Rank-based deactivation model for networks with age*

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We study the impact of age on network evolution which couples addition of new nodes and deactivation of old ones. During evolution, each node experiences two stages: active and inactive. The transition from the active state to the inactive one is based on the rank of the node. In this paper, we adopt age as a criterion of ranking, and propose two deactivation models that generalize previous research. In model A, the older active node possesses the higher rank, whereas in model B, the younger active node takes the higher rank. We make a comparative study between the two models through the node-degree distribution.

Keywords: complex networks, deactivation model, rank

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1. Introduction

Over the past decade, we have witnessed the emergence of a new science of networks, which has a powerful function of describing structures[1,2] and dynamics[3,4] of many real systems. With nodes representing individuals and edges mimicking interactions among them, systems can be simplified into networks. How to precisely analyze all kinds of realistic networks and then build general models is of high interest. Much attention has been paid to model networks, such as small-world networks,[5,6] scale-free networks,[7,8] community networks.[9] Among those flourishing models, evolving networks are of particular importance. The basic premise is that the network will continue to grow at a constant rate and new nodes attach to old ones with some possibility. When the newly added nodes connect with equal probability to nodes already present in the network, the degree distribution of the nodes of the resulting network is exponential. Whereas for newcomers connecting to old ones with linear preference of the node degree, the stationary distribution of connectivity is the power-law.

In the real world, agents, represented by nodes, always age after growth. For instance, in scientific citation networks there is a half-life effect: old papers are rarely cited since they are no longer sufficiently topical (or they are more often referenced through secondary literature). On the World Wide Web, popular web sites (for example, search engines) often lose favor to old links. To study the effect of this phenomenon on network evolution, the BA model has been modified by incorporating time dependence in the network.[10–27] Dorogovtsev and Mendes studied the case when the connection probability of the new node with an old one is not only proportional to the degree k but also to a power of its present age $\tau^{-\alpha}$ (τ is the age of a node).[10] They found that the network shows scale-free (SF) behavior only in the region $\alpha < 1$. For $\alpha > 1$, the distribution $P(k)$ is exponential. Yet, the gradual aging model shows lower clustering than realistic networks, except for Ref. [15] which introduced the clique neighborhood attachment to aging networks. On the contrary, Klemm and Eguíluz considered evolving networks based on the memory of nodes and proposed a degree-dependent deactivation network model,[16] which is highly clustered and retains the power-law distribution of the degree (without considering the exponential degree distributions).

Previous models of evolving networks always consider the preferential attachment to guarantee scale-free properties. However, the absolute importance (popularity or fitness) of an object is often difficult or impossible for newcomers to measure in social networks. Instead, it is quite common to have a clear knowledge about the relative values of two objects, i.e., who is more popular or richer between two individuals. Motivated by this, Fortunato *et al.* proposed a criterion of network growth that explicitly relies on the ranking of nodes.[28] The rank-driven mechanism generates networks with the scale-free degree distribution when the probability to link a target node is any power-law function of its rank, even when one has only partial information of node ranks.[28,29] Since the perception of how items are ranked requires far less information than their actual importance, the rank-driven mechanism can effectively mimic the reality in many cases where the relative values of agents are easier to access than their absolute values.

In this paper, we integrate rank with deactivation and study their influence on network aging, which generalizes pre-

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vious work.[22] We propose two models for rank-based deactivation networks and simulate the node-degree distributions for both cases. Results show that interesting properties of the generated networks display good features which have been observed in realistic systems.

2. Model

The deactivation model introduced by Klemm and Eguíluz[16] is defined as follows. First, start from an initial network of m completely connected seeds, whose states are all active. The number of incoming edges of a node is denoted by the in-degree k'. At each time step, add a new node n with m outgoing edges that are attached to the previously existing m active nodes. The newcomer is disconnected at this point, so $k'_n = 0$. Each node i of the m active nodes receives exactly one incoming edge, thereby $k'_i \to k'_i + 1$. Then, deactivate one (denoted by j) of the m active nodes has a probability

$$\pi(j) \sim \frac{1}{a + k'_j}, \tag{1}$$

where $a \geq 0$ is a constant bias, and then activate the new node n. In this way, the newly added node is always in the active state first and receives edges from subsequently added nodes until it is deactivated.

In the present work, the transition of a node from the active state to the inactive one is based on its rank, and the deactivation probability can be rewritten as

$$\pi(j) = \frac{\gamma}{a + R_j}, \tag{2}$$

where $a > 0$ is a constant bias factor, $R_j \in [1, m]$ is the rank of node j among the m active nodes, and $\gamma = [\sum_{j \in \Lambda} 1/(a + R_j)]^{-1}$ is a normalization factor. The summation runs over the set Λ of the current m active nodes. The factor γ can also be expressed as a function of m, namely, $\gamma(m) = [\sum_{R=1}^{m} 1/(a + R)]^{-1}$. Note that the larger rank a node possesses, the more difficult it is for it to be deactivated. The choice of prestige measure can be arbitrary, either topological or physical. For simplicity, we sort nodes with age and it is better to discriminate the following two cases. In model A, the older active node possesses the higher rank, i.e., $R_j = \varphi(\tau(j))$. Here $\varphi(\tau)$ is a ranking function, which formulates the older active node corresponding to the larger rank. In model B, the younger active node takes the larger rank, i.e., $R_j = \phi(\tau(j))$. Here $\phi(\tau)$ is also a ranking function, which formulates the younger active node corresponding to the larger rank. In both cases, $\tau(j) = t - t_j$ is the age of the active node j, where t_j represents the time that the node j added to the network. As we shall show below, the two models can be solved in the continuous degree approximation.

3. Degree distribution

Denoting with $p(k', t)$ the probability of the in-degree of active nodes at time t, we can write out the master equation, following ideas of Klemm and Eguíluz[16]

$$p(k' + 1, t + 1) = [1 - \pi(k')] p(k', t), \tag{3}$$

where $\pi(k')$ is the deactivation probability of a node with in-degree k'. For further calculation, we should derive the relation between R and k', which is quite sensitive to the rank of age.

3.1. Model A

In model A, $R_j = \varphi(\tau(j))$, which implies that old nodes have a higher possibility to gain new edges. Figure 1 shows that the relation between R and k' results from Monte–Carlo simulations of the deactivated model. In order to reduce statistical error, the in-degrees of the nodes are calculated as an average. The solid lines are least-squares fits in the form

$$R \simeq [a_0 + a_1 \Gamma(a_2, k'/a_3)]^{-1/\gamma}, \tag{4}$$

where $\Gamma(s, x)$ is the incomplete gamma function. Then one easily obtains

$$\begin{aligned} \pi(k') &\simeq \frac{a_1}{a_3^{a_2}\gamma} k'^{-1+a_2} \exp\left(-\frac{k'}{a_3}\right) \\ &\quad \times \left[a_0 + a_1 \Gamma\left(a_2, \frac{k'}{a_3}\right)\right]^{-1-1/\gamma} \pi(R) \\ &= \lambda(k') \pi(R), \end{aligned} \tag{5}$$

where $\pi(R)$ is the deactivation probability of a node with rank R. Substituting them into Eq. (3), we have

$$\begin{aligned} & p(k' + 1, t + 1) \\ &= [1 - \lambda(k')\pi(R)] p(k', t) \\ &= \left[1 - \frac{\gamma \lambda(k')}{a + [a_0 + a_1 \Gamma(a_2, k'/a_3)]^{-1/\gamma}}\right] p(k', t). \end{aligned} \tag{6}$$

The subsequent thing is just to follow the analytical method in Ref. [16]. Imposing the stationarity condition $p(k', t + 1) = p(k', t)$ yields

$$\begin{aligned} & p(k' + 1) - p(k') \\ &= -\frac{\gamma \lambda(k')}{a + [a_0 + a_1 \Gamma(a_2, k'/a_3)]^{-1/\gamma}} p(k'). \end{aligned} \tag{7}$$

Treating k' as continuous, we have

$$\frac{\mathrm{d}p(k')}{\mathrm{d}k'} = -\frac{\gamma \lambda(k')}{a + [a_0 + a_1 \Gamma(a_2, k'/a_3)]^{-1/\gamma}} p(k'), \tag{8}$$

and accordingly obtain the solution

$$p(k') = c_1 \left\{ a + \left[a_0 + a_1 \Gamma\left(a_2, \frac{k'}{a_3}\right)\right]^{-1/\gamma} \right\}^{-\gamma}, \tag{9}$$

with c_1 being the appropriate normalization constant. If the total number N of nodes in the network is large compared with

the number m of active nodes, the overall in-degree distribution $N(k')$ can be calculated as the rate of the change of the in-degree distribution $p(k')$ of the active nodes, which gives

$$N(k') = -\frac{\mathrm{d}p(k')}{\mathrm{d}k'} = d_1\gamma\lambda(k')\left\{a+\left[a_0+a_1\Gamma\left(a_2,\frac{k'}{a_3}\right)\right]^{-1/\gamma}\right\}^{-\gamma-1}, \quad (10)$$

where d_1 is the normalized factor. When choosing the value of the bias $a=0$, we obtain

$$N(k') = d_1\frac{a_1}{a_3^{a_2}}k'^{-1+a_2}\exp\left(-\frac{k'}{a_3}\right). \quad (11)$$

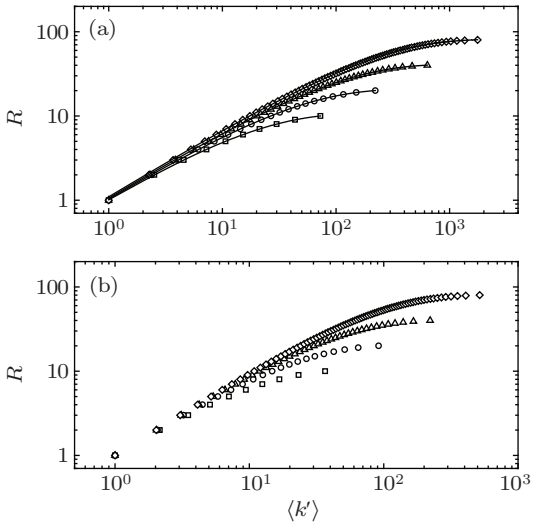


Fig. 1. Illustration of R versus average k' of the generated networks with (a) $a=0$, and (b) $a=m$, for various values of m: $m=10$ (square), 20 (circle), 40 (triangle), and 80 (diamond), respectively. The solid lines are least-squares fits in the form of $R\simeq[a_0+a_1\Gamma(a_2,k'/a_3)]^{-1/\gamma}$. The size of the network is $N=10^5$.

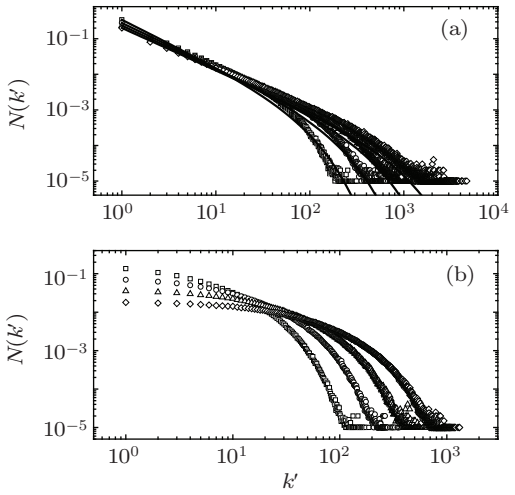


Fig. 2. Log–log plots of the node in-degree distribution of the generated networks for different values of a: (a) $a=0$; (b) $a=m$. The solid lines are the predicted in-degree distribution. The symbols as shown in Fig. 1.

In model A, the rank of a newly added node always takes the lowest value because of $R_i=\varphi(\tau(i))$. At each time step, if the node is active, its rank may increase by 1 with the addition of subsequent nodes, until it reaches the largest m. If a node j, which enters the network after the node i, is deactivated at a subsequent step, the rank of the node i will not increase by 1. In Fig. 2, we show the Monte–Carlo simulations of the in-degree distributions of the resulting networks with different values of a. All the plots display good right-skewed behavior, which is consistent with the empirical study of citation distributions of scientific papers.[26] We notice clear exponential cut-offs. The solid lines are the predicted in-degree distribution. All these predictions have been confirmed by computational simulations.

3.2. Model B

In model B, $R_j=\phi(\tau(j))$, which implies that newcomers have a higher possibility to gain new edges. Figure 3 shows the numerical relation between R and k'. The best polynomial fit gives $R\simeq b_1+b_2k'+b_3k'^2$, and one obtains $\pi(k')\simeq-(b_2+2b_3k')\pi(R)$. Substituting it into Eq. (3) yields

$$p(k'+1,t+1) = [1+(b_2+2b_3k')\pi(R)]p(k',t) = \left[1+\frac{\gamma(b_2+2b_3k')}{a+b_1+b_2k'+b_3k'^2}\right]p(k',t). \quad (12)$$

Similarly, we can obtain the solution in the stationary state

$$p(k') = c_2[a+b_1+b_2k'+b_3k'^2]^{\gamma}, \quad (13)$$

where c_2 is the appropriate normalization constant. Thus, the in-degree distribution of model B takes the form

$$N(k') = -\frac{\mathrm{d}p(k')}{\mathrm{d}k'} = d_2\gamma(b_2+2b_3k')(a+b_1+b_2k'+b_3k'^2)^{\gamma-1}, \quad (14)$$

with d_2 being the normalized factor.

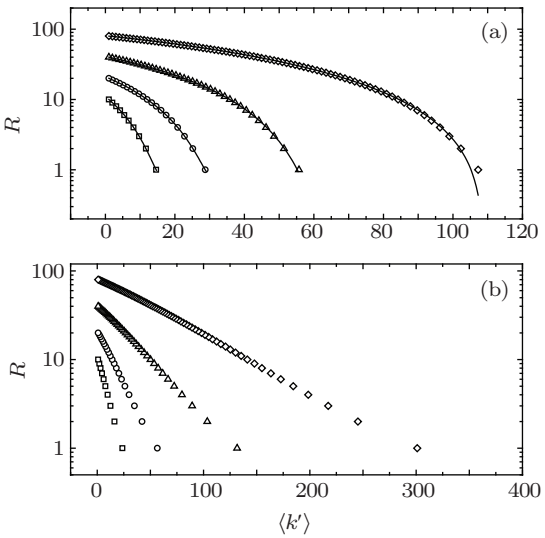


Fig. 3. Illustration of R versus average k' of the generated networks with (a) $a=0$ and (b) $a=m$, for various values of m: $m=10$ (square), 20 (circle), 40 (triangle), and 80 (diamond), respectively. The lines are least-squares fits in the form of $R\simeq b_1+b_2k'+b_3k'^2$. The size of the network is $N=10^5$.

In model B, the rank of the newly added node always takes the largest value because of $R_i = \phi(\tau(i))$. At each time step, if the node is active, its rank may decrease by 1 with the addition of subsequent nodes, until it reaches the smallest value of 1. If a node j, which enters the network after the node i, is deactivated at a subsequent step, the rank of the node i will not decrease by 1 at that point. Then, we can write the probability $q(k_i')$ for active node i of in-degree k' as

$$\begin{aligned}
q(1) &= \frac{\gamma}{m},\\
q(2) &= \left(1-\frac{\gamma}{m}\right)\frac{\gamma}{m-1},\\
q(3) &= \left(1-\frac{\gamma}{m}\right)\left(1-\frac{\gamma}{m-1}\right)\\
&\quad\times\left[\frac{\gamma}{m}\frac{\gamma}{m-1}+\left(1-\frac{\gamma}{m}\right)\frac{\gamma}{m-2}\right],\\
&\vdots\\
q(k_i') &= \overline{F}(k_i'-1)\left[\frac{\gamma}{m}\frac{\gamma}{R_i}+\left(1-\frac{\gamma}{m}\right)\frac{\gamma}{R_{\rm i}-1}\right],\\
q(k_{\rm i}'+1) &= \overline{F}(k_{\rm i}'-1)\left[\left(\frac{\gamma}{m}\right)^2\frac{\gamma}{R_{\rm i}-2}+\left(1-\frac{\gamma}{m}\right)^2\frac{\gamma}{R_{\rm i}}\right.\\
&\quad\left.+2\frac{\gamma}{m}\left(1-\frac{\gamma}{m}\right)\frac{\gamma}{R_{\rm i}-1}\right],\\
&\vdots\\
q(k_{\rm e}) &= \overline{F}(k_{\rm e}-1)\gamma,\\
q(k_{\rm e}+1) &= \overline{F}(k_{\rm e}-1)(1-\gamma)\gamma,\\
&\vdots\\
q(k_{\rm e}+b) &= \overline{F}(k_{\rm e}-1)(1-\gamma)^b\gamma,
\end{aligned} \tag{15}$$

where $\bar{F}(k)$ is the complementary cumulative distribution function, and $k_{\rm e}$ is the in-degree of a node when the node becomes the oldest active one, namely, its rank is equal to 1. Therefore, the in-degree distribution will have an extremum at $k_{\rm e}$. In Fig. 4, we plot the node in-degree distribution of the generated networks for different values of a. The solid lines in Fig. 4(a) are the predicted in-degree distribution, which consists of two parts. In the case of $k' < k_{\rm e}$, we can always obtain $q(k_{\rm i}+1) > q(k_{\rm i})$, and the in-degree distribution $N(k')$ (Eq. (14)) is monotonously increasing. For $k' \geq k_{\rm e}$, the node i becomes the oldest one of all the active nodes and its rank equals 1. Therefore, the in-degree distribution decreases exponentially in the form of $N(k_{\rm e})(1-\gamma)^{(k'-k_{\rm e})}$. This behavior has also been observed by Camacho *et al.* in the study of food webs.[30,31]

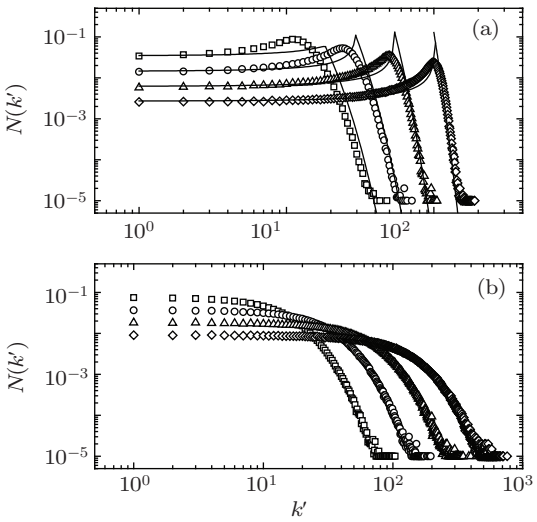


Fig. 4. Log–log plots of the node in-degree distribution of the generated networks for different values of a: (a) $a = 0$; (b) $a = m$. The solid lines are the predicted in-degree distribution, which consists of two parts. The symbols as shown in Fig. 3.

3.3. Limit case of the model

We have investigated the case of finite bias a for both models A and B. Now we consider the case of infinite bias a. In the limit $a \to \infty$, the deactivation probability is no longer dependent on the rank of node and each of the m active nodes will be deactivated with the equal probability $1/m$. Thus, equation (3) can be simplified into

$$p(k'+1,t+1) = \left(1-\frac{1}{m}\right)p(k',t). \tag{16}$$

Again assuming k' as continuous, we have

$$\frac{{\rm d}p(k')}{{\rm d}k'} = -\frac{1}{m}p(k'), \tag{17}$$

in the stationary state. Its solution reads

$$p(k') = c_3\exp\left(-\frac{k'}{m}\right), \tag{18}$$

with c_3 being the appropriate normalization constant. Finally, we obtain the overall in-degree distribution as

$$N(k') = -\frac{{\rm d}p(k')}{{\rm d}k'} = d_3\exp\left(-\frac{k'}{m}\right), \tag{19}$$

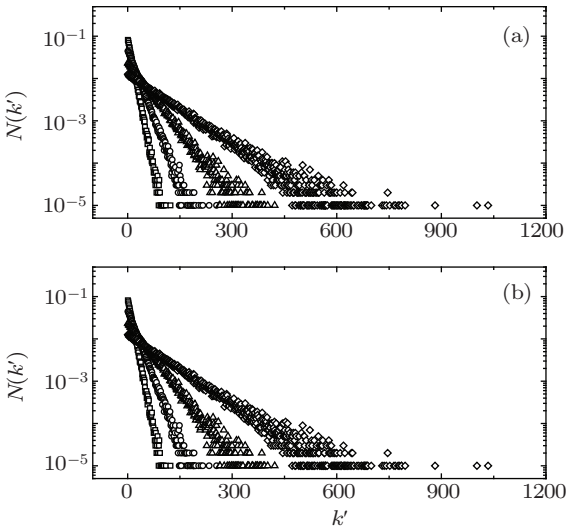


Fig. 5. Illustration of the in-degree distribution for (a) model A and (b) model B with $a = N$. The symbols are the same as those in Fig. 1.

with normalized factor d_3. In Fig. 5, we show the Monte–Carlo computational simulations of the in-degree distribution of both models for $a = N$ as an illustration of the limit. Obviously, all the plots decay exponentially.

4. Conclusion

To summarize, we have considered an alternatively simple and intuitive mechanism to generate networks to model a large and important class of networks with age. The network evolution couples the addition of new active nodes and deactivation of old ones. The deactivating probability of an active node is proportional to the inverse of its rank, which characterizes the individual capability of obtaining further links. We adopt age as a criterion of ranking and provide a comparable analysis of two deactivation models that generalize previous research. In model A, the rank of a node i is defined by $R_j = \varphi(\tau(j))$, whereas for model B, the rank of the node j is defined by $R_j = \phi(\tau(j))$. The node-degree distribution of the generated networks shows rich behavior of connectivity, which depends on the classification of ranking and the bias factor a. For a is finite, the degree distribution follows either a power law (for model A) or a polynomial law (for model B). Both distributions are truncated by adding an exponent. In model A, the older node possesses the larger rank, which implies that old nodes have a high possibility to gain new edges. Statistically old nodes have a long time to acquire edges, and finally the in-degree distribution is the power law based on the cumulative advantage. On the contrary, for model B, the rank of the newly added node always takes the largest value, which implies that newcomers have a high possibility to gain new edges. Hence, old nodes no longer have the cumulative advantage and large degree. This mechanism is responsible for a polynomial in-degree distribution. In the limit $a \to \infty$, we obtain a structured exponential network for both models. These results are consistent with those that have been empirically observed in real-world networks.[26,30,31]

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