x_{m,j})$ is the coupling function of the m-th, the last node, in the network.

The errors among the state variables of the network can be defined as

$$e_{i,j} = x_{i+1,j} - x_{i,j}, \quad (i = 1, 2, \ldots, m-1, \quad j = 1, 2, \ldots, n). \tag{3}$$

Based on the sliding mode control strategy, the switching surface of the network can be designed as[18]

$$s(t) = e_{m-1,1}(t) + \varphi(t), \tag{4}$$

where $\varphi(t)$ is given by

$$\dot{\varphi}(t) = \frac{1}{e_{m-1,1}} \left[\sum_{i=1}^{m-2} \sum_{j=1}^{n} e_{i,j} \dot{e}_{i,j} + \sum_{j=2}^{n} e_{m-1,j} \dot{e}_{m-1,j} + \sum_{j=1}^{n} \alpha_j (e_{m-1,j})^2 \right] \tag{5}$$

with α_j being an arbitrary positive coefficient. When the network operates in the sliding mode, it satisfies

$$s(t) = e_{m-1,1}(t) + \varphi(t) = 0. \tag{6}$$

When the dynamic network is in the sliding mode, it satisfies $\dot{s}(t) = 0$. The time derivative of Eq. (6) is in the form of

$$\begin{aligned} \dot{s} &= \dot{e}_{m-1,1} + \dot{\varphi} = 0 \Rightarrow \dot{e}_{m-1,1} = -\dot{\varphi} \\ &= -\frac{1}{e_{m-1,1}} \left[\sum_{i=1}^{m-2} \sum_{j=1}^{n} e_{i,j} \dot{e}_{i,j} + \sum_{j=2}^{n} e_{m-1,j} \dot{e}_{m-1,j} + \sum_{j=1}^{n} \alpha_j (e_{m-1,j})^2 \right]. \end{aligned} \tag{7}$$

To analyze the stability of the sliding mode dynamics Eq. (7) based on the Lyapunov stability theory, the Lyapunov function candidate is selected as

$$V(t) = \frac{1}{2} \left[\sum_{i=1}^{m-1} \sum_{j=1}^{n} (e_{i,j})^2 \right],$$

which leads to

$$\begin{aligned} \dot{V}(t) &= \sum_{i=1}^{m-1} \sum_{j=1}^{n} e_{i,j} \dot{e}_{i,j} \\ &= \sum_{i=1}^{m-2} \sum_{j=1}^{n} e_{i,j} \dot{e}_{i,j} + \sum_{j=2}^{n} e_{m-1,j} \dot{e}_{m-1,j} + e_{m-1,1} \dot{e}_{m-1,1} \\ &= \sum_{i=1}^{m-2} \sum_{j=1}^{n} e_{i,j} \dot{e}_{i,j} + \sum_{j=2}^{n} e_{m-1,j} \dot{e}_{m-1,j} \\ &\quad + e_{m-1,1} \left\{ -\frac{1}{e_{m-1,1}} \left[\sum_{i=1}^{m-2} \sum_{j=1}^{n} e_{i,j} \dot{e}_{i,j} + \sum_{j=2}^{n} e_{m-1,j} \dot{e}_{m-1,j} + \sum_{j=1}^{n} \alpha_j (e_{m-1,j})^2 \right] \right\} \\ &= -\sum_{j=1}^{n} \alpha_j (e_{m-1,j})^2 \le 0. \end{aligned} \tag{8}$$

To develop the equivalent control law, which is used in the sliding mode, it is assumed that $s = 0$ holds true. Then the following equivalent control law is obtained:

$$\begin{aligned} u_{\mathrm{eq}} &= F_1(x_{m-1,1}, x_{m-1,2}, \ldots, x_{m-1,n}) + h_1(x_{m-1,1}) \\ &\quad - F_1(x_{m,1}, x_{m,2}, \ldots, x_{m,n}) - h_1(x_{m,1}) \end{aligned}$$

$$-\frac{1}{e_{m-1,1}}\left[\sum_{i=1}^{m-2}\sum_{j=1}^{n}e_{i,j}\dot{e}_{i,j}+\sum_{j=2}^{n}e_{m-1,j}\dot{e}_{m-1,j}+\sum_{j=1}^{n}\alpha_j(e_{m-1,j})^2\right]. \tag{9}$$

The reaching control law is selected as $u_{\mathrm{r}}=\beta_s\mathrm{sgn}(s)$, and the overall controller is determined by

$$u=u_{\mathrm{eq}}+u_{\mathrm{r}}=u_{\mathrm{eq}}+\beta_s\mathrm{sgn}(s), \tag{10}$$

where β_s is the reaching gain, and calculated by

$$\dot{\beta}_s=-\zeta|s|. \tag{11}$$

From Eqs. (9) and (11), the above equation (10) can be further simplified into

$$\begin{aligned}u=&F_1(x_{m-1,1},x_{m-1,2},\ldots,x_{m-1,n})+h_1(x_{m-1,1})\\&-F_1(x_{m,1},x_{m,2},\ldots,x_{m,n})-h_1(x_{m,1})\\&-\frac{1}{e_{m-1,1}}\left[\sum_{i=1}^{m-2}\sum_{j=1}^{n}e_{i,j}\dot{e}_{i,j}+\sum_{j=2}^{n}e_{m-1,j}\dot{e}_{m-1,j}\right.\\&\left.+\sum_{j=1}^{n}\alpha_j(e_{m-1,j})^2\right]+\beta_s\mathrm{sgn}(s).\end{aligned} \tag{12}$$

The stability of the scheme occurring in Eq. (12) will be proved. Define a new Lyapunov function candidate as

$$V(t)=\frac{1}{2}s^2+\frac{1}{2\zeta}(\hat{\beta}-\beta_s)^2, \tag{13}$$

then the time derivative of Eq. (13) can be calculated by

$$\begin{aligned}\dot{V}(t)=&s(\dot{e}_{m-1,1}+\dot{\varphi})+\frac{1}{\zeta}(\hat{\beta}-\beta_s)(-\dot{\beta}_s)\\=&s\left\{F_1(x_{m,1},x_{m,2},\ldots,x_{m,n})+h_1(x_{m,1})+u\right.\\&-F_1(x_{m-1,1},x_{m-1,2},\ldots,x_{m-1,n})-h_1(x_{m-1,1})\\&+\frac{1}{e_{m-1,1}}\left[\sum_{i=1}^{m-2}\sum_{j=1}^{n}e_{i,j}\dot{e}_{i,j}+\sum_{j=2}^{n}e_{m-1,j}\dot{e}_{m-1,j}\right.\\&\left.\left.+\sum_{j=1}^{n}\alpha_j(e_{m-1,j})^2\right]\right\}+\frac{1}{\zeta}(\hat{\beta}-\beta_s)(-\dot{\beta}_s)\\=&s[\beta_s\mathrm{sgn}(s)]+(\hat{\beta}-\beta_s)\left(-\frac{1}{\zeta}\dot{\beta}_s\right)\\=&\beta_s|s|+\hat{\beta}|s|-\hat{\beta}|s|+(\hat{\beta}-\beta_s)\left(-\frac{1}{\zeta}\dot{\beta}_s\right)\\=&-(\hat{\beta}-\beta_s)|s|+(\hat{\beta}-\beta_s)\left(-\frac{1}{\zeta}\dot{\beta}_s\right)+\hat{\beta}|s|\\=&(\hat{\beta}-\beta_s)\left(-\frac{1}{\zeta}\dot{\beta}_s-|s|\right)+\hat{\beta}|s|.\end{aligned} \tag{14}$$

Consider that $\dot{\beta}_s=-\zeta|s|$, the value of scalar $\hat{\beta}$ is a negative constant, $\hat{\beta}=-\lambda<0$ where $\lambda>0$, then

$$\dot{V}(t)=-\lambda|s|\leq 0. \tag{15}$$

According to the Lyapunov stability theory and sliding mode control scheme, the reaching condition will be satisfied, and equation (13) is asymptotically stable. This completes the proof.

3. Numerical results

The chain network is treated as an example to be discussed in the following parts. The Cayley tree (Fig. 1) is a small world if the degree of the vertices exceeds 2.[30] It can be considered to be composed of a series of chained network couplings (Fig. 2), which not only achieves the synchronization effectively, but also reduces the area to readjust, when any part of the network changes.

The coupling matrix G_{il} of chain network is set to be

$$G_{il}=\begin{bmatrix}0&0&0&\ldots&0\\1&-1&0&\ldots&0\\\ldots&\ddots&\ddots&\ldots&\ldots\\0&0&\ddots&-1&0\\0&0&\ldots&1&-1\end{bmatrix}. \tag{16}$$

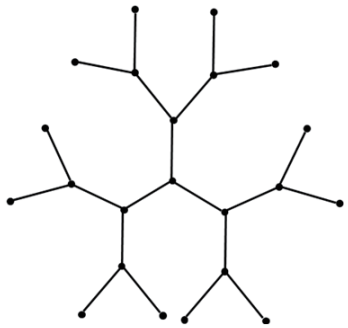


Fig. 1. Cayley tree network.

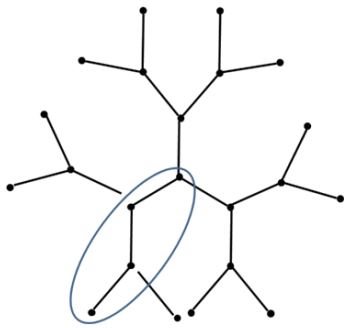


Fig. 2. (color online) Segmentation into chain network.

The double-scroll circuit chaos systems are treated as nodes of the chain network to demonstrate the synchronization mechanism mentioned above. Simulation is made with the number of the nodes being $m=4$.

The dynamic equation of double-scroll circuit chaos system is[31]

$$\begin{cases}\dot{x}=y,\\\dot{y}=z,\\\dot{z}=-\rho(x+y+z)+\varepsilon f(x),\end{cases} \tag{17}$$

where ρ and ε are parameters of the system, and $f(x)$ is the saturation function and defined by the following expression:

$$f(x)=\begin{cases}1, & x>k,\\ \dfrac{1}{k}x, & -k\le x\le k,\\ -1, & x<-k.\end{cases} \tag{18}$$

When the parameters are given as $\rho=0.5$, $\varepsilon=0.8$, and $k=0.7$, the chaos attractor of Eq. (17) is shown in Fig. 3.

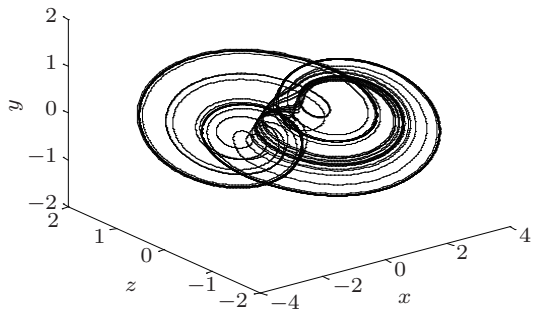


Fig. 3. Trajectories for double-scroll system.

The problem of selecting the type of nodes that should be coupled in order to improve the synchronization performance of complex networks by minimizing synchronization time,[32] and driving the systems to a desired state is still an important problem. In Ref. [33], the inverse lag synchronization of two identical nonlinear double scroll circuits was implemented via a resistor linearly coupled. The coupling functions of the network are given in the same way as those in Ref. [33]. The complex network is constructed by the following four chaotic systems according to chained network coupling:[34,35]

$$\begin{cases}\dot{x}_1=y_1,\\ \dot{y}_1=z_1,\\ \dot{z}_1=-\rho(x_1+y_1+z_1)+\varepsilon f(x_1),\end{cases} \tag{19}$$

$$\begin{cases}\dot{x}_2=y_2+\xi(x_1-x_2),\\ \dot{y}_2=z_2+\xi(y_1-y_2),\\ \dot{z}_2=-\rho(x_2+y_2+z_2)+\varepsilon f(x_2)+\xi(z_1-z_2),\end{cases} \tag{20}$$

$$\begin{cases}\dot{x}_3=y_3+\xi(x_2-x_3),\\ \dot{y}_3=z_3+\xi(y_2-y_3),\\ \dot{z}_3=-\rho(x_3+y_3+z_3)+\varepsilon f(x_3)+\xi(z_2-z_3),\end{cases} \tag{21}$$

$$\begin{cases}\dot{x}_4=y_4+\xi(x_3-x_4)+u,\\ \dot{y}_4=z_4+\xi(y_3-y_4),\\ \dot{z}_4=-\rho(x_4+y_4+z_4)+\varepsilon f(x_4)+\xi(z_3-z_4).\end{cases} \tag{22}$$

To reach the sliding surface and finally stabilize the surface, the only one proper $u(t)$ is selected as follows:

$$\begin{aligned}s(t) &= e_{31}(t)+\varphi(t)=x_4-x_3+\varphi(t), \qquad (23)\\ \dot{\varphi} &= \frac{1}{e_{31}}\{e_{11}e_{12}+e_{12}e_{13}-\rho e_{13}(e_{11}+e_{12}+e_{13})\\ &\quad +\varepsilon e_{13}[(f(x_2)-f(x_1)]+e_{21}(e_{22}+\xi e_{11})\\ &\quad +e_{22}(e_{23}+\xi e_{12})-\rho e_{23}(e_{21}+e_{22}+e_{23})\\ &\quad +\varepsilon e_{23}[(f(x_3)-f(x_2)]\\ &\quad +\xi e_{13}e_{23}+e_{32}(e_{33}+\xi e_{22})\\ &\quad -\rho e_{33}(e_{31}+e_{32}+e_{33})\\ &\quad +\varepsilon e_{33}[(f(x_4)-f(x_3)]+\xi e_{23}e_{33}\}, \qquad (24)\\ u &= -e_{32}+\xi(e_{31}-e_{21})\\ &\quad -\frac{1}{e_{31}}\{e_{11}e_{12}+e_{12}e_{13}-\rho e_{13}(e_{11}+e_{12}+e_{13})\\ &\quad +\varepsilon e_{13}[(f(x_2)-f(x_1)]+e_{21}(e_{22}+\xi e_{11})\\ &\quad +e_{22}(e_{23}+\xi e_{12})-\rho e_{23}(e_{21}+e_{22}+e_{23})\\ &\quad +\varepsilon e_{23}[(f(x_3)-f(x_2)]+\xi e_{13}e_{23}\\ &\quad +e_{32}(e_{33}+\xi e_{22})-\rho e_{33}(e_{31}+e_{32}+e_{33})\\ &\quad +\varepsilon e_{33}[(f(x_4)-f(x_3)]+\xi e_{23}e_{33}\}-\beta_s\mathrm{sgn}(s), \qquad (25)\end{aligned}$$

with $\dot{\beta}_s=-0.1\,|s|$ and starting from $\dot{\beta}_s=10$.

The relative weights of the coupling strength among the nodes of the network are all treated as ξ=1. Simulation results are shown in Figs. 4–10.

It can be seen from Figs. 4–6 that the state variable trajectories of the nodes are exactly the same after adding controllers. From Figs. 7–10, error signals and the switching surface approach to zero smoothly and rapidly in a short series of time after adding appropriate controllers and selecting the switching surface, then the synchronization of network could be achieved.

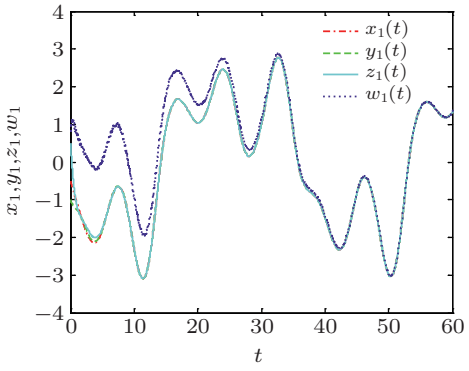


Fig. 4. (color online) Trajectories of the first dimension.

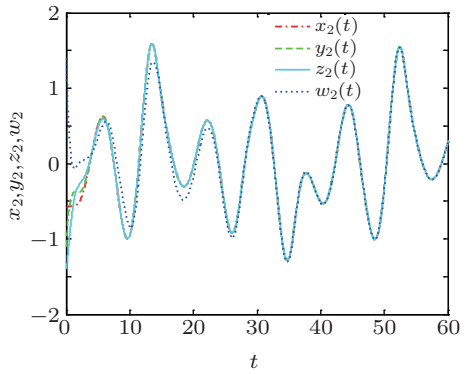


Fig. 5. (color online) Trajectories of the second dimension.

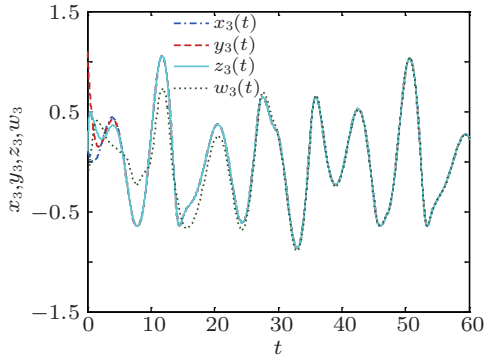


Fig. 6. (color online) Trajectories of the third dimension.

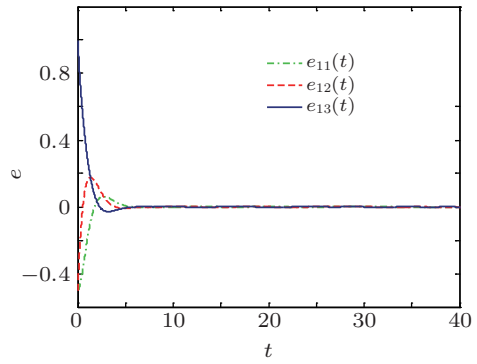


Fig. 7. (color online) Error variables e_{1j} $(j = 1, 2, 3)$ versus time t.

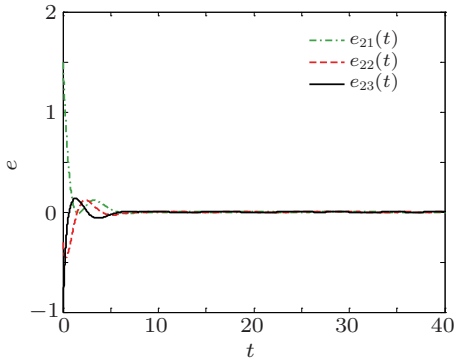


Fig. 8. (color online) Error variables e_{2j} $(j = 1, 2, 3)$ versus time t.

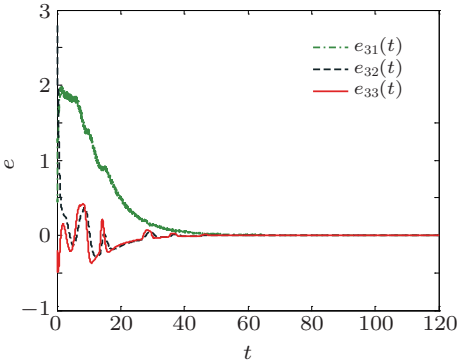


Fig. 9. (color online) Error variables e_{3j} $(j = 1, 2, 3)$ versus time t.

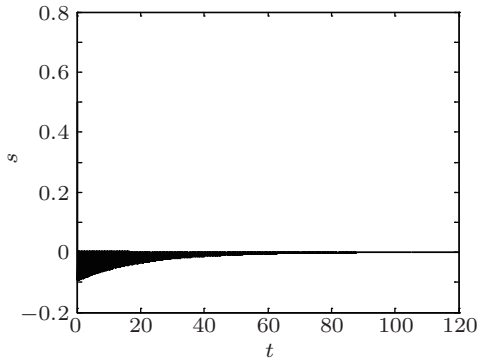


Fig. 10. Switching surface versus time t.

4. Conclusions

In this paper, a sliding mode control approach is proposed for the synchronization of dynamic networks. Based on the Lyapunov stability theory, the switching surface and the only one control input are designed to implement the synchronization. To verify the efficiency of the proposed approach, the chain network is constructed with double-scroll circuit chaos systems which are selected as an example. It is shown that this proposed sliding mode control approach has two main advantages: one is that it has no special requirements for the structure of the nodes, and the other is that it needs only one control input. The numerical simulation results show that the error signals approach to zero smoothly and rapidly in a short series of time after starting the controller, which demonstrates the effectiveness of the sliding mode control approach for the dynamical networks.

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