Investigating mesoscopic circuits with entangled state representations*

Liang Bao-Long(梁宝龙)[a)†], Wang Ji-Suo(王继锁)[a)b)], Meng Xiang-Guo(孟祥国)[a)], and Yang Qin-Ying(杨芹英)[a)]

[a)] School of Physics Science and Information Engineering, Liaocheng University, Shandong 252059, China
[b)] College of Physics and Engineering, Qufu Normal University, Qufu 273165, China

(Received 7 February 2012; revised manuscript received 31 May 2012)

By employing the continuous parameter entangled state representations, we investigate the energy level and the wave function for a capacitively and mutual-inductively coupled LC mesoscopic circuit. It is found that investigating the mesoscopic circuit in such representations can bring us the following conveniences. Firstly, the dynamical equation is naturally transformed into a single-variable differential equation. Secondly, the center-of-mass kinetic energy is included in the energy level of the system. Thus it is instructive to introduce the entangled state representation into the investigation of mesoscopic circuits.

Keywords: mesoscopic circuit, energy level, wave function, entangled state

PACS: 68.65.–k, 03.65.–w, 05.30.–d **DOI:** 10.1088/1674-1056/22/1/016804

1. Introduction

Due to the rapid progress in quantum computation and quantum information, a substantial interest has revived the physical research on quantum systems.[1–7] Among all kinds of possible schemes for quantum computation realization, the small solid-state devices are promising because they can be integrated easily to form large-scale circuits. So it is important to investigate the mesoscopic circuit including elementary components such as capacitance, inductance, and electric resistance. The single inductance–capacitance (LC) nondissipative mesoscopic circuit is the fundamental cell in mesoscopic circuits. It is constructive to investigate such a fundamental circuit. Its quantization and quantum effects were first discussed by Louisell,[8] and some further progresses have been made in this field in recent years.[9–13] To deal with those problems, one often chooses a suitable representation to simplify the calculation according to the Hamiltonian operators. The representation theory in quantum mechanics was invented by Dirac.[14] Dirac emphasized that choosing an appropriate representation for tackling definite dynamical problems can save time and labor. Based on the idea of quantum entanglement of Einstein, Podolsky, and Rosen,[15] Fan constructed the entangled states, which have many applications in quantum optics (see a review in Ref. [16]). In this work, we explain how to introduce the entangled state representations into the investigation of mesoscopic circuits by using the capacitively and mutual-inductively coupled LC mesoscopic circuit as an example and how to solve the dynamical problem by means of such representations. The procedure is arranged as follows. Firstly, for convenience, we introduce the entangled state representation. Secondly, we give the Hamiltonian operator for the mesoscopic circuit. Finally, we will obtain the energy level and the wave function by means of the entangled state representations.

2. Two entangled state representations

For two interaction particles with momentum operators $\hat{P}_1$, $\hat{P}_2$ and the corresponding coordinate operators $\hat{Q}_1$, $\hat{Q}_2$, in order to reduce the two-body problem, we introduce the center-of-mass coordinate

$$\hat{Q}_{\rm cm} = \mu_1 \hat{Q}_1 + \mu_2 \hat{Q}_2, \tag{1}$$

and the mass-weighted relative momentum

$$\hat{P}_{\rm r} = \mu_2 \hat{P}_1 - \mu_1 \hat{P}_2, \tag{2}$$

where $\mu_1 + \mu_2 = 1$, $\mu_j = m_j/M$, $M = m_1 + m_2$, and m_j is the mass of the j-th particle. The $\hat{Q}_j$, $\hat{P}_j$ are related to Bose creation and annihilation operators by

$$\hat{Q}_j = \sqrt{\frac{\hbar}{2m_j\omega_j}}\left(\hat{a}_j + \hat{a}_j^\dagger\right), \quad \hat{P}_j = {\rm i}\sqrt{\frac{m_j\omega_j\hbar}{2}}\left(\hat{a}_j - \hat{a}_j^\dagger\right), \tag{3}$$

where $[\hat{a}_l, \hat{a}_j^\dagger] = \delta_{lj}$, and ω_j is the vibration frequency of the j-th particle. Due to the fact that operators $\hat{Q}_{\rm cm}$ and $\hat{P}_{\rm r}$ are commutative, i.e., $\left[\hat{Q}_{\rm cm}, \hat{P}_{\rm r}\right] = 0$, they have a common eigenvector[16]

$$\begin{aligned}|\xi\rangle = \exp\Bigg\{ & -\frac{1}{2}|\xi|^2 + \frac{1}{\sqrt{\lambda}}\left[\xi + (\mu_1 - \mu_2)\,\xi^*\right]\hat{a}_1^\dagger \\ & + \frac{1}{\sqrt{\lambda}}\left[\xi^* - (\mu_1 - \mu_2)\,\xi\right]\hat{a}_2^\dagger \\ & + \frac{1}{\lambda}\left[(\mu_1 - \mu_2)(\hat{a}_1^{\dagger 2} - \hat{a}_2^{\dagger 2}) - 4\mu_1\mu_2\hat{a}_1^\dagger\hat{a}_2^\dagger\right]\Bigg\}|00\rangle, \end{aligned} \tag{4}$$

*Project supported by the National Natural Science Foundation of China (Grant Nos. 11147009 and 11244005), and the Natural Science Foundation of Shandong Province, China (Grant Nos. ZR2010AQ027 and ZR2012AM004)

†Corresponding author. E-mail: baolong-liang951@sohu.com; blliang@lcu.edu.cn

© 2013 Chinese Physical Society and IOP Publishing Ltd

where $|00\rangle$ is the two-mode vacuum state, ξ is complex, $\xi = \xi_1 + \mathrm{i}\xi_2$, and $\lambda = 2\left(\mu_1^2 + \mu_2^2\right)$. Particularly, when $m_1 = m_2$, $|\xi\rangle$ will degenerate to the following form:

$$|\xi\rangle = \exp\left(-\frac{1}{2}|\xi|^2 + \xi\hat{a}_1^\dagger + \xi^*\hat{a}_2^\dagger - \hat{a}_1^\dagger\hat{a}_2^\dagger\right)|00\rangle, \quad (5)$$

which is the common eigenvector of operators $\hat{Q}_1 + \hat{Q}_2$ and $\hat{P}_1 - \hat{P}_2$. From Eqs. (1)–(5), the eigenvector equations can be obtained as

$$\hat{Q}_{\mathrm{cm}}|\xi\rangle = \sqrt{\frac{\lambda}{2}}\xi_1|\xi\rangle = \sqrt{\mu_1^2+\mu_2^2}\xi_1|\xi\rangle, \quad (6)$$

$$\hat{P}_{\mathrm{r}}|\xi\rangle = \sqrt{\frac{\lambda}{2}}\xi_2|\xi\rangle = \sqrt{\mu_1^2+\mu_2^2}\xi_2|\xi\rangle. \quad (7)$$

By using the technique of integration within an ordered product (IWOP) of operators,[17,18] the following completeness relation can be proved:

$$\int \frac{\mathrm{d}^2\xi}{\pi}|\xi\rangle\langle\xi| = 1, \quad \mathrm{d}^2\xi = \mathrm{d}\xi_1 d\xi_2. \quad (8)$$

On the other hand, for two particles' relative position operator $\hat{Q}_{\mathrm{r}} = \hat{Q}_1 - \hat{Q}_2$ and total momentum operator $\hat{P} = \hat{P}_1 + \hat{P}_2$, based on the idea of quantum entanglement of Einstein, Podolsky, and Rosen,[15] who firstly noticed the fact that $[\hat{Q}_{\mathrm{r}}, \hat{P}] = 0$, their common eigenstate can be constructed as[17,18]

$$|\eta\rangle = \exp\left(-\frac{|\eta|^2}{2} + \eta\hat{a}_1^\dagger - \eta^*\hat{a}_2^\dagger + \hat{a}_1^\dagger\hat{a}_2^\dagger\right)|00\rangle, \quad (9)$$

where η is a complex, $\eta = \eta_1 + \mathrm{i}\eta_2$, and $|\eta\rangle$ obeys the completeness relation

$$\int \frac{\mathrm{d}^2\eta}{\pi}|\eta\rangle\langle\eta| = 1, \quad \mathrm{d}^2\eta = \mathrm{d}\eta_1\,\mathrm{d}\eta_2. \quad (10)$$

The eigenvector equations are

$$\hat{P}|\eta\rangle = \sqrt{2}\eta_2|\eta\rangle, \quad (11)$$

$$\hat{Q}_{\mathrm{r}}|\eta\rangle = \sqrt{2}\eta_1|\eta\rangle. \quad (12)$$

The internal product between $|\eta\rangle$ and $|\xi\rangle$ can be also calculated by virtue of Eqs. (4) and (9)

$$\langle\eta|\xi\rangle = \sqrt{\frac{\lambda}{4}}\exp\{\mathrm{i}[(\mu_1-\mu_2)(\eta_1\eta_2 - \xi_1\xi_2) + \sqrt{\lambda}(\eta_1\xi_2 - \eta_2\xi_1)]\}. \quad (13)$$

From Eqs. (1), (2), and (6)–(9), we can deduce

$$\langle\eta|\hat{P}_{\mathrm{r}} = \langle\eta|\hat{p}_{\mathrm{r}}\int\frac{\mathrm{d}^2\xi}{\pi}|\xi\rangle\langle\xi| = \int\frac{\mathrm{d}^2\xi}{\pi}\sqrt{\frac{\lambda}{2}}\xi_1\langle\eta|\,\xi\rangle\langle\xi| = -\frac{1}{\sqrt{2}}\left[\mathrm{i}\frac{\partial}{\partial\eta_1} + (\mu_1-\mu_2)\eta_2\right]\langle\eta|, \quad (14)$$

$$\langle\eta|\hat{Q}_{\mathrm{cm}} = \frac{1}{\sqrt{2}}\left[\mathrm{i}\frac{\partial}{\partial\eta_2} + (\mu_1-\mu_2)\eta_1\right]\langle\eta|. \quad (15)$$

Similarly, we have

$$\langle\xi|\hat{P} = \sqrt{\frac{2}{\lambda}}\left[-\mathrm{i}\frac{\partial}{\partial\xi_1} - (\mu_1-\mu_2)\xi_2\right]\langle\xi|, \quad (16)$$

$$\langle\xi|\hat{Q}_{\mathrm{r}} = \sqrt{\frac{2}{\lambda}}\left[i\frac{\partial}{\partial\xi_2} + (\mu_1-\mu_2)\xi_1\right]\langle\xi|. \quad (17)$$

In the following, we will investigate the capacitively and inductively-coupled LC mesoscopic circuit by means of the above relations.

3. Hamilton dynamical analysis for the capacitively and mutual-inductively coupled LC mesoscopic circuit

For the mesoscopic capacitively and mutual-inductively coupled LC circuit shown in Fig. 1, if using the branch charge Q_j ($j = 1,2$) as the generalized coordinate, the potential energy of the system is given by

$$\mathscr{V} = \frac{(Q_1 - Q_2)^2}{2C}, \quad (18)$$

where C is the coupling capacitance. On the other hand, the kinetic energy related to $\dot{q}_i$ is

$$\mathscr{T} = \frac{1}{2}L_1\dot{Q}_1^2 + \frac{1}{2}L_2\dot{Q}_2^2 + M\dot{Q}_1\dot{Q}_2, \quad (19)$$

where $M = k\sqrt{L_1L_2}$ represents the mutual-inductance between the two inductors. Thus the Lagrangian of the system is

$$\mathscr{L} = \mathscr{T} - \mathscr{V}. \quad (20)$$

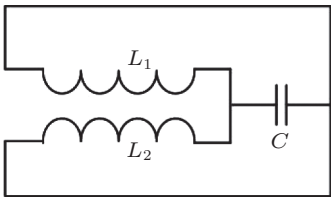


Fig. 1. Mesoscopic LC circuit coupled capacitively and mutual-inductively.

From Eq. (20), we can calculate the generalized momenta conjugated to q_i

$$P_1 \equiv \frac{\partial\mathscr{L}}{\partial\dot{q}_1} = L_1\dot{Q}_1 + M\dot{Q}_2, \quad P_2 \equiv \frac{\partial\mathscr{L}}{\partial\dot{Q}_2} = L_2\dot{Q}_2 + M\dot{Q}_1. \quad (21)$$

We notice that P_j is the magnetic flux through the j-th ($j = 1,2$) inductor. Via the Legendre transformation and Eqs. (20) and (21), we can obtain the Hamiltonian of the system

$$\mathscr{H} = \frac{P_1^2}{2\gamma_1} + \frac{P_2^2}{2\gamma_2} + \tau P_1P_2 + \nu(Q_1 - Q_2)^2, \quad (22)$$

where $\gamma_j = L_j(1-k^2)$, $\tau = \frac{k}{\sqrt{L_1L_2}(1-k^2)}$, and $\nu = \frac{1}{2C}$. Equation (22) implies that the system is equivalent to two charged particles coupled by a spring with elastic constant ν. From the point of view of molecular dynamics, the term $\nu(Q_1 - Q_2)^2$ represents the internal bond potential, and the term τP_1P_2 is the kinetic coupling energy, which represents the current–current magnetic interaction since the motion of charged particles produces the electric current. Accoding to the standard canonical quantization principle,[14] a pair of conjugate quantities Q_j and P_j are associated with a pair of Hermitian operators $\hat{Q}_j$ and $\hat{P}_j$, and they satisfy the commutation relation $[\hat{Q}_j, \hat{P}_j] = \mathrm{i}\hbar$. Thus

the Hamilton operator of the system is

$$\hat{\mathscr{H}} = \frac{\hat{P}_1^2}{2\gamma_1} + \frac{\hat{P}_2^2}{2\gamma_2} + \tau\hat{P}_1\hat{P}_2 + \nu(\hat{Q}_1 - \hat{Q}_2)^2. \tag{23}$$

4. Energy level and wave function of the system

In order to deal with the system in the entangled state representation, the Hamilton operator $\hat{\mathscr{H}}$ needs to be transformed. Noting

$$\begin{aligned} &\hat{Q}_1 - \hat{Q}_2 \equiv \hat{Q}_\mathrm{r}, \quad \hat{P}_1 + \hat{P}_2 \equiv \hat{P}, \quad [\hat{Q}_\mathrm{r}, \hat{P}] = 0, \\ &\mu_1\hat{Q}_1 + \mu_2\hat{Q}_2 \equiv \hat{Q}_\mathrm{c}, \quad \mu_2\hat{P}_1 - \mu_1\hat{P}_2 \equiv \hat{P}_\mathrm{r}, \quad [\hat{Q}_\mathrm{c}, \hat{P}_\mathrm{r}] = 0, \end{aligned} \tag{24}$$

where $\gamma = \gamma_1 + \gamma_2$ is the total mass, $\mu = \gamma_1\gamma_2/\gamma$ is the reduced mass, and $\mu_j = \gamma_j/\gamma$, we have

$$\begin{aligned} &\hat{Q}_1 = \hat{Q}_\mathrm{c} + \mu_2\hat{Q}_\mathrm{r}, \quad \hat{Q}_2 = \hat{Q}_\mathrm{c} - \mu_1\hat{Q}_\mathrm{r}, \\ &\hat{P}_1 = \hat{P}_\mathrm{r} + \mu_1\hat{P}, \quad \hat{P}_2 = \mu_2\hat{P} - \hat{P}_\mathrm{r}, \\ &[\hat{Q}_\mathrm{r}, \hat{P}_\mathrm{r}] = \mathrm{i}\hbar, \quad [\hat{Q}_\mathrm{c}, \quad \hat{p}] = \mathrm{i}\hbar. \end{aligned} \tag{25}$$

Substituting Eq. (25) into Eq. (23) leads to

$$\begin{aligned} \hat{\mathscr{H}} \;=\; &\left(\frac{1}{2\gamma} + \tau\mu_1\mu_2\right)\hat{P}^2 + \left(\frac{1}{2\mu} - \tau\right)\hat{P}_\mathrm{r}^2 \\ &+ \tau(\mu_2 - \mu_1)\hat{P}\hat{P}_\mathrm{r} + \nu\hat{Q}_\mathrm{r}^2. \end{aligned} \tag{26}$$

We introduce a double-mode Fock space, in which $\hat{a}_j^\dagger$, $\hat{a}_j$ and $\hat{Q}_j, \hat{P}_j$ satisfies

$$\hat{Q}_j = \sqrt{\frac{\hbar}{2\gamma_j\omega_j}}\left(\hat{a}_j + \hat{a}_j^\dagger\right), \quad \hat{P}_j = \mathrm{i}\sqrt{\frac{\gamma_j\omega_j\hbar}{2}}\left(\hat{a}_j - \hat{a}_j^\dagger\right), \tag{27}$$

where $\omega_j = \sqrt{2\nu/\gamma_j}$ is the vibration frequency of the j-th charged particle. Next, we investigate the system by virtue of eigenvector $|\xi\rangle$ corresponding to $\{\hat{Q}_\mathrm{cm}, \hat{P}_\mathrm{r}\}$ and $|\eta\rangle$ corresponding to $\{\hat{Q}_\mathrm{r}, \hat{P}\}$. Here, we consider $|E_n\rangle$ as the eigenstate and E_n as the eigenvalue of Hamilton $\hat{\mathscr{H}}$. Using Eqs. (11), (12), and (14), we have

$$\begin{aligned} &\langle\eta|\hat{\mathscr{H}}|E_n\rangle \\ &= E_n\langle\eta|E_n\rangle = \left\{\frac{1}{2}\left(\tau - \frac{1}{2\mu}\right)\left[\frac{\partial}{\partial\eta_1} - \mathrm{i}(\mu_1 - \mu_2)\eta_2\right]^2\right. \\ &\quad - \mathrm{i}\eta_2\tau(\mu_2 - \mu_1)\left[\frac{\partial}{\partial\eta_1} - \mathrm{i}(\mu_1 - \mu_2)\eta_2\right] \\ &\quad \left. + \left[\left(\frac{1}{\gamma} + 2\tau\mu_1\mu_2\right)\eta_2^2 + 2\nu\eta_1^2\right]\right\}\langle\eta|E_n\rangle, \end{aligned} \tag{28}$$

from which we can see that the dynamical problem of the more complicate two-body system can be solved by employing simultaneous $\langle\eta|$ and $\langle\xi|$ representations. Suppose that, in the $\langle\eta|$ representation, the wave function of energy eigenstate $|E_n\rangle$ takes the form

$$\langle\eta|E_n\rangle = \exp[\mathrm{i}(\mu_1 - \mu_2)\eta_1\eta_2]\psi_n, \tag{29}$$

where ψ_n is undetermined. From the following operator relation:

$$\exp[-\mathrm{i}(\mu_1 - \mu_2)\eta_1\eta_2]\left[\frac{\partial}{\partial\eta_1} - \mathrm{i}(\mu_1 - \mu_2)\eta_2\right]$$

$$\times \exp[\mathrm{i}(\mu_1 - \mu_2)\eta_1\eta_2] = \frac{\partial}{\partial\eta_1}, \tag{30}$$

the differential equation about ψ_n can be obtained

$$\begin{aligned} &\left\{\frac{1}{2}\left(\tau - \frac{1}{2\mu}\right)\frac{\partial^2}{\partial\eta_1^2} - \mathrm{i}\eta_2\tau(\mu_2 - \mu_1)\frac{\partial}{\partial\eta_1}\right. \\ &\left. + \left[\left(\frac{1}{\gamma} + 2\tau\mu_1\mu_2\right)\eta_2^2 + 2\nu\eta_1^2 - E_n\right]\right\}\psi_n = 0. \end{aligned} \tag{31}$$

Letting ψ_n take the form

$$\psi_n = \exp[2\mathrm{i}\eta_1\eta_2\tau(\mu_1 - \mu_2)\mu/(1 - 2\mu\tau)]\varphi_n \equiv \mathrm{e}^{\mathrm{i}\eta_1\rho}\varphi_n, \tag{32}$$

where $\rho = 2\eta_2\tau\mu(\mu_1 - \mu_2)/(1 - 2\mu\tau)$, and making use of the relation

$$\mathrm{e}^{-\mathrm{i}\eta_1\rho}\frac{\partial}{\partial\eta_1}\mathrm{e}^{\mathrm{i}\eta_1\rho} = \frac{\partial}{\partial\eta_1} + \mathrm{i}\rho, \tag{33}$$

we can obtain the differential equation about φ_n from Eq. (31)

$$\begin{aligned} &\left\{-\frac{1}{2}\left(\frac{1}{2\mu} - \tau\right)\frac{\partial^2}{\partial\eta_1^2} + \frac{1 - \tau^2\mu\gamma}{\gamma(1 - 2\mu\tau)}\eta_2^2\right. \\ &\left. + 2\nu\eta_1^2 - E_n\right\}\varphi_n = 0. \end{aligned} \tag{34}$$

Comparing the above equation with the stationary-state equation of the standard harmonic oscillator

$$-\frac{\hbar^2}{2\mu}\frac{\mathrm{d}^2\varphi_n}{\mathrm{d}\eta_1^2} + \frac{1}{2}\mu\omega^2\eta_1^2\varphi_n = \mathscr{E}_n\varphi_n, \tag{35}$$

where $\mathscr{E}_n = (n + \frac{1}{2})\hbar\omega$ $(n = 0, 1, 2, \ldots)$ and $\omega = 2\sqrt{\nu/\mu}$, the eigenenergy belonging to Eq. (34) is

$$E_n = \frac{1 - \tau^2\mu\gamma}{1 - 2\mu\tau}\frac{\eta_2^2}{\gamma} + \left(n + \frac{1}{2}\right)\hbar\sqrt{\frac{1 - 2\mu\tau}{2}}\omega, \tag{36}$$

which implies that because of the existence of mutual-inductance coupling term $\tau\hat{P}_1\hat{P}_2$, not only the center-of-mass kinetic eneygy is included in the energy level of the system, but also the frequency of the energy-level transition varies. In order to obtain the solution, comparing Eq. (34) with the standard Hermite equation whose solution is known, we have

$$\varphi_n = N_n\exp\left(-\frac{1}{2}\alpha^2\eta_1^2\right)\mathrm{H}_n(\alpha\eta_1), \tag{37}$$

where

$$\alpha^2 = (2/\hbar)\sqrt{2\mu\nu/(1 - 2\mu\tau)}, \quad N_n = \sqrt{\alpha/\sqrt{\pi}2^n n!}$$

represents the normalization constant, and $\mathrm{H}_n(\alpha\eta_1)$ is the Hermitian polynomial. Combining Eqs. (32) and (37) with Eq. (29), we deduce the wave function of energy eigenstate $|E_n\rangle$ in the $\langle\eta|$ representation

$$\begin{aligned} \langle\eta|E_n\rangle \;=\; &N_n\exp\left\{\mathrm{i}\eta_1[(\mu_1 - \mu_2)\eta_2 + \rho]\right. \\ &\left. - \frac{1}{2}\alpha^2\eta_1^2\right\}\mathrm{H}_n(\alpha\eta_1). \end{aligned} \tag{38}$$

In order to evaluate $\langle\eta|E_n\rangle$ intuitively, we give an example by choosing the following mesoscopic parameters: $L_1 = L_2 = 10^{-8}$ H, $C = 5 \times 10^{-9}$ F, and coupling coefficient $k = 0.3$ between the two inductors, which can be easily archived with present-day technologies. Based on the above param-

eters, we show respectively $\langle\eta\ |E_n\rangle$ (here, we take the real part $\mathrm{Re}[\langle\eta\ |E_n\rangle]$) and the corresponding probability density $|\langle\eta\ |E_n\rangle|^2$ as a function of η_1 for different quantum number n, which corresponds to different wave functions (see Figs. 2 and 3). Combining Fig. 2 with Fig. 3, we can draw the conclusion that $\mathrm{Re}[\langle\eta\ |E_n\rangle]-\eta_1$ intersects n times with the η_1 axis in a finite η_1 range. That is to say, the equation $\mathrm{Re}[\langle\eta\ |E_n\rangle]=0$ has n solutions, or $\mathrm{Re}[\langle\eta\ |E_n\rangle]$ possesses n nodes. When n takes a smaller quantum number, which means a lower energy of the system, in some quantum states for different η_1, the probability of finding particles (here, electrons) is different distinctively. This just shows that the system exhibits strong quantum characters in the lower energy state. But with n increasing, the width of the single peak becomes narrow, which tells us that the probability tends to be equal. The classical characters prevail against the quantum ones gradually.

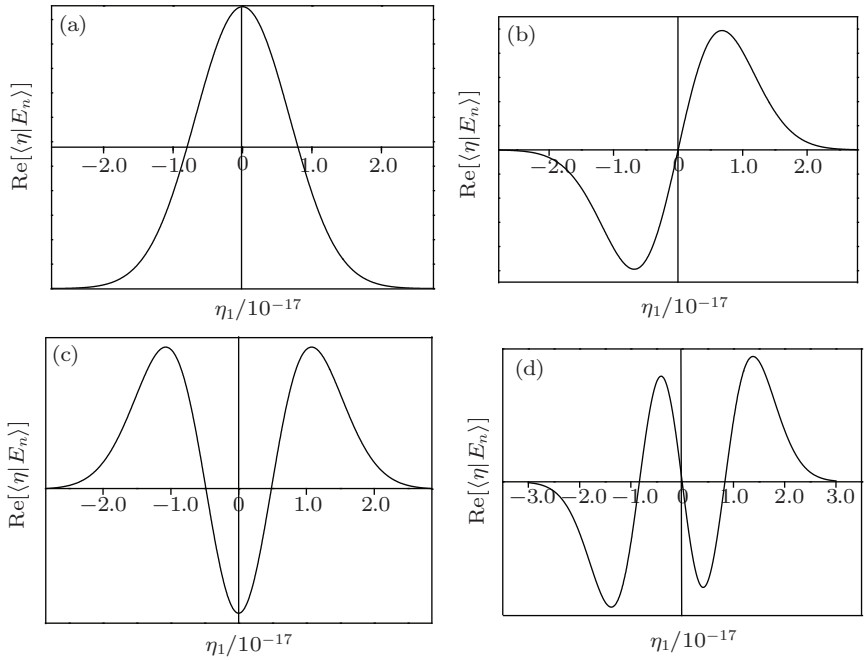


Fig. 2. $\mathrm{Re}[\langle\eta\ |E_n\rangle]$ as a function of η_1 for (a) $n=0$, (b) $n=1$, (c) $n=2$, and (d) $n=3$.

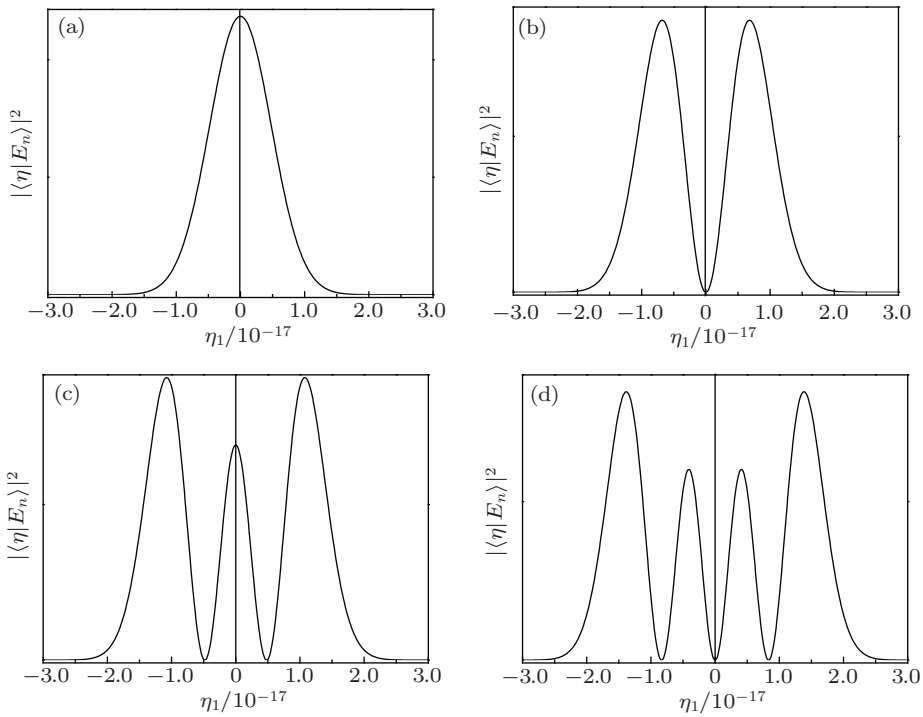


Fig. 3. The $|\langle\eta\ |E_n\rangle|^2$ as a function of η_1 for (a) $n=0$, (b) $n=1$, (c) $n=2$, and (d) $n=3$.

5. Conclusion

In summary, by employing the continuous parameter entangled state representations, we investigate the energy level and the wave function for the capacitively and inductively-coupled LC mesoscopic circuit. We find that investigating the mesoscopic circuit problem in such representations can bring us the following conveniences. Firstly, the dynamical equation is naturally transformed into a single-variable differential equation by denoting the other variable as a parameter. Secondly, the center-of-mass kinetic energy is included in the energy-level of the system. Thus it is instructive to introduce the entangled state representation to the investigation of mesoscopic circuits.

References

[1] Fan H Y and Pan X Y 1998 *Chin. Phys. Lett.* **15** 625

[2] Vandersypen L M K, Steffen M, Breyta G, Yannoni C S, Sherwood M H and Chuang I L 2001 *Nature* **414** 883

[3] Gulde S, Riebe M, Lancaster G P T, Becher C, Eschner J, Häffner H, Schmidt-Kaler F, Chuang I L and Blatt R 2003 *Nature* **421** 48

[4] Turchette Q A, Hood C J, Lange W, Mabuchi H and Kimble H J 1995 *Phys. Rev. Lett.* **75** 4710

[5] Liang B L, Wang J S, Meng X G and Su J 2010 *Chin. Phys.* B **19** 010315

[6] Liang B L and Wang J S 2007 *Chin. Phys.* **16** 3097

[7] Xu C L 2012 *Chin. Phys.* B **21** 020402

[8] Louisell W H 1973 *Quantum Statistical Properties of Radiation* (New York: John Wiley) Chapter 4

[9] Makhlin Y, Schön G and Shnirman A 2001 *Rev. Mod. Phys.* **73** 357

[10] Nakamura Y, Pashkin Y A and Tsai J S 1999 *Nature* **398** 786

[11] Deppe F, Mariantoni M, Menzel E P, Saito S, Kakuyanagi K, Tanaka H, Meno T, Semba K, Takayanagi H and Gross R 2007 *Phys. Rev.* B **76** 214503

[12] van der Ploeg S H W, Izmalkov A, van den Brink A M, Hübner U, Grajcar M, Il'ichev E, Meyer H G and Zagoskin A M 2007 *Phys. Rev. Lett.* **98** 057004

[13] Meng X G, Wang J S and Liang B L 2008 *J. Phys.* A: *Math. Theor.* **41** 235208

[14] Dirac P A M 1958 *The Principles of Quantum Mechanics* (Oxford: Oxford Univ. Press)

[15] Einstein A, Podolsky B and Rosen N 1935 *Phys. Rev.* **47** 777

[16] Fan H Y and Xiong Y 1995 *Phys. Rev.* A **51** 3343

[17] Fan H Y, Zaidi H R and Klauder J R 1987 *Phys. Rev.* D **35** 1831

[18] Fan H Y and Zaidi H R 1988 *Phys. Rev.* A **37** 2985