Traffic resource allocation for complex networks*

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In this paper, an optimal resource allocation strategy is proposed to enhance traffic dynamics in complex networks. The network resources are the total node packet-delivering capacity and the total link bandwidth. An analytical method is developed to estimate the overall network capacity by using the concept of efficient betweenness (ratio of algorithmic betweenness and local processing capacity). Three network structures (scale-free, small-world, and random networks) and two typical routing protocols (shortest path protocol and efficient routing protocol) are adopted to demonstrate the performance of the proposed strategy. Our results show that the network capacity is reversely proportional to the average path length for a particular routing protocol and the shortest path protocol can achieve the largest network capacity when the proposed resource allocation strategy is adopted.

Keywords: complex network, shortest path protocol, efficient routing protocol, resource allocation strategy

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1. Introduction

The dynamic properties of network traffic systems have attracted much attention from physical and engineering communities. The network prototypes include data transfer in the Internet, the airlines between airports, the motion of vehicles in urban traffic systems, the migration of carbon in biosystems, etc. Since the discovery of small-world phenomena[1] and scale-free properties,[2] it is widely proved that the topology and degree distribution of networks have profound effects on the processes taking place on these networks, including the traffic flow.[3–6]

In the past few years, the phenomena of phase transition,[7,8] the process of epidemic spreading,[9,10] the scaling of traffic fluctuation,[11,12] and the routing strategies[13–21] of traffic dynamics have been widely studied. Recently, the effects of node capacity allocation on the performance of traffic systems have drawn much attention.[22–25] In many realistic systems, a widely used method to increase the network capacity is to enlarge the delivery capacity of nodes or the bandwidth of links. However, it is very expensive to do so. An alternative way is to maximize network capacity by optimizing the allocation of the limited traffic resources. Obviously, the capacity of a node should match its traffic supply. Yang *et al.*[22] proposed a strategy in which node capacities are assigned proportionally to their degrees with an exponent ϕ. It is shown that the system performs best when ϕ is slightly larger than 1.0. Gong *et al.*[23] found that there exists an optimal allocation for node delivery capability to achieve the best capacity of the network, and the achievable network capacity limit is inversely proportional to the average distance of the network measured according to the same routing rule. Xia *et al.*[25] showed that the betweenness may not be an accurate measurement of node traffic load when the traffic generation rate is not uniform for all sources.

In this paper, we address the resource allocation problems in traffic systems with complex networks. Firstly, we define the efficient betweenness as the ratio of algorithmic betweenness and local processing capacity, and propose an optimal strategy to allocate the node delivery capacity. The strategy relies on the algorithmic betweenness of the nodes. Then, we develop an analytical method to estimate the overall traffic capacity by using the concept of efficient betweenness. In addition, we also extend the proposed strategy to optimize the allocation of link bandwidth. Both theoretical analysis and numerical results show that the network performance is positively correlated with the reverse of the average path length of the network, which implies that the shortest path protocol is optimal for traffic dynamics when the proposed strategy is implemented.

This paper is organized as follows. In Section 2, the traffic model of complex networks is presented. In Section 3, we propose an optimal resource allocation strategy and provide both theoretical analysis and numerical results to illustrate the performance of the proposed strategy. We conclude this paper in Section 4.

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2. Network traffic model

We study the network traffic behavior on three representative network models: (i) the Barabási–Albert (BA) scale-free network,[2] (ii) the Newman–Watts (NW) small-world network,[26] and (iii) the Erdös–Rényi (ER) random graph.[27] The BA model can generate a scale-free network with power-law degree distribution by the growth and preferential attachment mechanisms. This model starts with m_0 fully connected nodes; a new node with m ($m \leq m_0$) links is added to the existing graph at each time step according to the preferential attachment. The probability Π_i for the new node to connect to existing node i is proportional to the degree k_i of the node, $\Pi_i = k_i/\Sigma_j k_j$, where the sum runs over all existing nodes. The NW small-world networks are generated by randomly adding long-range links to a one-dimensional lattice with nearest and next-nearest interactions with periodic boundary conditions. The ER random graphs are generated by connecting couples of randomly selected nodes while prohibiting multiple connections.

The network traffic model is described as follows. At each time step, there are R packets generated in the system with randomly chosen sources and destinations. The packets are routed in the system with given routing strategies. Once arriving at its destination, the packet will be removed from the system. The queue length of each node is assumed to be unlimited, and the first-in-first-out (FIFO) discipline is applied at each queue. In order to describe the phase transition of traffic flow in the network, we adopt the order parameter[7]

$$\eta(R) = \lim_{t\to\infty} \frac{\langle \Delta N_{\rm p} \rangle}{R\Delta t}, \tag{1}$$

where $\Delta N_{\rm p} = N_p(t+\Delta t) - N_p(t)$, $\langle \cdots \rangle$ indicates the average over a time window of width Δt, and $N_p(t)$ is the total packet number within the network at time t. The order parameter represents the balance between the outflow and inflow of packets. In the free-flow state, due to the balance of created and removed packets, η is around zero. With increasing packet generation rate R, there will be a critical value of $R_{\rm c}$ which characterizes the phase transition from free flow to congestion. When R exceeds $R_{\rm c}$, the packets continuously accumulate in the network, and η will become a constant larger than zero. The network capacity can be measured by the maximal generating rate $R_{\rm c}$ at the phase transition point. Theoretically, when the nodes have the same delivery capacity, $R_{\rm c}$ can be estimated by using the maximal algorithmic betweenness of the nodes as[4]

$$R_{\rm c} = \frac{N(N-1)}{B^{\max}}, \tag{2}$$

where $B^{\max}$ is the largest algorithmic betweenness of nodes in the system, and the node's algorithmic betweenness is defined as the number of routing paths going through the node. Equation (2) denotes that congestion firstly appears at the node(s) with the maximal algorithmic betweenness and then propagates to the whole system.

We consider the shortest path routing strategy and the efficient routing strategy.[18] The shortest path routing strategy is widely used in many modern communication networks. However, it often leads to the collapse of hub nodes, which may greatly restrict its overall performance. The efficient routing strategy is proposed to avoid the collapse of hub nodes in a hierarchical network like the scale-free networks. In the efficient routing strategy, the efficient path between a source node and a destination node is defined as the path with the sum of node degrees at its minimum.[18] For scale-free networks, the network traffic capacity can be greatly improved with the shortest path strategy because the packets are propelled to the periphery of the network.

3. Theoretical analysis and numerical results

3.1. Optimal allocation of a node's delivery capacity

Firstly, we investigate the optimal resource allocation strategy. At this stage, each link's bandwidth is considered as infinity. The node's delivering capacity is considered in the following way. At each time step, a node i can deliver C_i packets at most to its neighboring nodes. For simplicity, we suppose that the nodes' total packet-delivering capacity is a constant, $C = \sum_{i=1}^{N} C_i$. It is set to $C = N$, where N is network's size. We consider two allocation strategies of C_i for the network traffic system. The first strategy allocates the delivery resource evenly among the nodes

$$C_i = \frac{C}{N} = 1. \tag{3}$$

This strategy is adopted in most previous studies. The second strategy allocates the delivery resource according to the betweenness of the nodes

$$C_i = \frac{B_i}{\sum_{j=1}^{N} B_j} C, \tag{4}$$

where B_i is the algorithmic betweenness of node i. Therefore, the nodes with more paths going through them will have more delivery capacity to forward packets. We can also allocate the delivery resource by the node's degree information, $C_i = (K_i/\sum_{j=1}^{N} K_j)C$, where K_i is the degree of node i. In the following, we will show that the optimal strategy is exactly Eq. (4).

To find the optimal allocation strategy, we introduce the efficient betweenness of nodes $B_i^{\rm eff} = B_i/C_i$. Obviously, if the node's packet-delivering capacity is unequal, congestion occurs at the node with the largest efficient betweenness. We can revise Eq. (2) to estimate $R_{\rm c}$

$$R_{\rm c} = \frac{N(N-1)}{B_{\max}^{\rm eff}}, \tag{5}$$

where $B_{\max}^{\text{eff}}$ is the largest efficient betweenness of nodes in the system.

With the resource allocation strategy of Eq. (4), we get $B_i^{\text{eff}} = \sum_{j=1}^{N} B_j / C$. Since $\sum_{j=1}^{N} B_j = N(N-1)\bar{d}$ (here $\bar{d}$ is the average path length), B_i^{eff} turns out to be

$$B_i^{\text{eff}} = \frac{N(N-1)\bar{d}}{C} = \text{const.} \tag{6}$$

This indicates that all nodes in the system have an equal efficient betweenness. Thus congestion cannot develop earlier in any specific node, and thus the network capacity can be best optimized. We can also obtain

$$R_{\text{c}} = \frac{N(N-1)}{B_{\max}^{\text{eff}}} = \frac{N(N-1)}{B_i^{\text{eff}}} = \frac{C}{\bar{d}}. \tag{7}$$

From the above equation, we can see that using the allocation strategy of Eq. (4), the network capacity is inversely proportional to the average path length $\bar{d}$. In all existing routing protocols, we know that the shortest path strategy has the shortest average distance $\bar{d}_{\min}$. So we can see that Eq. (4) is the optimal allocation strategy, and the maximal network capacity $R_{\text{c}}^{\max}$ appears when using the shortest path protocol.

This is also proved by numerical simulations. We carry out network traffic simulations on Barabási–Albert (BA) scale-free networks[2] with the resource allocation strategy of Eq. (4). For network size $N = 1000$, the average degree $\langle k \rangle = 8$, and the total resource of $C = N$, the average path length is $\bar{d} \approx 4.50$ for the efficient routing protocol and is $\bar{d} \approx 3.16$ for the shortest path protocol. Using Eq. (7), we can estimate that $R_{\text{c}} \approx 221$ and $R_{\text{c}} \approx 315$ for the efficient routing protocol and the shortest path protocol, respectively. Figure 1 shows the order parameter η vs R in the two routing protocols. We can see that the simulation results of R_{c} are consistent with the analytical results.

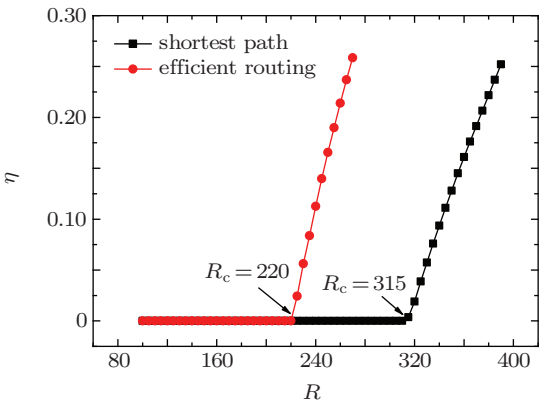


Fig. 1. (color online) Order parameter η vs R with the shortest path protocol and the efficient routing protocol in BA scale-free networks. Other parameters are network size $N = 1000$, average degree $\langle k \rangle = 8$, the total delivery resource is $C = N$ and allocated with Eq. (4).

Figure 2 depicts the network capacities for networks with different sizes. We can see that with the allocation strategy of Eq. (4), in both routing protocols, network capacity R_{c} is larger for different network sizes. Meanwhile, with Eq. (4), the system capacity is larger with the shortest path than that with the efficient path.

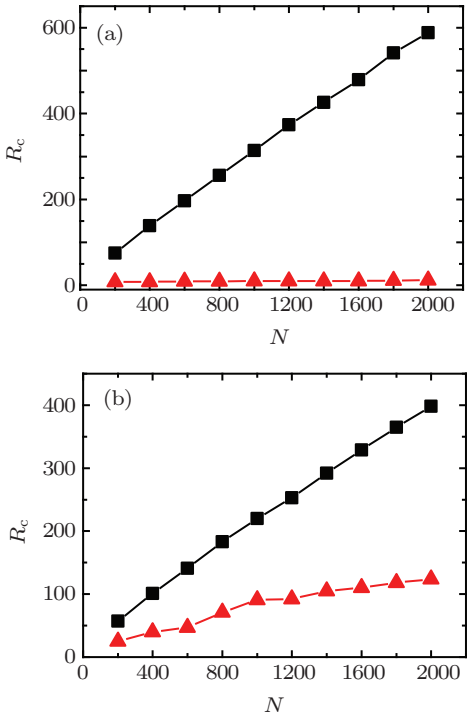


Fig. 2. (color online) The R_{c} vs network size N in (a) the shortest path protocol and (b) the efficient routing protocol for BA scale-free networks with average degree $\langle k \rangle = 8$ and total delivery resource $C = N$. The delivery resource is allocated with Eqs. (3) (▲) and (4) (■), respectively.

We note that our allocation strategy performs better than that of Yang *et al.*[22] Although it is found that the node's algorithmic betweenness scales with its degree;[28] some nodes in the system have the same degree but different algorithmic betweenness. Our preliminary simulations show that for BA networks of $N = 1000$ and $\langle k \rangle = 8$, the system can reach an optimum of $R_{\text{c}} \approx 193$ with Yang *et al.*'s strategy and the shortest path protocol. But with our new allocation strategy, we get a maximum $R_{\text{c}} \approx 315$. Therefore, the present allocation strategy can out perform the previous one.

In Fig. 3, we investigate the traffic capacities in Newman–Watts small-world networks[26] and Erdös–Rényi random graphs.[27] In both types of networks, the network capacity with the allocation strategy of Eq. (4) is larger than that with Eq. (3). Moreover, with the resource allocation strategy of Eq. (4), the network capacity with the shortest path protocol is slightly larger than that with the efficient path protocol. With Eq. (3), the network capacity with the efficient path protocol is larger than that with the shortest path protocol. This is because with Eq. (4), the delivery capacity of nodes is allocated to avoid the collapse in the hub nodes. While with Eq. (3), the efficient routing strategy can route the packets to the periphery of the system to avoid collapse in the hub nodes.

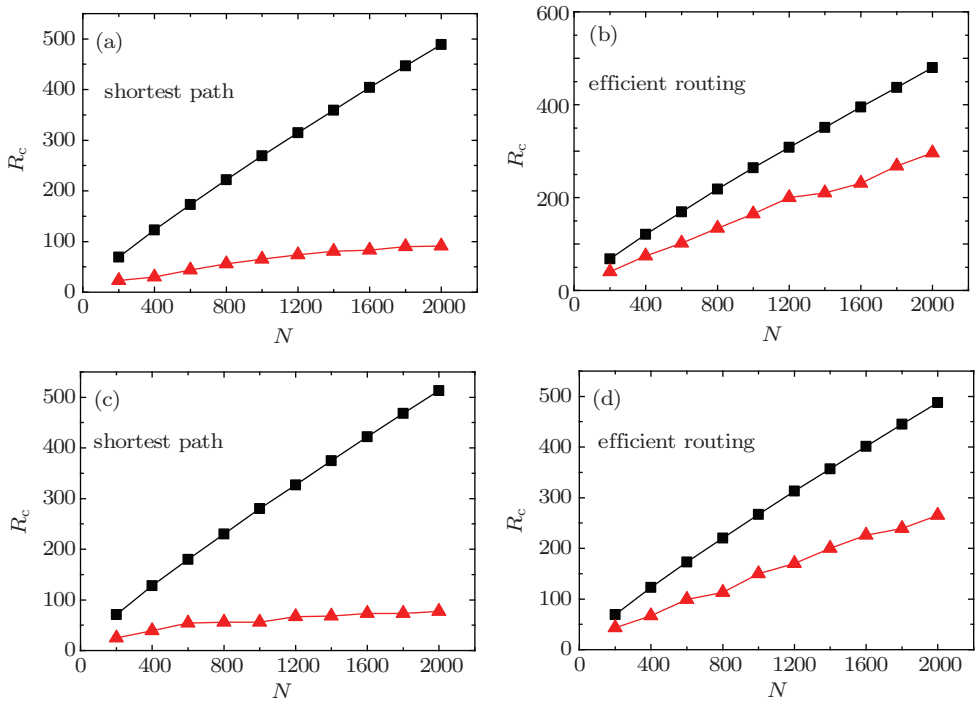


Fig. 3. (color online) The R_c vs network size N in the shortest path protocol and the efficient routing protocol for ((a), (b)) Newman–Watts small world networks and ((c), (d)) Erdös–Rényi random networks with average degree $\langle k\rangle = 8$ and $C = N$. The delivery resource is allocated with Eqs. (3) (▲) and (4) (■), respectively.

3.2. Optimal allocation of a link's bandwidth

Next we extend our study to investigate the optimal resource allocation strategy for a link's bandwidth. Bandwidth D_{ij} is the maximal number of packets that can be forwarded from node i to node j by link l_{ij} per time step. We denote D as the total resource of bandwidth. If all links have the same bandwidth $D_{ij} = 1$, the total bandwidth resource is $D = \sum_{i=1,j=1,i\neq j}^{N} A_{ij} \cdot 1 = \langle k\rangle N$, where A_{ij} is the element of adjacency matrix, and $\langle k\rangle$ is the average node degree of the network. For simplicity, the total bandwidth resource is set to $D = \langle k\rangle N$.

We also consider two kinds of bandwidth allocation strategies. The first strategy allocates the bandwidth resource evenly among the links

$$D_{ij} = \frac{D}{\sum_{i=1,j=1,i\neq j}^{N} A_{ij}} = 1. \tag{8}$$

The second strategy allocates the bandwidth resource according to the algorithmic betweenness of the links

$$D_{ij} = \frac{B_{ij}}{\sum_{i=1,j=1,i\neq j}^{N-1} A_{ij}B_{ij}} D, \tag{9}$$

where B_{ij} is the algorithmic betweenness of link l_{ij}, which is defined as the number of routing paths passing through link l_{ij}.

The node's delivery capacity is $C_i = \sum_{j=1}^{k} D_{ij}$, where the sum runs over all its neighbors. And the node's algorithmic betweenness is $B_i = \sum_{j=1}^{k} B_{ij}$. So the efficient betweenness of node i is

$$B_i^{\text{eff}} = \frac{B_i}{C_i} = \frac{\sum_{j=1}^{k} B_{ij}}{\sum_{j=1}^{k} D_{ij}}. \tag{10}$$

With the bandwidth allocation strategy of Eq. (9), the node's efficient betweenness is

$$B_i^{\text{eff}} = \frac{\sum_{j=1}^{k} B_{ij}}{\sum_{j=1}^{k} D_{ij}} = \frac{\sum_{i=1,j=1,i\neq j}^{N} A_{ij}B_{ij}}{D}. \tag{11}$$

Since $\sum_{i=1}^{N} B_i = \sum_{i=1,j=1,i\neq j}^{N} A_{ij}B_{ij}$, we can get $\sum_{i=1,j=1,i\neq j}^{N} A_{ij}B_{ij} = N(N-1)\bar{d}$, where $\bar{d}$ is the average path length of the network. Hence, we get

$$B_i^{\text{eff}} = \frac{N(N-1)\bar{d}}{D} = \text{const.} \tag{12}$$

This equation indicates that all nodes in the system have the same efficient betweenness. And the value is proportional to the average path length of the network. With this bandwidth allocation strategy, $B_{\max}^{\text{eff}}$ can reach the minimum. By using a similar analysis, R_c can be estimated as in the previous discussion

$$R_c = \frac{D}{\bar{d}}. \tag{13}$$

Thus we can see that with a finite bandwidth resource, Eq. (9) gives an optimal allocation strategy for all routing protocols. We also find that the network capacity can be largest when the shortest path protocol is adopted because it has the shortest average path length $\bar{d}$.

In BA scale-free networks with size $N = 1000$, average degree $\langle k\rangle = 8$, and $D = \sum_{i=1,j=1,i\neq j}^{N} A_{ij} = 8000$, the average path length is $\bar{d} = 3.16$ for the shortest path routing strategy, and $\bar{d} = 4.50$ for the efficient routing strategy. When the bandwidth allocation follows Eq. (9), we can get $R_c \approx 1775$ for the

efficient routing strategy and $R_c \approx 2526$ for the shortest path strategy. Figure 4 shows the simulation results of order parameter η vs R for BA model networks under the two routing protocols. We can see that the simulation results are basically consistent with the analytical ones.

In Fig. 5, we study the network capacities of BA, NW, and ER networks of different sizes with the two bandwidth allocation strategies. As is shown, with a finite total bandwidth resource D, in both routing protocols, the network capacity with the allocation strategy of Eq. (9) is larger than that with Eq. (8). This result is also consistent with the theoretical analysis.

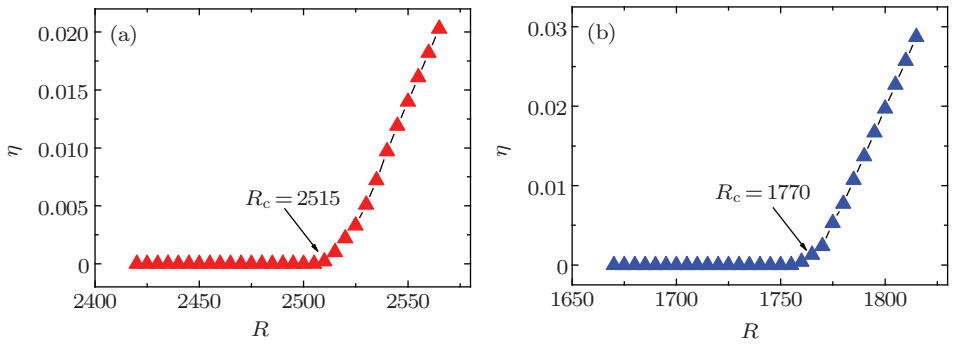


Fig. 4. (color online) Order parameter η vs R in the shortest path protocol and the efficient routing protocol for BA scale-free networks. Other parameters are network size $N = 1000$, average degree $\langle k \rangle = 8$, total bandwidth resource $D = \sum_{i=1,j=1,i\neq j}^{N} A_{ij} = 8000$. The bandwidth resource is allocated according to Eq. (9).

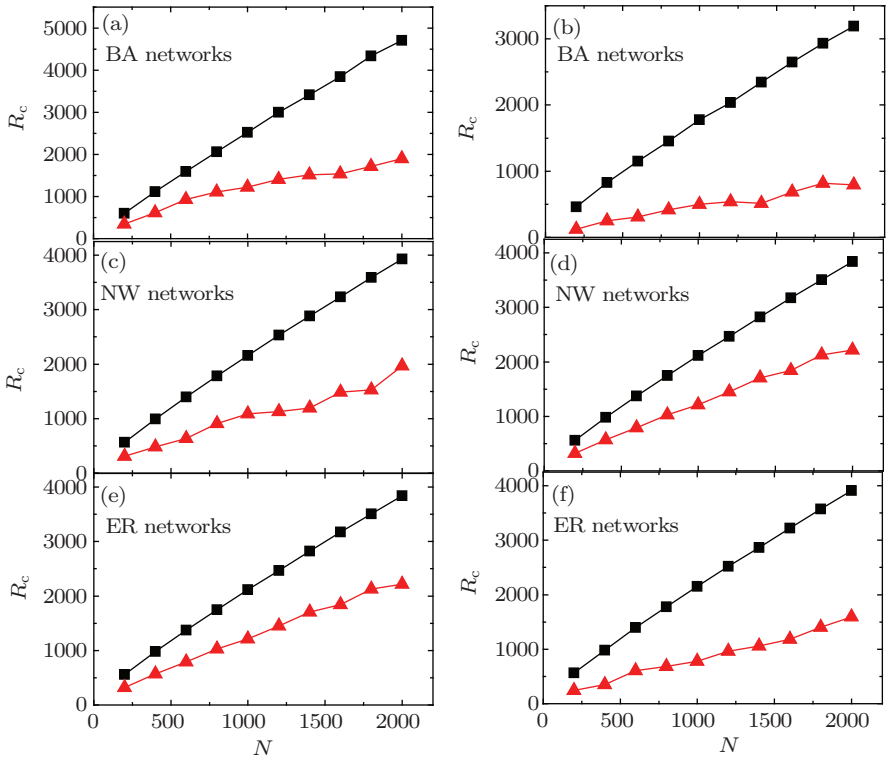


Fig. 5. (color online) The R_c vs network size N in ((a),(c),(e)) the shortest path protocol and ((b),(d),(f)) the efficient routing protocol for BA scale-free networks, NW small-world networks, and ER random graphs with average degree $\langle k \rangle = 8$ and total bandwidth resource $D = \sum_{i=1,j=1,i\neq j}^{N} A_{ij}$. The bandwidth resource is allocated with Eqs. (8) (▲) and (9) (■), respectively.

3.3. Optimal link and node resource allocation

For a comprehensive study, we continue to investigate the allocation strategy for both link and node resources. We consider $C = D = \sum_{i=1,j=1,i\neq j}^{N} A_{ij}$ for BA scale-free networks and use the shortest path protocol. We consider three situations of node and link resources allocation strategies:

(i) Eq. (4) together with Eq. (9);

(ii) Eq. (4) together with Eq. (8); and

(iii) $C_i = \langle k \rangle$ together with Eq. (9).

From Fig. 6, we can see that allocation strategy (i) is the best, as it gives the largest traffic capacity for each network size. We also study these three resource allocation strategies in

Newman–Watts small-world networks and Erdös–Rényi random graphs. Strategy (i) is the best for both the shortest path strategy and the most efficient routing strategy.

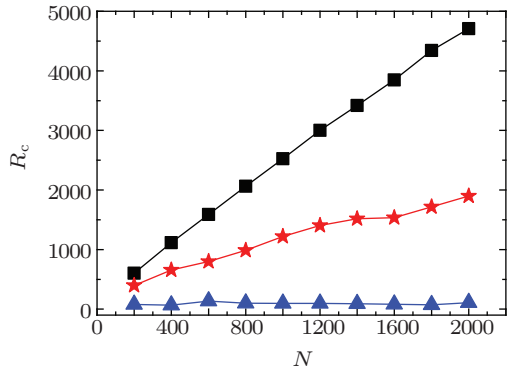


Fig. 6. (color online) The R_c vs network size N in the shortest path protocol for BA networks with average degree $\langle k\rangle = 8$. The allocation strategies for a node's delivery capacity and a link's bandwidth are Eq. (4) with Eq. (9) (■), Eq. (4) with Eq. (8) (★), and $C_i = \langle k\rangle$ with Eq. (9) (▲), respectively.

From the above theoretical analysis and simulation results, we can see that with a finite total resource of a node's delivery capacity C or a link's bandwidth D, the optimal allocation strategy depends on the algorithmic betweenness. With the allocation strategy of Eqs. (4) and (9), the system's traffic capacity can be optimized. When $C = D$, the network's optimal capacity can be estimated by

$$R_c^{\max} = \frac{C}{\bar{d}} = \frac{D}{\bar{d}}. \tag{14}$$

If $C \neq D$, the network's capacity can be estimated by

$$R_c^{\max} = \min\left(\frac{C}{\bar{d}}, \frac{D}{\bar{d}}\right). \tag{15}$$

The system's traffic capacity is reversely proportional to the average path length of the given routing strategy. To best optimize the traffic capacity, we should choose the shortest path routing strategy with the minimum average path length.

4. Conclusion

In summary, we have comprehensively investigated the resource allocation strategy for traffic systems of complex networks. We consider the allocation of node delivery capacity and link bandwith in BA scale-free networks, NW small-world networks, and ER random graphs. It is found that the optimal resource allocation strategy depends on the node's and/or link's algorithmic betweenness. With the optimal resource allocation strategies of Eqs. (4) and (9), the network traffic capacity is reversely proportional to the average path length of a specified routing protocol. Therefore, the network capacity is maximized when the shortest path routing protocol is adopted. Numerical results also confirm that the systems can perform better with the shortest path routing protocol than with the efficient routing protocol.

The main advances of this work are given as follows. (i) An analytical method is derived to estimate the network capacity considering local node/link processing power. (ii) Three cases of local processing capacity allocation are studied: node delivering capacity, link bandwidth, both node capacity, and link bandwidth. Since the previous studies mainly consider the node resource factor and traffic congestion often occurs on links, our study should be of importance for some communication and transportation networks. (iii) When considering node/link capacities, we find that the overall system capacity is reversely proportional to the average path length of a given routing protocol. (iv) Three types of network infrastructures and two types of routing protocols are studied. Since some transportation networks (such as urban traffic networks) are not scale-free, our study is more general, and the results can be of practical importance. Although some previous studies have presented models in which the node capacity is proportional to the betweenness,[29] our work is worthwhile by including the comparison of the performances of shortest path protocol and efficient routing protocol.

This study can be helpful for designing realistic traffic and communication networks, such as urban traffic systems, peer-to-peer networks, and wireless sensor networks.

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