Enhanced electron–positron pair creation by the frequency chirped laser pulse*

Jiang Min(姜 敏), Xie Bai-Song(谢柏松)†, Sang Hai-Bo(桑海波), and Li Zi-Liang(李子良)

Key Laboratory of Beam Technology and Materials Modification of the Ministry of Education, College of Nuclear Science and Technology, Beijing Normal University, Beijing 100875, *China*

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Based on the quantum Vlasov equation, the effect of frequency chirp on electron–positron pair production is investigated. The cycle parameter, which characterizes the laser field cycle degree within the pulse, is also considered. In both supercycle and subcycle laser pulses the frequency chirp can greatly enhance the momentum distribution function of created pairs and the pair number density. The pair number density created by a supercycle laser pulse is larger than that by a subcycle pulse under the same laser frequency and chirping. There exists an optimal cycle parameter corresponding to the maximum value of the created pair number density for different chirp rates. It is found that the pair number density is sensitive/insensitive to chirping rate when the cycle parameter lies below/above the optimal one.

Keywords: electron–positron, quantum nonlocalization, quantum statistics and Vlasov equation, X-ray laser

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Electron–positron pair creation from the vacuum induced by a strong external field is a nonperturbative phenomenon in quantum field theory. In the presence of a strong external electric field the quantum electrodynamics (QED) vacuum will be unstable and decay via spontaneous electron–position pair production,[1,2] which is well known as the Schwinger mechanism. Since scientists have calculated the pair production rate in a constant field,[3] there has been much research about the electron–positron pair creation from the vacuum. The Schwinger mechanism has not been directly observed yet, because the critical field $E_{\rm cr} = m_{\rm e}^2 c^3/e\hbar \sim 1.3 \times 10^{18}$ V/m is still difficult to realize today. However, with the advance of high-intensity laser technology, in particular X-ray free electron laser (XFEL) facilities, it will be possible to produce an electric field strength $E = 0.1E_{\rm cr}$ in the near future. On the other hand due to the tunneling property of the Schwinger mechanism for pair production some research is performed in involved problems at much lower intensities by choosing appropriate external fields.[4–9] The momentum spectrum of the produced particles was found to be extremely sensitive to the physical pulse parameters such as frequency chirp.[10]

We know that in many QED processes, for example, Lamb shift of QED and quantum games in cavity QED, etc.,[11–14] the researched systems exhibit some nonlinear behaviors. Here the electron–positron pair creation from the QED vacuum is a nonequilibrium and time-dependent process, the quantum kinetic method such as quantum Vlasov equation (QVE) will be an appropriate approach.[15–19] By solving QVE, one can get the pair production rate and the electron/positron momentum information. To simplify our calculations, the laser pulse field is treated as classical background field, the spatial focusing scale is much larger than the Compton wavelength,[20] the magnetic field is assumed to be zero, and the electric field is time-dependent. In general the total electric field is $\boldsymbol{E} = \boldsymbol{E}_{\rm ext} + \boldsymbol{E}_{\rm int}$, but under the XFEL condition the inner field $\boldsymbol{E}_{\rm int}$ can be neglected so that $\boldsymbol{E} = \boldsymbol{E}_{\rm ext}$.

In a recent study,[21] we have used the QVE solving approach to study the dynamically assisted effect on pair production in the combined electric field

$$E(t) = E_0 \exp\left(-\frac{t^2}{\tau_0^2}\right) \sin(\omega_0 t), \tag{1}$$

where $E_0 = 0.1E_{\rm cr}$, ω_0 is the laser frequency associated to the laser wavelength $\lambda = 0.15 \times 10^{-9}$ m, τ_0 is the laser pulse length and $b = \omega_0 \tau_0$ denotes as a cycle parameter, which characterizes the laser field cycle degree within the pulse. Note that the natural units ($\hbar = c = m_{\rm e} = 1$) are used throughout this paper and all quantities are normalized by the electron corresponding ones which are the same as those in Ref. [21].

In this paper, we will extend our previous research along two directions. One is to consider the frequency chirped laser pulse and study the effect of frequency chirp on the momentum spectrum of the created pairs and produced particle number density. The other is that we consider both $b > 1$, a supercycle laser pulse, and $b < 1$, a subcycle laser pulse. Some new phenomena and results are presented. For example, the pair momentum and number density depend strongly on the frequency chirping in the subcycle situation and in the moderate supercycle situation. They are enhanced greatly by the rapid

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†Corresponding author. E-mail: bsxie@bnu.edu.cn

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frequency chirping. There exists approximately a universal optimal cycle parameter b that makes the maximum created pair number density, i.e., this optimal cycle parameter and the corresponding maximum pair number density are insensitive to frequency chirping.

We employ the quantum kinetic method to describe pair production in a homogeneous, time-dependent electric field; the time-evolution of the one-particle distribution function $f(\boldsymbol{p},t)$ can be described by the Boltzmann equation:

$$\frac{\mathrm{d}f(\boldsymbol{p},t)}{\mathrm{d}t} = s(\boldsymbol{p},t) + c(\boldsymbol{p},t), \tag{2}$$

where $s(\boldsymbol{p},t)$ is the source term of pair production and $c(\boldsymbol{p},t)$ is the collision term. As the created particle density is sufficiently low under subcritical electric fields, consequently, $s(\boldsymbol{p},t) \gg c(\boldsymbol{p},t)$. So the collision term can be ignored in the Boltzmann equation which would result in the quantum Vlasov equation as

$$\frac{\mathrm{d}f(\boldsymbol{p},t)}{\mathrm{d}t} = s(\boldsymbol{p},t).$$

From this equation, we get the quantum kinetic equation for $f(\boldsymbol{p},t)$ in the following[22]

$$\begin{aligned}\frac{\mathrm{d}f(\boldsymbol{p},t)}{\mathrm{d}t} &= \frac{eE(t)\varepsilon_\perp^2}{2\omega^2(\boldsymbol{p},t)}\int_{t_0}^{t}\mathrm{d}t'\frac{eE(t')[1-2f(\boldsymbol{p},t')]}{\omega^2(\boldsymbol{p},t')} \\ &\quad\times\cos\left[2\int_{t'}^{t}\mathrm{d}\tau\omega(\boldsymbol{p},\tau)\right],\end{aligned} \tag{3}$$

where $\boldsymbol{p} = (\boldsymbol{p}_\perp, p_\parallel)$ is the three-vector momentum, $\varepsilon_\perp^2 = m_\mathrm{e}^2 + p_\perp^2$ is the the transverse energy squared, $\omega^2(\boldsymbol{p},t) = \varepsilon_\perp^2 + p_\parallel^2$ is the total energy-squared, $p_\parallel = p_3 - eA(t)$ associated to laser field vector potential $A(t)$ is the longitudinal momentum. It should be noted that this equation is valid only for subcritical external applied fields. If the applied fields approach or/and exceed the critical field E_cr the internal field caused by the created electron–positron pairs current would have to be included in the problem and the corresponding quantum Vlasov equation would have to be modified. Because the field we considered in this paper is kept as $E_0 = 0.1E_\mathrm{cr}$ equation (3) is still valid. For convenience two auxiliary quantities are defined as $q(\boldsymbol{p},t) = eE(t)\varepsilon_\perp/\omega^2(\boldsymbol{p},t)$ and $\Theta(\boldsymbol{p},t',t) = \int_{t'}^{t}\omega(\boldsymbol{p},\tau)\mathrm{d}\tau$, then equation (3) becomes

$$\begin{aligned}\frac{\mathrm{d}f(\boldsymbol{p},t)}{\mathrm{d}t} &= \frac{1}{2}q(\boldsymbol{p},t)\int_{t_0}^{t}\mathrm{d}t'q(\boldsymbol{p},t')[1-2f(\boldsymbol{p},t')] \\ &\quad\times\cos[2\Theta(\boldsymbol{p},t',t)].\end{aligned} \tag{4}$$

This integral-differential equation can be rewritten as a set of first order ordinary differential equations (ODE), which are

$$\begin{aligned}\dot{f}(\boldsymbol{p},t) &= \frac{1}{2}q(\boldsymbol{p},t)g(\boldsymbol{p},t), \\ \dot{g}(\boldsymbol{p},t) &= q(\boldsymbol{p},t)[1-2f(\boldsymbol{p},t)] - 2\omega(\boldsymbol{p},t)w(\boldsymbol{p},t), \\ \dot{w}(\boldsymbol{p},t) &= 2\omega(\boldsymbol{p},t)g(\boldsymbol{p},t).\end{aligned} \tag{5}$$

Note that the integral-differential equation or the ODE system is valid for any time-dependent electric field. By integrating the distribution function to momentum, we can get the time-dependent pair number density

$$n(t) = \int\frac{\mathrm{d}^3p}{(2\pi)^3}f(\boldsymbol{p},t). \tag{6}$$

This is a very useful quantity to help us to compare the effects of different electric field parameters on the pair creation. It is also noted that the single particle distribution function $f(\boldsymbol{p},t)$ has a definite physical meaning at $t\to\infty$ when the external electric field is zero. Thus what we are really interested in are $f(\boldsymbol{p},\infty)$ and the produced particle number density $n(\infty)$.

In order to see the influence of the field frequency variation on the pair production, a frequency chirp $\Delta\omega$ is introduced so that the field form becomes

$$E(t) = E_0\exp\left(-\frac{t^2}{\tau_0^2}\right)\sin[(\omega_0+\Delta\omega)t], \tag{7}$$

where $\Delta\omega$ is chosen as

$$\Delta\omega = \begin{cases} 0, & t < -\tau, \\ \dfrac{\omega_0}{10}\left(1+\dfrac{t}{\tau}\right), & -\tau \le t < \tau, \\ \dfrac{\omega_0}{5}, & t \ge \tau \end{cases}$$

with $\tau = k\tau_0$. Note that k can be used to describe the chirping rate. Obviously, smaller k means a rapid chirping. In the following, the effects of the frequency chirp on the electron–positron pair creation under the conditions of both supercycle ($b > 1$) and subcycle ($b < 1$) laser pulse are studied. The cycle parameter b impacts especially on the pair production in different chirp rates, for example, $k = 3$, 2, 1, and 0.5, are also studied.

First, let us consider the supercycle laser pulse by setting $b = 10$. Figure 1 gives the momentum dependence of the single particle distribution function $f(\boldsymbol{p},\infty)$ for different chirping $k = 3$, 2, 1, and 0.5, respectively. From Fig. 1(a) and Fig. 1(b), one can see that when the chirp rate is relatively small, the momentum spectrum does not change obviously by comparing with the result without the chirp (refer to Ref. [21]). In this case the maximum $f(\boldsymbol{p},\infty)$ is of the same order of magnitude $\sim 10^{-11}$. However, when the chirp rate is increased to $k = 1$, the maximum $f(\boldsymbol{p},\infty)$ is $\sim 10^{-8}$, which is about three orders of magnitude larger than that without or with small chirping, as shown in Fig. 1(c). As the chirping rate increases further to $k = 0.5$, the maximum $f(\boldsymbol{p},\infty)$ is about $\sim 10^{-7}$, four orders of magnitude greater than that of without or with small chirping, see Fig. 1(d).

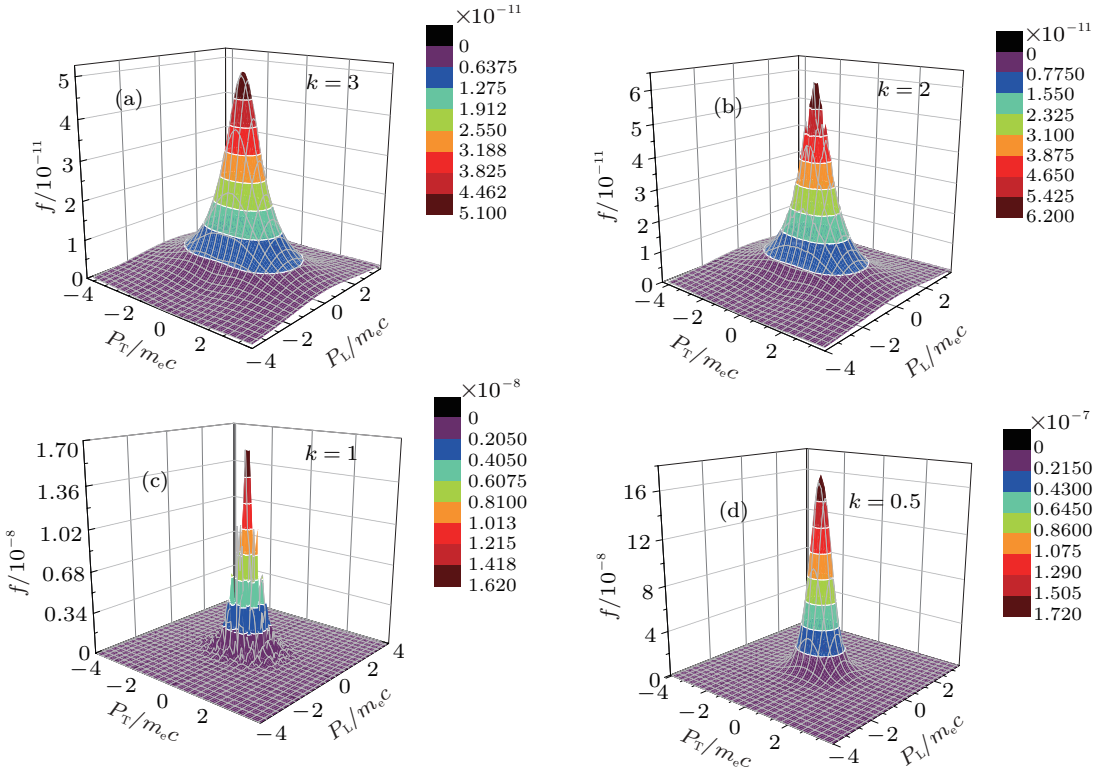


Fig. 1. (color online) Momentum dependence of the distribution function of the created pairs when $t \to \infty$ for different chirp structures in supercycle $b = 10$. Here the values of k are chosen as $k = 3$, 2, 1, and 0.5.

Although the momentum distribution function, especially its peak value, increases with chirping rate, the momentum domain is decreasing. This fact can be seen by comparing Figs. 1(c) and 1(d) to Figs. 1(a) and 1(b). Therefore, it is suspected that the created pairs are enhanced greatly by rapid frequency chirping. To give a credible result, the number density of the produced particles is calculated numerically and is given in Fig. 2. From this figure, it is clear that the produced pair number density can be indeed increased greatly when k is decreasing, i.e., the chirp rate is increasing. For the electric field without frequency chirp, the created particle number density is only $n(\infty) = 1.66 \times 10^{-14}$. After the laser pulse chirp structure is added, for example, when $k = 3$ one gets $n(\infty) \approx 2.30 \times 10^{-14}$, which is still a much smaller growth than that without the chirp structure. However when $k = 2$, the values of $n(\infty)$ is 6.52×10^{-13}. It is more than one order of magnitude larger than that of the above case. Furthermore, when $k = 1$, corresponding to a larger chirp rate, the created $e^- e^+$ pair number density is enhanced a lot and is about three orders of magnitude greater than that of $k = 2$. And finally when $k = 0.5$, the number density $n(\infty)$ reaches 1.65×10^{-8}, which is about six orders of magnitude larger than that without the chirp. Therefore, it can be concluded that the chirp structure plays an important role in enhancing pair production. With the smaller and smaller k, i.e., as the chirp rate becomes faster and faster, the produced particles number density increases more and more greatly.

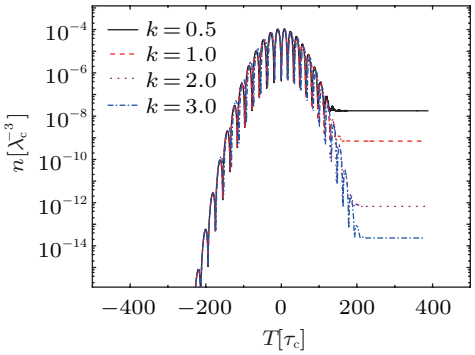


Fig. 2. (color online) Time evolution of the produced particles number density $n(t)$ for four different chirp rates $k = 3$, 2, 1, and 0.5 in supercycle $b = 10$. The abscissa unit of time $\tau_c = \lambda_c/c = 1.287 \times 10^{-21}$ s, where $\lambda_c = \hbar/m_e c = 3.869 \times 10^{-13}$ m is the Compton wavelength.

Now let us study the effect of chirp structure on pair production in a subcycle laser pulse by setting $b = 0.5$. The corresponding momentum dependence of single particle distribution function $f(\boldsymbol{p}, \infty)$ of the created pairs for different chirping $k = 3$, 2, 1, and 0.5 is shown in Fig. 3. As k decreases, i.e., the growth of chirp rate, the maximum values of $f(\boldsymbol{p}, \infty)$ increase by more than six orders of magnitude, from 2.28×10^{-16} to 4.10×10^{-10}. This variation of $f(\boldsymbol{p}, \infty)$ changing with the chirp rate is similar to that in the supercycle structure. But there are still some small differences. Obviously, with the same k, the maximum $f(\boldsymbol{p}, \infty)$ in the subcycle situation is at least three orders smaller than that in the supercycle situation. Furthermore, the momentum domain is reduced faster than that in supercycle as k decreases. These differences maybe

attributed to the difference of cycle parameters b. Here we also plot the time evolution of created pair number density $n(t)$ with four different values of k in the subcycle situation as shown in Fig. 4. It is found that the same trend of the stationary number density $n(\infty)$ changing with k occurs in the subcycle situation but with smaller values compared to the corresponding supercycle situation. For example, when $k = 0.5$, the number density $n(\infty)$ reaches $\sim 10^{-10}$, the maximum of four different values of the chirp rate k, which is still about two orders of magnitude smaller than that of the supercycle situation. By comparing the supercycle and subcycle results it concludes that a supercycle parameter b can enhance the pair creation greatly in comparison with a subcycle situation if the laser frequency as well its chirping is kept the same. It is not surprising because the former laser field acting on the vacuum has a longer time so that more electron–positron pair can tunnel out from the vacuum. By the way from comparisons of Fig. 2 with Fig. 4, we see that there is also greater oscillation in the supercycle situation for the number density $n(t)$ before it becomes the steady value $n(\infty)$, which seems to inherit the characteristic of the laser field.

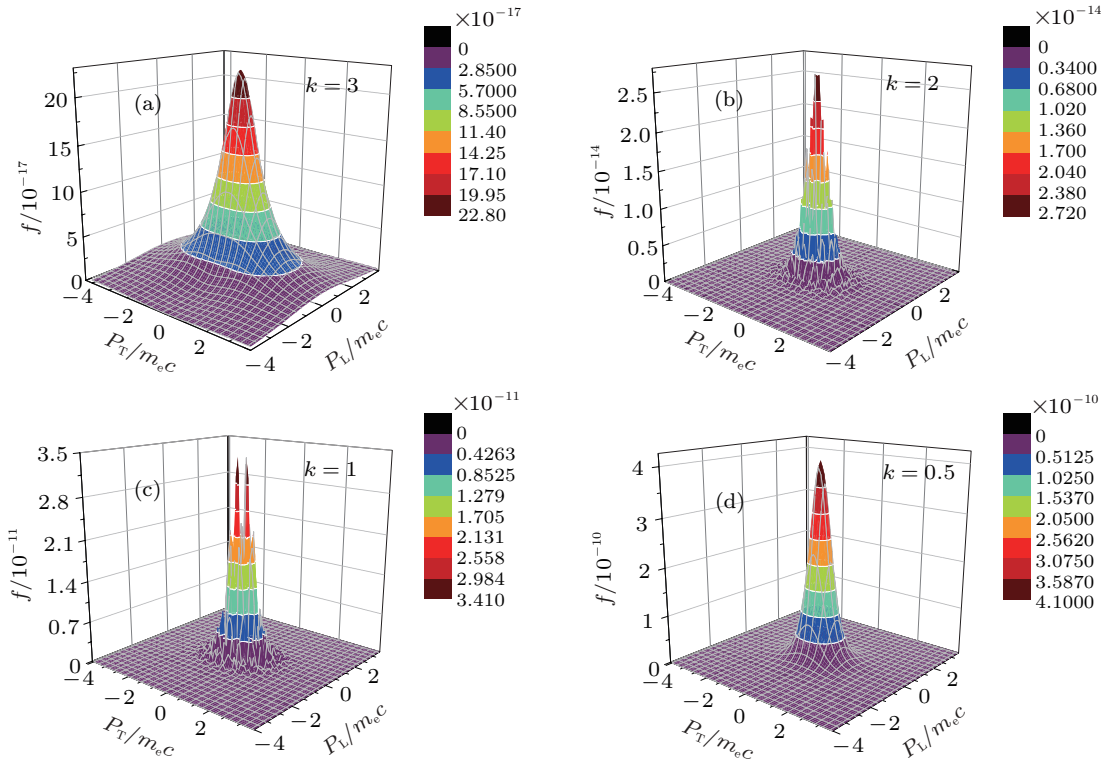


Fig. 3. (color online) Momentum dependence of the distribution function of the produced particles when $t \to \infty$ in subcycle $b = 0.5$. Here the values of k are chosen as $k = 3$, 2, 1, and 0.5.

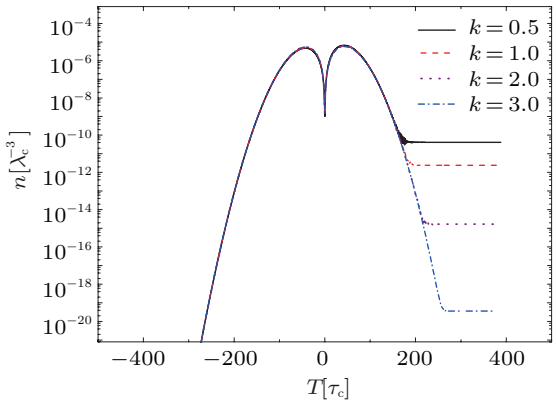


Fig. 4. (color online) Time evolution of the created pairs number density $n(t)$ for four different chirp rates $k = 3$, 2, 1, and 0.5 in subcycle $b = 0.5$. The abscissa unit of time $\tau_c = \lambda_c/c = 1.287 \times 10^{-21}$ s, where $\lambda_c = \hbar/m_e c = 3.869 \times 10^{-13}$ m is the Compton wavelength.

Now we turn to investigate the effect of the cycle parameter b on the pair creation under the conditions of four different frequency chirping rates. For a simple illustration, let the b change from ~ 0 to 600.

In Fig. 5 we display the dependence of produced particle number density $n(\infty)$ with b for $k = 3$, 2, 1, and 0.5. It is found that there is a maximum $n(\infty)$ at about 0.05 for each chirp rate and the corresponding b is near 125. The detailed results of four chirping rates are listed as follows:

(i) $k = 3$, $n_{\max} = 0.044$ at $b_{\text{opt}} = 120$;

(ii) $k = 2$, $n_{\max} = 0.045$ at $b_{\text{opt}} = 123$;

(iii) $k = 1$, $n_{\max} = 0.048$ at $b_{\text{opt}} = 129$;

(iv) $k = 0.5$, $n_{\max} = 0.053$ at $b_{\text{opt}} = 132$.

From these data, we can see that $n_{\max}$ and b_{opt} have almost the same values for different values of k which exhibits different chirp rates. In other words the maximum values of

the created particles number density and the corresponding optimal cycle parameter $b_{\rm opt}$ are not sensitive to the chirp structures. On the other hand from Fig. 5 we see that when $b < b_{\rm opt}$ the stationary number densities are very different with each other greatly at the same b for different values of k. The variations of $n(\infty)$ at $b = 10$ and $b = 0.5$ for different k have been researched in detail in the above discussion and they can also be seen here in Fig. 5. When the cycle parameter is greater than $b_{\rm opt}$, however, the created pair number density keeps the same changing trend with the increase of b for different chirp rates. This surprising fact indicates that it seems to have a universal behavior of pair number density dependence of b and k when $b \geq b_{\rm opt}$.

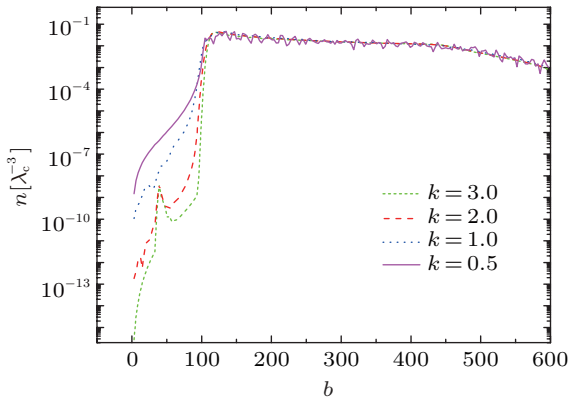


Fig. 5. (color online) The relationship between the created pariticle number density and the cycle parameter b for $k = 3$, 2, 1, and 0.5.

Through detailed data analysis by numerical solving QVE, we can now conclude that the chirp structure has a great effect on the created particle momentum and number density for $b < b_{\rm opt}$ while when $b > b_{\rm opt}$ the stationary number density is not sensitive to the chirp rate. In addition, the produced pair number density $n_{\rm max}$ and the corresponding optimal cycle parameter $b_{\rm opt}$ seems to have a universal behavior, which is very insensitive to the chirp structure. These results may be helpful to guide future pair creation experimentally in the laboratory. On the other hand we note that in Fig. 5 for $b <\sim 100$ there is strong nonlinear dependence of $n(\infty)$ on b, in particular for small chirping rate $k = 3$ and $k = 2$. We guess that the peaks of $n(\infty)$ maybe associated to the quantum mechanics resonances among the levels driven by laser field. It may be understood in the frame of complex WKB similarly in Ref. [9]. Certainly the physical mechanism on the combined effects of frequency chirping and cycle parameter needs further research.

In summary, in this paper the effects of a field with frequency chirping and with different cycle pulses on the pair creation are investigated. Impacts of the chirp rates in both of supercycle and subcycle laser pulses and the cycle parameters in different chirp rates on electron–positron pair production in a vacuum are studied respectively. For a fixed $b = 10$, a typical supercycle structure, the number of pair creation with a large chirp rate is greatly enhanced comparable to that without or with a small chirp rate. In subcycle situation, the results are very similar with that in supercycle structure but a relatively smaller $n(\infty)$ is obtained for the same chirp rate. So in order to get high number density it is better to use larger b. On the other hand, for a fixed k, there exists an optimal combined field parameter $b_{\rm opt}$. It shows that the effect of chirp rate on the created particles number density is sensitive when $b < b_{\rm opt}$ while it is approximately insensitive when $b > b_{\rm opt}$. Furthermore, the optimal cycle parameter and the maximum value of the created pair number density are also insensitive to chirp rates. These results are helpful to design the laser pulse including the appropriate frequency chirping that can be used to observe significant pair creation with high number density in possible future experiments.

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