A cellular automata traffic flow modeling of desired speed variability

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The satisfaction rate of desired velocity in the case of a mixture of fast and slow vehicles is studied by using a cellular automaton method. It is found that at low density the satisfaction rate depends on the maximal velocity. However, the behavior of the satisfaction rate as a function of the coefficient of variance is independent of the maximal velocity. This is in good agreement with empirical results obtained by Lipshtat [*Phys. Rev.* E **79** 066110 (2009)]. Furthermore, our numerical result demonstrates that at low density the satisfaction rate takes higher values, whereas the coefficient of variance is close to zero. The coefficient of variance increases with increasing density, while the satisfaction rate decreases to zero. Moreover, we have also shown that, at low density the coefficient variance depends strongly on the probability of overtaking.

Keywords: traffic, celluar automata, satisfaction coefficient, desired speed

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1. Introduction

Transport systems have been playing an important role in the development of countries. However, it was only from the second half of the last century that they became a source of problems. Indeed, because of increasing requirements of transport of goods and people, the number of vehicles has seen a significant rise, giving rise to more and more frequent congestion that generates direct and indirect costs. Due to the lack of space, construction of new infrastructures can no longer be the appropriate response to such problems. The only feasible solution in the short and medium term relies on optimizing the use of existing infrastructures by designing techniques for dynamic management of traffic flows. Nevertheless, the design and implementation of such techniques require a good understanding of the phenomena which govern traffic. From this perspective, a variety of models have been proposed.

The task of traffic modeling is to simplify complex phenomena on the roads. Several types of models have been proposed. They are distinguished by the finesse at which they can reproduce the characteristics of the flow of vehicles.[1] We can classify most of the models into three major families: the microscopic, mesoscopic, and macroscopic models.

The macroscopic traffic models represent the average characteristics of the flow, which consists of vehicles each having a stochastic (or random) characteristic. These models are based on a fluid-dynamical description.

Mesoscopic models represent a compromise between the accuracy of microscopic models and the computational efficiency of macroscopic ones. These models are often used when real-time simulation with a high level of detail is needed.[2]

Microscopic models consider individual vehicles and human driving behaviors. Indeed, the movement of individual vehicles is governed by the drivers? behaviors, the road topology, and the headway distribution. Each vehicle may be described by a set of parameters, including position, actual speed, desired speed, and route choice.

Many of the microscopic approaches developed in recent years are formulated by using cellular automata (CA), which has emerged in the last few years as a very promising alternative to existing traffic flow models.[3–6] Recently, several studies have been introduced to study new traffic situations using cellular automata.[7–11]

Cellular automata models quantify a complex behavior into simple individual components. Indeed, the freeway is split into homogeneous cells of equal length, and time is divided into time steps of equal duration. These cells can be either empty or occupied by at most one vehicle.

The Nagel and Schreckenberg (NaSch) model is one of the most popular CA[12]. This model was the first one to take into account human behavior to explain spontaneous jams. However, additional rules have been added, in order to capture more complex situations. In this context, an extension of this model was proposed by Ez-Zahraouy *et al.*[13] to investigate the effect of the length of a vehicle on traffic flow behaviors. To explore the effect of different maximal velocities, Ez-Zahraouy *et al.*[14] proposed a CA traffic flow model, in which

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the vehicles are filled according to their maximal speed. Using the same rules,[14] Jetto *et al.*[16] extended the NaSch model to treat both mixtures of length and maximal velocities.

Recently, Lipshtat[15] studied the effect of distribution of desired speeds on traffic using a digital simulation Matlab. He also studied the variation related to new traffic parameters, such as satisfaction rate, coefficient of variance, and standard deviation. Our aim in this paper is to make a numerical study based on the cellular automata model proposed by Ez-Zahraouy *et al.*,[14] in order to expand the study by Lipshtat.[15] This paper is organized as follows: Section 2 is devoted to explaining our computational model in detail. In Section 3, we will present the main results obtained through simulation with critical discussion. In the last section, we close with a general conclusion.

2. Model

It is well known that the CA NaSch model is based on four steps which are necessary to reproduce some basic features of real traffic. However, additional rules should be added, in order to capture more complex situations. An interesting extension is, for example, the models in which a mixture of fast and slow vehicles is taken into account. However, the disorder of the maximal velocities leads to platoon phenomena.[17] In order to overcome this situation, fast vehicles must overtake the slow ones. The results obtained by Ez-Zahraouy *et al.*[14] have shown a competition between platoon formation and their dissociation by overtaking. The overtaking rules were described as follows:

C1 Vehicles of type 1 follow those of type 2.

C2 The distance between one vehicle of type 1 and one vehicle of type 2, Gap 1, is not greater than the maximal velocities of vehicles of type 2.

C3 The number of empty sites in front of type-2 vehicles is greater than the minimal safety distance required for overtaking.

C4 If the overtaking probability is greater than a random number, then type-1 vehicles overtake with the velocities of min(Gap 1+Gap 2, Vmax1), with Gap 1 being the distance between one vehicle of type 1 and one vehicle of type 2, Gap 2 the number of empty sites in front of type-2 vehicles. To avoid collision with type-2 vehicles in front, Gap1+1+Gap2 is the new gap required for overtaking.

3. Simulation results

In our computational studies, we have considered a chain with 1000 sites, and periodic boundary conditions.

We define a satisfaction rate η as

$$\eta = \left(\frac{v_{\mathrm{a}}}{V_{\max}}\right), \tag{1}$$

where v_{a} is the actual speed, and $V_{\max}$ is the desired velocity.

The coefficient of variation C_v is defined as a measure of relative dispersion. It is calculated by

$$C_v = \frac{\sigma}{V}, \tag{2}$$

where σ is the standard deviation in the form of

$$\sigma = \sqrt{\langle v^2\rangle - \langle v\rangle^2}, \tag{3}$$

and V is the mean velocity expressed by

$$V = \langle v\rangle. \tag{4}$$

3.1. Case of the Nash model

In this subsection, we address the case of the NaSch model. Figure 1 displays the variation of the satisfaction rate according to the coefficient of variance for different maximal velocities ($V_{\max} = 3, 4, 5$). It is clear that the satisfaction rate is independent of the maximum velocities. The same results were obtained by Lipshtat (see Fig. 1(a) in Ref. [15]).

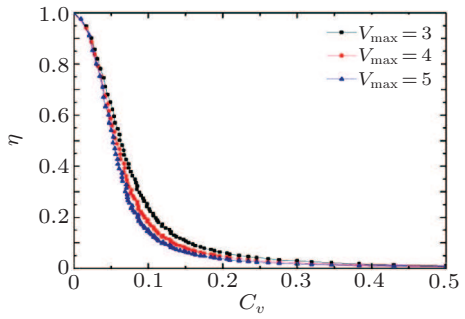


Fig. 1. (color online) Variation of the satisfaction rate with the coefficient of variance for different maximum speeds.

3.2. Study of disorder of the maximum velocity

In fact, the traffic is heterogeneous (i.e., different speeds and different lengths). Hence, the variation of the satisfaction rate versus the coefficient of variance in the case of a mixture of fast ($V_{\max 1} = 5$) and slow vehicles ($V_{\max 2} = 1$) without overtaking is given in Fig. 2. At low density, all slow vehicles are satisfied as they move at their desired speed ($\eta = 1$). However fast vehicles are not satisfied ($\eta < 1$), since they are not authorized to overcome the slow ones. In this range of density, we observe the same values of the coefficient of variance, leading to different values of satisfaction rate. Only if the density increases, both types of vehicles are forced to slow down, which decreases the velocity fluctuation. At high density (the jamming phase), the coefficient of variance vanishes, since the vehicles stop.

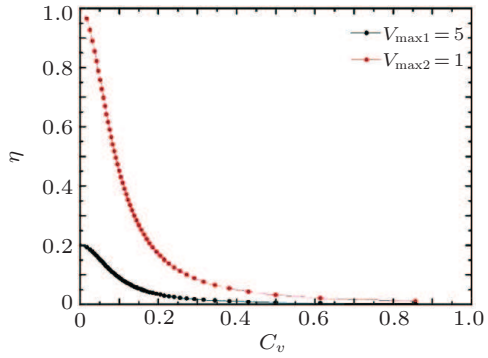


Fig. 2. (color online) Variation of the satisfaction rate with the coefficient of variance for both types of vehicles without overflow, with $V_{max1} = 5$ for type 1 and $V_{max2} = 1$ for type 2.

3.3. Cases of disorder of the maximum velocity with overtaking

As mentioned in the Introduction, the disorder on the maximal velocities leads to platoon phenomena. To be closer to the reality, the fast vehicles should be allowed to overtake the slow ones. To simulate such situation, we use the model proposed by Ez-Zahraouy *et al.*,[15] which considers the case of disorder on the maximal velocities. In the following, the fraction of fast vehicles is 0.75, and the fraction of slow ones is 0.25. Figure 3 shows that both types of vehicles are satisfied at low density $\rho < 0.2$ (free flow phase). Indeed, fast vehicles overtake the slow ones without hesitation the since there is enough empty space; the slow vehicles move at their desired speed without any hindrance. When the density increases ($0.2 < \rho < 0.3$), the overtaking becomes more difficult due to the lack of empty space. Hence, η falls from 1 to 0.2, i.e., 80% of the drivers of fast vehicles are not satisfied, whereas all drivers of slow vehicles are still satisfied ($\eta = 1$). After that the fast vehicles are forced to follow the movement of slow ones (platoon phase, $0.3 < \rho < 0.5$). When the density reaches the critical value $\rho = 0.5$, the drivers of slow vehicles are not satisfied either. Hence, η starts to decrease until it vanishes when the density reaches its maximum value (Jamming phase).

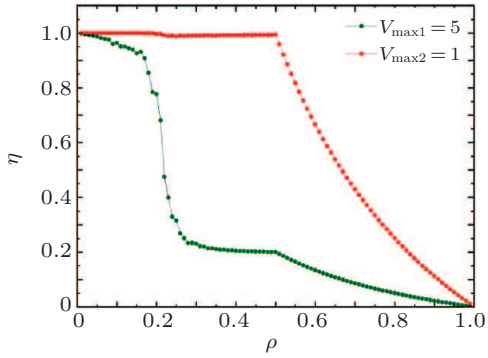


Fig. 3. (color online) Variation of the satisfaction rate with the density for the mixture of two velocities.

Figure 4 presents the variation of the standard deviation as a function of the density for two types of vehicles in the case of overtaking. Indeed, at low density, the standard deviation (σ) of the two kinds of vehicles is null, hence each type of vehicle is moving on average with their maximum speed. From a given density, the standard deviation (σ) of both types of vehicles increases until it reaches its first maximum which is equal to $\sigma_1 = 1.75$ for fast vehicles and $\sigma_2 = 0.10$ for slow ones. After that, the values of the standard deviation of type 1 fall continuously with density ρ. Indeed, the situation of overtaking becomes more difficult as long as ρ increases (reduction in empty spaces), and thus not all fast vehicles can overtake. They are in their congested phase, whereas the slow ones are still in their free flow regime. From the critical density of slow vehicles ($\rho = 0.5$), the standard deviation (σ) of both velocities undergoes an increase with similar behavior to reach a second maximum value ($\sigma = 0.50$). In this case, both types of vehicles run with the same average velocity (the platoon phase). Starting from a higher density ($\rho = 0.75$), the standard deviations of both types decrease as they move with a low average velocity, which tends to zero. Both types are in the jamming phase.

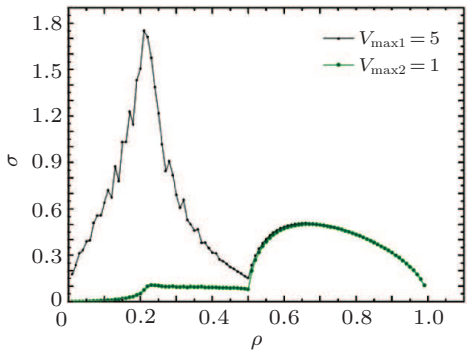


Fig. 4. (color online) Variation of the standard deviation with the density for two types of vehicles in the case of overtaking.

Figure 5 gives the coefficient of variance as a function of the density in the case of a mixture of the two maximum velocities ($V_{max1} = 5$, $V_{max2} = 1$) with overtaking. Indeed, we present the coefficient of variance depending on the density, in other words, the CV as a function of time (each density corresponds to a period of time). This behavior is similar to what is found empirically by Lipshtat (see Fig. 5(a) in Ref. [15]).

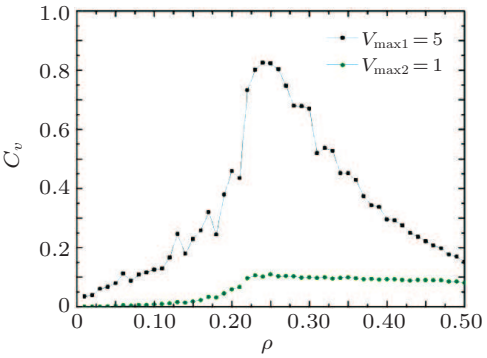


Fig. 5. (color online) Variation of coefficient of variance with the density for two types of vehicles in the case of overtaking ($P_s = 1$).

Figure 6 focuses on the variation of satisfaction rate and the coefficient of variance versus the density in the case of a mixture of fast and slow vehicles, with overtaking. In fact, at low density (free flow phase), as there is enough empty space, all vehicles are satisfied until the critical density, at which the satisfaction rate begins to decrease (a proportion of fast vehicles cannot overtake the slow ones).

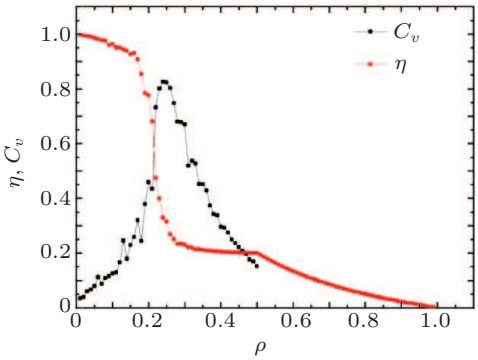


Fig. 6. (color online) Variation of satisfaction rate and the coefficient of variance with the density for the fast type ($V_{\max 1} = 5$) in the case of a mixture with slow ones.

However, in the free flow phase, the coefficient of variance is null since all the vehicles run with their desired speed. Thus, in this case, the maximum velocity is equal to the mean velocity. Nevertheless, when the fast vehicles cannot easily overtake the slow ones, the coefficient of variance undergoes a significant increase until reaching a maximum value. After that, the coefficient of variance begins to decrease due to the platoon phase density.

4. Conclusion

We have studied the satisfaction rate of desired velocity in the case of a mixture of fast and slow vehicles using a cellular automaton method. We have shown, using parallel dynamics update, that on the one hand the satisfaction rate depends on the maximal velocity and the probability of overtaking. On the other hand, the coefficient variance increases when the total density of vehicles on the road increases. Our results are in good agreement with those obtained empirically by Lipshtat.[15]

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