Consensus protocol for heterogeneous multi-agent systems: A Markov chain approach*

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This paper deals with the consensus problem for heterogeneous multi-agent systems. Different from most existing consensus protocols, we consider the consensus seeking of two types of agents, namely, active agents and passive agents. The objective is to directly control the active agents such that the states of all the agents would achieve consensus. In order to obtain a computational approach, we subtly introduce an appropriate Markov chain to cast the heterogeneous systems into a unified framework. Such a framework is helpful for tackling the constraints from passive agents. Furthermore, a sufficient and necessary condition is established to guarantee the consensus in heterogeneous multi-agent systems. Finally, simulation results are provided to verify the theoretical analysis and the effectiveness of the proposed protocol.

Keywords: heterogeneous multi-agent system, consensus protocol, Markov chain

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1. Introduction

The consensus problem for multi-agent systems concerns how a team of agents can reach agreement on a common value by negotiating with their neighbors.[1] It arises in civilian and military applications including air traffic control, sensor networks, mobile robots, social networks, and so on. Recently, it has received considerable research interest from different areas such as communications, control, and mechanics.[2–14]

The theoretical framework for posing and solving consensus problems was introduced by Olfati-Saber and Murray in Ref. [1], where a consensus problem was studied under strongly connected fixed topology. The results were then extended to more general topologies with a spanning tree[2] and even disconnected ones.[9] Recently, there have been many works with close ties to consensus problems, to name a few, formation control, flocking, and so on.[3–7,11,12] For example, Lin[3] considered the formation control of kinematic unicycles. Stabilization of point formation, line formation and any geometric formation were explored. Luo *et al.*[7] investigated the leader-following formation control of mobile agents by using distributed observer techniques. Furthermore, communication delay between agents was considered. Chen and Zhang[11] studied the circular motion control of nonholonomic vehicles. It was shown that collective circular motion can be formed provided that the topology is jointly connected. In Ref. [12], Li *et al.* proposed a distributed observer-type consensus protocol under fixed topology, which put the consensus problem and synchronization of complex networks in a unified framework.

In all the aforementioned references, the agent dynamics is the same for all agents, which means that all the agents have the same capability. They are considered as homogeneous multi-agent systems. However, in the real world, multi-agent systems most likely consist of different types of agents with different capabilities.[13] In Ref. [15], Yang *et al.* considered the consensus problem of mobile agents with different communication capabilities. Simulation results showed that the communication capability heterogeneity do benefit the consensus. Yan *et al.*[8] studied the cooperative pursuit problem of multiple agents by using a frequency domain approach. Sufficient condition to guarantee the stability of the multi-agent system was given. However, it is hard to check the stability conditions for practical multi-agent systems and the conditions are not intuitional. In Ref. [16], we proposed a distributed estimator based on consensus protocols for heterogeneous sensor networks by taking the computational heterogeneity of two types of sensors into account.

Motivated by some previous works,[8,13,15,16] we consider in this paper a consensus problem of multi-agent systems with two types of agents, namely, active agents and passive agents. Only the active agents can be directly incented to control action, while the states of the passive agents are determined by that of the active agents directly or indirectly. As a result, the dynamics of these two types of agents are significantly different, which cannot be treated in a unified framework as in homogeneous multi-agent systems.[2–14] It makes protocol

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design quite difficult since the constraints from passive agents confine the controllability of active agents and thus the achievability of consensus. To the best of our knowledge, few results on the theoretical analysis of this kind of heterogeneous multi-agent system have been reported in the literature. How to deal with the heterogeneity of multi-agent systems remains largely unexplored. The objective of this paper is to establish sufficient and necessary conditions for the consensus of heterogeneous multi-agent systems.

2. Problem statement and consensus protocol

We consider a heterogeneous multi-agent system composed of two types of agents: active agents are the directly controlled agents and passive agents are the uncontrolled ones, whose states are determined by the active agents. We assume that all the agents have the same interaction range r, i.e., both active and passive agents can interact with those lying in its range of r. We denote the set of active and passive agents as $\mathscr{I}$ and $\mathscr{I}^{\rm c}$, respectively. It is convenient to model the interaction network of all agents as an undirected graph $\mathscr{G}=(\mathscr{V},\mathscr{E})$, where $\mathscr{V}=\mathscr{I}\cup\mathscr{I}^{\rm c}=\{1,2,\ldots,N\}$ is the vertex set consisting of all agents, N is the total number of agents, and $\mathscr{E}\subset\mathscr{V}\times\mathscr{V}$ is the edge set of communication links between neighboring agents. An edge of $\mathscr{E}$ is denoted by (i,j), which means that the information can be exchanged between agents i and j. Thus, the neighbors $\mathscr{N}_i$ of agent i can be expressed as $\mathscr{N}_i=\{j\in\mathscr{V}:(i,j)\in\mathscr{E}\}=\{j\in\mathscr{V}:d_{ij}\le r\}$, where d_{ij} is the distance between agents i and j. Furthermore, we denote the active and passive neighbors of agent i as $\mathscr{N}_i^{\rm a}=\mathscr{N}_i\cap\mathscr{I}$ and $\mathscr{N}_i^{\rm p}=\mathscr{N}_i\cap\mathscr{I}^{\rm c}$, respectively. A vertex i is said to be reachable from vertex j, if there exists a sequence of edges of the form $(i,i_1),(i_1,i_2),\ldots,(i_k,j)$. The vertex that is reachable from every other vertex of $\mathscr{G}$ is called a globally reachable vertex. Graph $\mathscr{G}$ is connected if any two vertices can be mutually reachable.

In this paper, we consider the consensus control of all the agents by only directly controlling the active agents. For an active agent, the dynamics is described by a single integrator of the form

$$\dot{x}_i(t)=u_i,\ (i\in\mathscr{I}), \tag{1}$$

where $x_i\in\Re$ is the state of agent i and $u_i\in\Re$ is the control input; while for each passive agent, its state is determined by that of its neighbors, i.e.,

$$x_i(t)=\sum_{j\in\mathscr{N}_i}\gamma_{ij}x_j(t),\ (i\in\mathscr{I}^{\rm c}), \tag{2}$$

where γ_{ij} is the nonnegative weight satisfying $\sum_{j\in\mathscr{N}_i}\gamma_{ij}=1$.

In this paper, our goal is to directly control only the active agents such that all the agents could achieve consensus, that is, $x_i-x_j\to 0$, for $i,j\in\mathscr{V}$. In order to enable the agents to reach an agreement with respect to the states of N agents, we choose the control law for active agent i as

$$u_i=\sum_{j\in\mathscr{N}_i}a_{ij}(x_j(t)-x_i(t)), \tag{3}$$

where a_{ij} is associated with the communication channel. Note that if there are no passive agents, then Eq. (1) coupled with Eq. (3) is the so-called consensus protocol widely used for multi-agent systems in Refs. [1]–[4].

Remark 1 As can be seen from Eqs. (1) and (2), the behaviors of active and passive agents are significantly different. The coexistence of two types of agents brings a challenge for the theoretical analysis, since equations (1) and (2) cannot be dealt with in a uniform framework as those in Refs. [1]–[5] and [7]. This is the distinct feature between our proposed consensus protocol and the existing ones.[1–5,7]

3. Analysis of consensus protocol

3.1. Markov chain based unified framework

Without loss of generality, we label the active agents in $\mathscr{I}$ from 1 to m. In order to tackle the difficulty originating from the heterogeneity of active and passive agents and get a computable approach, we will adopt the Markov chain approach following the next two steps to define a Markov chain $\mathscr{M}$.

1) Let the state x_i of passive agent i correspond to the state of $\mathscr{M}$ for all $i\in\mathscr{I}^{\rm c}$, and introduce an additional state z_0. Denote the set $\{x_i,i\in\mathscr{I}^{\rm c}\}$ by $\{x_{s_1},x_{s_2},\ldots,x_{s_{N-m}}\}$. Then the state z_k of $\mathscr{M}$ can be expressed as

$$z_k=\begin{cases}z_0, & (k=0),\\ x_{s_k}, & (k=1,2,\ldots,N-m).\end{cases}$$

2) In order to define p_{ij} from state z_j to state z_i, we consider two cases: i) $i\neq 0$, and ii) $i=0$. For case i), let

$$p_{ij}=\begin{cases}0, & (i=j),\\ 0, & (i\neq j,\ i\notin\mathscr{N}_j^{\rm p}),\\ \gamma_{ij}, & (i\neq j,\ i\in\mathscr{N}_j^{\rm p}),\end{cases}$$

while for case ii), we set p_{ij} to be

$$p_{0j}=\begin{cases}1, & (j=0),\\ 1-\sum\limits_{k\in\mathscr{N}_j^{\rm p}}\gamma_{kj}, & (j\neq 0).\end{cases}$$

Then, for every state z_j, we have $\sum\limits_{k=0}^{N-m}p_{kj}=1$.

Remark 2 The introduction of Markov chain $\mathscr{M}$ aims to present a computational treatment of the algebraic constraint (2). Without the transformation of $\mathscr{M}$, we need to compute a complex matrix inverse to achieve consensus. Moreover, it is sometimes impossible due to the information locality of each passive agent.

As for Markov chain $\mathscr{M}$, we can obtain the following important result.

Lemma 1 If graph $\mathscr{G}$ is connected and the weights satisfy $0 < \gamma_{kj} \leq 1$, $\forall k, j = m, m+1, \ldots, N$, then $\boldsymbol{I} - \boldsymbol{\Upsilon}$ is a nonsingular matrix, where $\boldsymbol{\Upsilon} = [\Upsilon_{ij}] \in \Re^{(N-m)\times(N-m)}$ with elements

$$\Upsilon_{ij} = \begin{cases} 0, & (j = i), \\ \gamma_{(m+i)(m+j)}, & (j \neq i). \end{cases} \tag{4}$$

Proof It suffices to show that $\rho(\Upsilon) < 1$, where $\rho(\cdot)$ is the spectral radius of a matrix.

Based on the construction of Markov chain $\mathscr{M}$, the transition probability matrix $\boldsymbol{P} = [p_{ij}] \in \Re^{(N-m+1)\times(N-m+1)}$ can be expressed as

$$\boldsymbol{P} = \begin{bmatrix} 1 & \boldsymbol{p}_0^{\mathrm{T}} \\ \boldsymbol{0} & \boldsymbol{\Upsilon} \end{bmatrix}, \tag{5}$$

where $\boldsymbol{p}_0 = [p_{0(m+1)}, \ldots, p_{0N}]^{\mathrm{T}}$ and $\boldsymbol{0}$ is a zero matrix with appropriate dimensions.

We claim that z_0 is a globally reachable state, namely, z_0 can be reached from all the other states in $\mathscr{M}$ in finite steps with positive probabilities.

We prove the claim by contradiction. Suppose on the contrary that there exist two sets $\mathscr{V}_1$ and $\mathscr{V}_2$ such that the states of $\mathscr{V}_1$ can be reached from z_0 and that of $\mathscr{V}_2$ are not reachable from z_0. Then for each state $z_{k_1} \in \mathscr{V}_1$, it is not reachable from any state in $\mathscr{V}_2$ with probability 1. If this is not the case, then z_0 can be reached from some state $z_{k_2} \in \mathscr{V}_2$ with nonzero probability, which is a contradiction to the definition of $\mathscr{V}_2$. Since graph $\mathscr{G}$ is undirected, this means that $\gamma_{ij} \neq 0$ if and only if $\gamma_{ji} \neq 0$. It thus follows that for any two states $z_{k_1} \in \mathscr{V}_1$ and $z_{k_2} \in \mathscr{V}_2$, they are not reachable from each other with nonzero probability. As a result, there exists a permutation matrix $\mathscr{P}$ such that $\boldsymbol{P}$ in Eq. (5) has the form

$$\mathscr{P}^{\mathrm{T}} P \mathscr{P} = \begin{bmatrix} 1 & \tilde{p}_0^{\mathrm{T}} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\Upsilon}_{\mathscr{V}_1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{\Upsilon}_{\mathscr{V}_2} \end{bmatrix},$$

where $\tilde{\boldsymbol{p}}_0$ is the stacked vector of p_{0i}, for all $z_i \in \mathscr{V}_1$.

By the assumption, graph $\mathscr{G}$ is connected, then every two vertices $k \in \mathscr{I}^{\mathrm{c}}$ and $i \in \mathscr{I}$ are reachable mutually, where k is the vertex associated with the state $z_k \in \mathscr{V}_2$. Noting that as the states of Markov chain $\mathscr{M}$, $z_k \in \mathscr{V}_2$ is not reachable from any state in $\mathscr{V}_1$. In consequence, there must be some vertex $i' \in \mathscr{I}$ and vertex $k' \in \mathscr{I}^{\mathrm{c}}$ such that i' and k' are adjacent in graph $\mathscr{G}$, i.e., $\gamma_{ki'} \neq 0$. According to the definition of $\mathscr{V}_2$ and the one-step transition probability of $\mathscr{M}$, for state $z_{k'} \in \mathscr{V}_2$, we have $p_{0k'} = 0$, i.e., $\sum_{j \in \mathscr{N}_{k'}^{\mathrm{p}}} \gamma_{jk'} = 1$. This is a contradiction.

Based on the above discussion, z_0 is a globally reachable state. Then it follows Theorem 2.1 in Ref. [2] that there is only one closed strong component in the Markov chain $\mathscr{M}$. Furthermore, since $\boldsymbol{P}$ is a stochastic matrix, we can show that $\rho(\boldsymbol{\Upsilon}) < 1$ according to Theorem 2.7 in Ref. [2]. The proof is thus complete.

Stacking x_i ($i \in \mathscr{V}$) into vectors $\boldsymbol{x}_{\mathrm{a}} = [x_1, x_2, \ldots, x_m]^{\mathrm{T}}$ and $\boldsymbol{x}_{\mathrm{p}} = [x_{m+1}, x_{m+2}, \ldots, x_N]^{\mathrm{T}}$, respectively, then using Lemma 1, we can obtain from Eq. (2) that

$$\boldsymbol{x}_{\mathrm{p}}(t) = (I - \boldsymbol{\Upsilon})^{-1} \tilde{\boldsymbol{\Upsilon}} x_{\mathrm{a}}(t), \tag{6}$$

where $\tilde{\boldsymbol{\Upsilon}} = [\tilde{\Upsilon}_{ij}] \in \Re^{(N-m)\times m}$ with elements

$$\tilde{\Upsilon}_{ij} = \begin{cases} 0, & (j \notin \mathscr{N}_i), \\ \gamma_{(m+i)j}, & (j \in \mathscr{N}_i). \end{cases} \tag{7}$$

Since $\mathscr{N}_i = \mathscr{N}_i^{\mathrm{a}} \cup \mathscr{N}_i^{\mathrm{p}}$, it then follows from Eqs. (1) and (3) that

$$\dot{x}_i(t) = \sum_{j \in \mathscr{N}_i^{\mathrm{a}}} a_{ij}(x_j(t) - x_i(t)) + \sum_{j \in \mathscr{N}_i^{\mathrm{p}}} a_{ij}(x_j(t) - x_i(t)). \tag{8}$$

Writing Eq. (8) in the vector form yields

$$\dot{\boldsymbol{x}}_{\mathrm{a}}(t) = -(\boldsymbol{L}^{\mathrm{a}} + \boldsymbol{D})\boldsymbol{x}_{\mathrm{a}}(t) + \tilde{\boldsymbol{A}}\boldsymbol{x}_{\mathrm{p}}(t), \tag{9}$$

where $\boldsymbol{L}_{\mathrm{a}} = [l_{ij}^{\mathrm{a}}] \in \Re^{m\times m}$ is the Laplacian matrix corresponding to the subgraph $\mathscr{G}_{\mathrm{sub}}$ induced by all active agents defined by

$$l_{ij}^{\mathrm{a}} = \begin{cases} \sum_{j \in \mathscr{N}_i^{\mathrm{a}}} a_{ij}, & (j = i), \\ -a_{ij}, & (j \neq i), \end{cases}$$

and

$$\boldsymbol{D} = \begin{bmatrix} \sum\limits_{j \in \mathscr{N}_1^{\mathrm{p}}} a_{1j} & & \\ & \ddots & \\ & & \sum\limits_{j \in \mathscr{N}_m^{\mathrm{p}}} a_{mj} \end{bmatrix}, \quad \tilde{\boldsymbol{A}} = \begin{bmatrix} a_{1(m+1)} & \cdots & a_{1N} \\ \vdots & \ddots & \vdots \\ a_{m(m+1)} & \cdots & a_{mN} \end{bmatrix}. \tag{10}$$

Substituting Eq. (6) into Eq.(9), we obtain

$$\dot{\boldsymbol{x}}_{\mathrm{a}}(t) = -\big(\boldsymbol{L}_{\mathrm{a}} + \boldsymbol{D} - \tilde{\boldsymbol{A}}(\boldsymbol{I} - \boldsymbol{\Upsilon})^{-1}\tilde{\boldsymbol{\Upsilon}}\big)x_{\mathrm{a}}(t). \tag{11}$$

Remark 3 Although graph $\mathscr{G}$ is undirected, $L_{\mathrm{a}} + D - \tilde{A}(I - \Upsilon)^{-1}\tilde{\Upsilon}$ is generally asymmetric, since there are two types of agents in the network and we do not assume that $a_{ij} = a_{ji}$, $\forall i \in \mathscr{I}$, and $\gamma_{ij} = \gamma_{ji}$, $\forall i \in \mathscr{I}^{\mathrm{c}}$.

3.2. Stability analysis

Now we investigate the convergence property of the transformed consensus protocol (11). We have the following theorem, which establishes a necessary and sufficient condition for consensus of states among all active and passive agents.

Theorem 1 With a time-invariant topology $\mathscr{G}$ and constant weights a_{ij}, $\forall i \in \mathscr{I}$ and γ_{ij}, $\forall i \in \mathscr{I}^{\mathrm{c}}$, under the consensus protocol (1) and (2) with control law (3), all the agents can achieve consensus asymptotically as $t \to \infty$ if and only if graph $\mathscr{G}$ is connected.

Proof of sufficiency Since $\sum_{j \in \mathscr{N}_i} \gamma_{ij} = 1$, for all $i \in \mathscr{I}^{\mathrm{c}}$ by the assumption, it follows from the definition of matrix $\boldsymbol{\Upsilon}$ in Eq. (4) and matrix $\tilde{\boldsymbol{\Upsilon}}$ in Eq. (7) that $(\boldsymbol{\Upsilon} + \tilde{\boldsymbol{\Upsilon}})\boldsymbol{1} = \boldsymbol{1}$, where $\boldsymbol{1}$ is the column vector with all ones. Note that $\mathscr{G}$ is connected. As a result, we obtain from Lemma 1 that $(\boldsymbol{I} - \boldsymbol{\Upsilon})^{-1}\tilde{\boldsymbol{\Upsilon}}\boldsymbol{1} = \boldsymbol{1}$. In

view of the definition of matrices D and $\tilde{A}$, we thus obtain

$$\boldsymbol{D}\mathbf{1} = \tilde{\boldsymbol{A}}(\boldsymbol{I}-\boldsymbol{\Upsilon})^{-1}\tilde{\boldsymbol{\Upsilon}}\mathbf{1}. \tag{12}$$

Since $\boldsymbol{L}_{\rm a}$ is the Laplacian matrix corresponding to the subgraph induced by $\mathscr{I}$ composed of all active agents, one has $\boldsymbol{L}_{\rm a}\mathbf{1} = \mathbf{1}$. This fact together with Eq. (12) implies that

$$\big(\boldsymbol{L}_{\rm a} + \boldsymbol{D} - \tilde{\boldsymbol{A}}(\boldsymbol{I}-\boldsymbol{\Upsilon})^{-1}\tilde{\boldsymbol{\Upsilon}}\big)\mathbf{1} = 0.$$

This means that zero is the eigenvalue of matrix $\boldsymbol{L}_{\rm a} + \boldsymbol{D} - \tilde{\boldsymbol{A}}(\boldsymbol{I}-\boldsymbol{\Upsilon})\tilde{\boldsymbol{\Upsilon}}$ associated with the eigenvector $\mathbf{1}$. By virtue of Gersgorin disc theorem,[17] we know that all the other eigenvalues are located in the open right half plane. In order to finish this proof, we only need to check the algebraic multiplicity of the zero eigenvalue.

Denote $\tilde{\boldsymbol{L}} = \boldsymbol{L}_{\rm a} + \boldsymbol{D} - \tilde{\boldsymbol{A}}(\boldsymbol{I}-\boldsymbol{\Upsilon})\tilde{\boldsymbol{\Upsilon}}$ and the graph associated with $\tilde{\boldsymbol{L}}$ as $\tilde{\mathscr{G}}$. Obviously, $\tilde{\mathscr{G}}$ is a graph based on the set $\mathscr{I}$. Furthermore, since $\boldsymbol{L}_{\rm a}$ is the Laplacian matrix corresponding to the subgraph $\mathscr{G}_{\rm sub}$ induced by $\mathscr{I}$, $\tilde{\mathscr{G}}$ can be regarded as the graph obtained by inserting some additional edges associated with the entries of matrix $\boldsymbol{D} - \tilde{\boldsymbol{A}}(\boldsymbol{I}-\boldsymbol{\Upsilon})\tilde{\boldsymbol{\Upsilon}}$ to subgraph $\mathscr{G}_{\rm sub}$. As a consequence, $\tilde{\mathscr{G}}$ is connected in that $\mathscr{G}$ is connected by the assumption. It thus follows from Theorem 1 in Ref. [1] that rank $(\tilde{\boldsymbol{L}}) = m-1$. This implies that $\tilde{\boldsymbol{L}}$ has exactly one zero eigenvalue.

The solution to Eq. (11) is given by $x_{\rm a}(t) = \exp(-\tilde{\boldsymbol{L}}t)x_{\rm a}(0)$. We know from the properties of matrix $\tilde{\boldsymbol{L}}$ that $\exp(-\tilde{\boldsymbol{L}}t)$ is a stochastic matrix. Therefore, in light of Lemma 3.7 in Ref. [2], all the active agents could achieve consensus asymptotically, that is,

$$x_i(t) \to \sum_{i=1}^{m} \nu_i x_i(0),\ (i = 1,2,\ldots,m),$$

as $t \to \infty$, where $\boldsymbol{\nu} = [\nu_1, \nu_2, \ldots, \nu_m]^{\rm T} \geq 0$ is the eigenvector of $(\exp(-\tilde{\boldsymbol{L}}))^{\rm T}$, which is associated with the one eigenvalue and satisfies $\mathbf{1}^{\rm T}\nu = 1$. Since $(\boldsymbol{I}-\boldsymbol{\Upsilon})^{-1}\tilde{\boldsymbol{\Upsilon}}\mathbf{1} = \mathbf{1}$, it follows from Eq. (6) that for all passive agents, we also have

$$x_i(t) \to \sum_{i=1}^{m} \nu_i x_i(0),\ (i = m+1, m+2, \ldots, N),$$

as $t \to \infty$. Therefore, all the agents could achieve consensus under the consensus protocol (1) and (2) with control law (3).

Proof of necessity If the graph $\mathscr{G}$ is not connected, then there exist at least two separate connected subgraphs $\mathscr{G}_1$ and $\mathscr{G}_2$. For these two subgraphs, from the sufficiency part, we know that the agents in $\mathscr{G}_1$ and $\mathscr{G}_2$ could achieve consensus respectively. However, there is no information exchange between agents in $\mathscr{G}_1$ and $\mathscr{G}_2$. Therefore, the consensus states $\sum_{i\in\mathscr{G}_1} \nu_i^{(1)} x_i(0)$ and $\sum_{i\in\mathscr{G}_2} \nu_i^{(2)} x_i(0)$ are in general different. Global consensus is not expected in this case. This completes the proof.

Remark 4 Theorem 1 claims that the consensus can be achieved with connected interaction topology. It is noted that the convergence rate for the homogeneous multi-agent systems heavily depends on the structure of the topology.[1,3] As for the heterogeneous system in this paper, it is characterized by not only the second smallest eigenvalue of matrix $\tilde{\boldsymbol{L}}$, but also the numbers of active and passive agents.

4. Simulation results

In this section, we conduct the simulation studies to validate the effectiveness of the proposed consensus protocol and the theoretical results in the previous section.

The simulations are performed with a group of 20 agents, consisting of 12 active agents and 8 passive agents, with the topology shown in Fig. 1, where the agents are randomly distributed in the square $[-20,20]\times[-20,20]$.

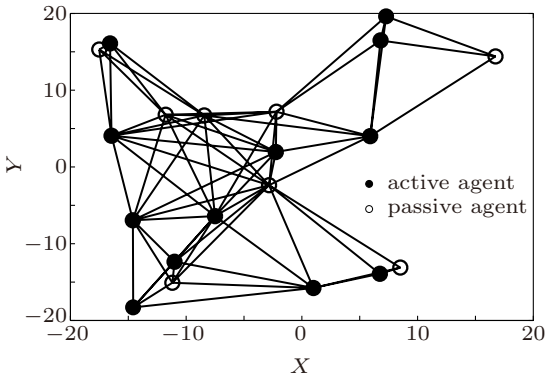


Fig. 1. An example of a heterogeneous topology graph consisting of two types of agents.

To enable the consensus control in Eq. (3), each agent i should transmit its state x_i to its neighbors in $\mathscr{N}_i$ via a wireless channel. It is known that wireless communication suffers from path loss when a wireless signal propagates in the environment.[18] In order to simulate the path loss, we set the weight a_{ij} in Eq. (3) to be $a_{ij} = \sqrt{1/(d_{ij}^{\eta}+1)}$, where d_{ij} represents the distance between agents i and j, and η (typically $2 \leq \eta \leq 6$) is the path loss exponent. Let $\gamma_{ij} = a_{ij}/\sum_{j\in\mathscr{N}_i} a_{ij}$, for passive agents, $i \in \mathscr{I}^{\rm c}$. Note that in this case, $a_{ij} = a_{ji}$ and $\gamma_{ij} = \gamma_{ji}$. Thus, it can be shown that the matrix $\boldsymbol{L}_{\rm a} + \boldsymbol{D} - \tilde{\boldsymbol{A}}(\boldsymbol{I}-\boldsymbol{\Upsilon})^{-1}\tilde{\boldsymbol{\Upsilon}}$ in Eq. (11) is symmetric.

According to Theorem 1, all the agents could achieve consensus on their states. To see this, we randomly choose the initial states $x_i(0)$ $(i = 1,2,\ldots,12)$ of all the active agents from the interval $[-10,10]$, and depict the state trajectories of all the active and passive agents in Fig. 2(a) in the case of $\eta = 2$.

In order to quantitatively measure the degree of consensus of all the agents, we make use of the mean square deviation among all agents defined by

$$\text{MSD} = N^{-1}\sum_{i=1}^{N} \|x_i - \text{Ave}(x)\|^2,$$

where $\mathrm{Ave}(x) = N^{-1}\sum_{i=1}^{N} x_i$ is the average of all states. The result of MSD is shown in Fig. 2(b), where the curve is obtained by averaging 10 independent simulation results. From Fig. 2, we can see that the proposed consensus control (3) can ensure the global consensus of all active and passive agents by directly controlling only the active agents.

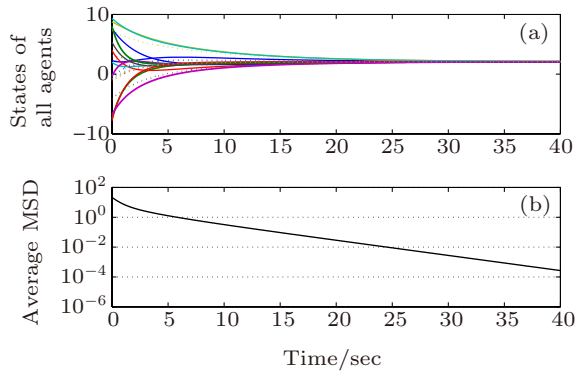


Fig. 2. (color online) (a) Trajectories of the states of all the 20 agents. (b) The average MSD performance of the proposed consensus protocol.

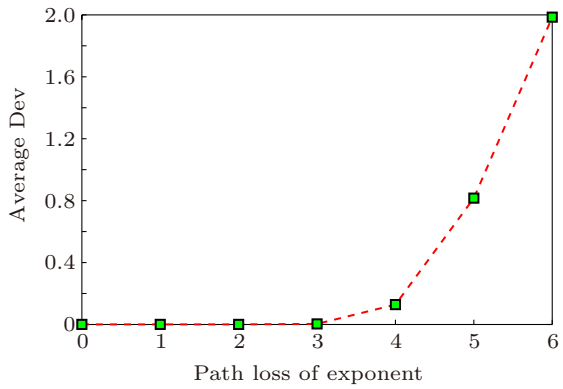


Fig. 3. (color online) Average Dev of states of all agents for different path loss exponent η.

Finally, we discuss the effect of path loss exponent η on the consensus of the multi-agent systems. To quantify the consensus of states of all agents, we consider the square root criterion of MSD as

$$\mathrm{Dev} = \sqrt{N^{-1}\sum_{i=1}^{N} \|x_i - \mathrm{Ave}(x)\|}.$$

For each simulation, Dev is averaged over the last half of simulation steps in order to avoid transient period. The simulation result of the average Dev of 10 independent runs is shown in Fig. 3. It can be observed that the path loss exponent has a great impact on the convergence time of the consensus. For Fig. 3, the simulation time is 400 sec. We can get much smaller Dev with a longer running time. This is because the larger the value of η is, the less influence is executed between neighboring agents. In the extreme case with infinite $\eta = \infty$, then no agents could receive any information from the others. In this case, consensus is absolutely not achieved. Therefore, in order to guarantee a shorter convergence time of consensus, the weights a_{ij} and γ_{ij} should be carefully designed.

5. Conclusion

In this paper, a consensus problem of heterogeneous multi-agent systems is investigated for a group of two types of agents. A consensus protocol by directly controlling only active agents is designed to enable all the agents to achieve consensus of their states. In order to tackle the difficulty arising from the heterogeneity of different agents, we introduce an appropriate Markov chain to cast the heterogeneous consensus protocol into a homogeneous one. Furthermore, it is shown that the states of all active and passive agents can achieve consensus with the proposed protocol. Simulation results are given to illustrate the effectiveness of the proposed heterogeneous consensus protocol.

References

[1] Olfati-Saber R and Murray R M 2004 *IEEE Trans. Autom. Control.* **49** 1520
[2] Ren W and Beard R W 2005 *IEEE Trans. Autom. Control.* **50** 655
[3] Lin Z Y 2006 *Coupled Dynamic Systems: From Structure toward Stability and Stabilizability* (PhD dissertation) (Toronto: University of Toronto)
[4] Ren W, Beard R and Atkins E 2007 *IEEE Control Syst. Mag.* **27** 71
[5] Zhang C X, Li H and Lin P 2008 *Chin. Phys.* B **17** 4458
[6] Shi Y and Yu B 2011 *Automatica.* **47** 754
[7] Luo X Y, Han N N and Guan X P 2010 *Chin. Phys.* B **19** 100202
[8] Yan J, Guan X P and Luo X Y 2011 *Chin. Phys.* B **20** 048901
[9] Song H Y, Yu L, Hu H X and Zhang W A 2012 *Chin. Phys.* B **21** 028901
[10] Bolognani S, Favero S D, Schenato L and Varagnolo D 2010 *Int. J. Robust Nonlin.* **20** 176
[11] Chen Z Y and Zhang H T 2011 *Automatica.* **47** 1929
[12] Li Z K, Duan Z S, Chen G R and Huang L 2010 *IEEE Trans. Circu. Systems I.* **57** 213
[13] Bicchi A, Danesi A, Dini G, La Porta S, Pallottino L, Savino I M and Schiavi R 2008 *IEEE Robot. Autom. Mag.* **15** 62
[14] Wu J and Shi Y 2011 *Syst. Control Lett.* **60** 863
[15] Yang W, Cao L, Wang X F and Li X 2006 *Phys. Rev.* E **74** 037101
[16] Zhu S Y, Chen C L, Guan X P and Long C N 2010 *IEEE International Symposium on a World of Wireless, Mobile and Multimedia Networks* June 14–17, 2010 Montreal, Canada
[17] Horn R A and Johnson C R 1985 *Matrix Analysis* (Cambridge: Cambridge University Press)
[18] Goldsmith A 2005 *Wireless Communications* (Cambridge: Cambridge University Press)