A dynamic rheological model for thin-film lubrication*

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In this study, the effects of the non-Newtonian rheological properties of the lubricant in a thin-film lubrication regime between smooth surfaces were investigated. The thin-film lubrication regime typically appears in Stribeck curves with a clearly observable minimum coefficient of friction (COF) and a low-COF region, which is desired for its lower energy dissipation. A dynamic rheology of the lubricant from the hydrodynamic lubrication regime to the thin-film lubrication regime was proposed based on the convected Maxwell constitutive equation. This rheology model includes the increased relaxation time and the yield stress of the confined lubricant thin film, as well as their dependences on the lubricant film thickness. The Deborah number (*De* number) was adopted to describe the liquid–solid transition of the confined lubricant thin film under shearing. Then a series of Stribeck curves were calculated based on Tichy's extended lubrication equations with a perturbation of the *De* number. The results show that the minimum COF points in the Stribeck curve correspond to a critical *De* number of 1.0, indicating a liquid-to-solid transition of the confined lubricant film. Furthermore, the two proposed parameters in the dynamic rheological model, namely negative slipping length b (indicating the lubricant interfacial effect) and the characteristic relaxation time λ_0, were found to determine the minimum COF and the width of the low-COF region, both of which were required to optimize the shape of the Stribeck curve. The developed dynamic rheological model interprets the correlation between the rheological and interfacial properties of lubricant and its lubrication behavior in the thin-film regime.

Keywords: rheology, viscoelasticity, Deborah number, thin-film lubrication

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1. Introduction

Liquid viscosity is a crucial lubricant parameter for the hydrodynamic lubrication of machines based on the conventional Reynolds equations. However, the conventional Reynolds equations are limited to describing and predicting the properties of boundary or thin-film lubrication,[1,2] since boundary lubrication is often defined as the lubrication regime in which liquid bulk properties like viscosity are no longer dominant over friction. Generally, the transition from hydrodynamic lubrication regime to boundary lubrication regime can be clarified in a typical Stribeck curve, as shown in Fig. 1.[3–5] With a constant normal load and a decreasing sliding velocity, the Stribeck curve indicates the variations in the coefficient of friction (COF) of the sliding pairs. The COF undergoes a transition from a viscosity-dominant hydrodynamic regime to a boundary-layer-dominant regime. The transition between these two regimes indicates that the lubrication regime may be thin-film lubrication or mixed lubrication. Generally, the transition region through the thin-film lubrication or mixed lubrication regime typically exhibits a clearly observable minimum COF and a low-COF region,[4] which is desired for many applications due to its lower energy dissipation.

For rough surfaces, many studies have proven that the transition from hydrodynamic lubrication to boundary lubrication is induced by mixed contact, i.e., a ratio of solid–solid contact area to the area separated by the lubricant.[3] Thus, the COF in the transition region is related to the surface roughness R_a, as shown by the experimental results in Fig. 1. However, for smooth surfaces, two general characteristics are observed: (i) the surfaces' COF is significantly lower than that of rough surfaces, with a lower minimum COF value (as indicated by the triangle points in Fig. 1), and (ii) the variation slope of the COF increases, resulting in a wider low-COF region. Huang *et al.*[4] attributed the transition area on the Stribeck curve for smooth surfaces to a thin-film lubrication state.

It can be seen from Fig. 1 that the Stribeck curve with lower surface roughness has a significantly lower minimum COF, a wider low-COF region, and sharper transition slopes than those of rough surfaces under the same normal load and sliding velocity. For a smooth surface in this transition regime, the film thickness typically ranges from 10 nm to 1000 nm. A lubricant's rheological and interfacial properties change when the confinement interspace is reduced to micro or nano scale.[6,7] It has been proved that the rheological properties of a liquid confined between smooth surfaces at a thickness of only several molecular diameters differ significantly from the bulk properties.[8,9] These dramatically different liquid prop-

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erties have been observed using experimental methods, such as SFA[8–10] and AFM,[11] and simulation methods, such as molecular dynamics (MD).[12] Many studies show that the apparent viscosity of the thin film can be several orders of magnitude greater than that of the bulk lubricant.[8,10] The confined thin films also demonstrate solid-like properties, including a critical yield stress or dynamic shear melting transition.[8,9] It is widely recognized that the confined film properties undergo a liquid-to-solid transition characterized by a noticeable non-Newtonian behavior.[8,9] However, only a limited number of studies have used quantitative rheological models to investigate the effects of the confined lubricant's solid-like properties and the interactions between the confined lubricant and solid substrate. A unified rheological model of the lubricant must be built to cover all the lubrication regions, including the hydrodynamic lubrication, boundary lubrication, and transition regions. However, the expansion of the current lubrication equations to include the experimentally proven rheological properties of thin-film lubricants has two limitations: the thickness/velocity dependence of rheological properties due to the lubricant viscoelasticity and the interfacial properties of the lubricant and solid substrate.

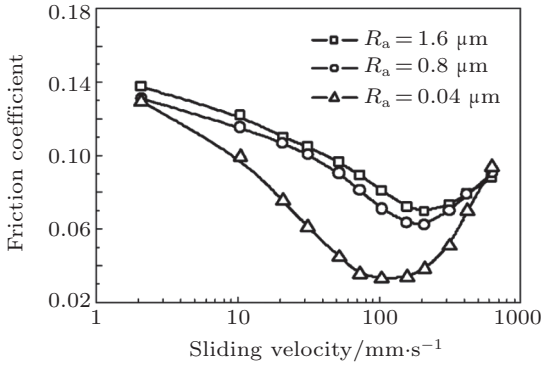


Fig. 1. Typical experimental Stribeck curves for tribopairs of different roughness.[3] The tribopairs are AISI-52100 steel balls versus AISI-1045 steel surfaces, lubricated by HVI500 base oil. As the roughness R_a decreases, the variation slopes sharpen (shown by the triangle points), resulting in a lower minimum COF and a wider low-COF region during the transition from hydrodynamic lubrication to boundary lubrication.

Tichy[1] provided a solution by combining the basic lubrication equation with a convected Maxwell model to describe the lubricant viscoelasticity and solved the requisite equations using a perturbation of the Deborah number (*De* number). The *De* number is defined as a relation between the characteristic relaxation time of the lubricant λ_0 and the time of the shearing action. The *De* number is zero for a Newtonian fluid and infinite for a Hookean elastic solid. Using the *De* number, the conventional lubrication equations are expanded to include the lubricant viscoelasticity. When the *De* number equals 1.0, the lubricant is considered to undergo a transition from liquid-like to solid-like. Tichy suggested that the modified equations are valid for higher *De* numbers. However, this model assumes a constant relaxation time for the thin-film lubricant.

This paper reports our investigation of the lubrication transition from the hydrodynamic lubrication regime through thin-film lubrication to the boundary lubrication regime using the lubricant rheology. We investigate the effect of the non-Newtonian behavior of the lubricant in the transition regime, including the relaxation time and the interfacial behaviors of the lubricant/substrate. The paper is organized as follows. In Subsection 2.1, we expand the lubrication equations, considering a convected Maxwell model and a perturbation of the *De* number. Subsection 2.2 describes the dynamic rheological model of thin-film lubricants in detail. Section 3 presents our calculation of the Stribeck curve and the effects of the rheological model.

2. Theoretical model

2.1. Modified lubrication equations based on the *De* number

In Tichy's work,[1] the *De* number was used to describe the viscoelasticity of a lubricant and modify the conventional lubrication equations for a typical Couette-geometry sliding pair, as shown in Fig. 2. In Tichy's model, a fluid layer is confined between two solid walls with non-slip boundary conditions.[1] The rubbing surfaces are assumed to be ideally smooth because the predominance of mixed lubrication for rough surfaces makes identifying the underlying mechanisms from the rheological properties of the lubricant difficult.[4,10]

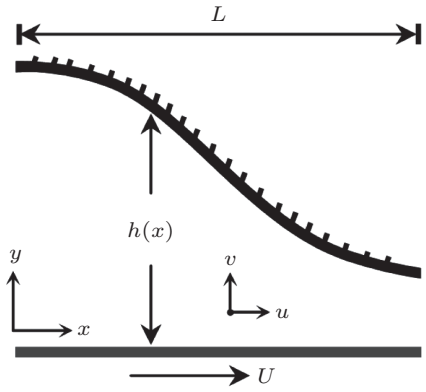


Fig. 2. Schematic diagram of the convergent wedge-shaped sliding pair, where x is the sliding direction, y is the cross-film direction, U is the sliding velocity, u and v are the lubricant flow velocities, $h(x)$ is the film thickness, and L is the width of the sliding area.

In the present work, the following Maxwell constitutive equation is adopted for the lubricant based on Tichy's work:[1]

$$\tau_{ij} + \lambda \frac{\mathrm{d}\tau_{ij}}{\mathrm{d}t} = -\eta \dot{\gamma}_{ij}, \tag{1}$$

where subscripts i and j are the Cartesian tensor indicial notations, which indicate x and y in Fig. 2, respectively; τ_{xy} is the

shear stress; $\dot{\gamma}_{xy}$ is the shear rate; η is the liquid viscosity; λ is the relaxation time; and t is the time. If the relaxation time λ is zero, the typical Newtonian liquid model is recovered.

Then, according to the convected Maxwell model from Tichy,[1]

$$\frac{\mathrm{D}}{\mathrm{D}t}\tau_{ij} = \frac{\partial}{\partial t}\tau_{ij} + v_m \frac{\partial}{\partial x_m}\tau_{ij}, \tag{2}$$

where subscripts i, j are the Cartesian tensor indicial notations with summation on repeated index m.

The following constitutive equations for the lubricant are obtained by combining Eqs. (1) and (2):

$$\begin{aligned}
&\tau_{xx} + \lambda\left(v_x\frac{\partial\tau_{xx}}{\partial x} + v_y\frac{\partial\tau_{xx}}{\partial y} - 2\frac{\partial v_x}{\partial y}\tau_{xy} - 2\frac{\partial v_x}{\partial x}\tau_{xx}\right)\\
&= -2\eta\frac{\partial v_x}{\partial x},\\
&\tau_{xy} + \lambda\left(v_x\frac{\partial\tau_{xy}}{\partial x} + v_y\frac{\partial\tau_{xy}}{\partial y} - \frac{\partial v_x}{\partial y}\tau_{yy} - \frac{\partial v_y}{\partial x}\tau_{xx}\right)\\
&= -\eta\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right),\\
&\tau_{yy} + \lambda\left(v_x\frac{\partial\tau_{yy}}{\partial x} + v_y\frac{\partial\tau_{yy}}{\partial y} - 2\frac{\partial v_y}{\partial y}\tau_{yy} - 2\frac{\partial v_y}{\partial x}\tau_{xy}\right)\\
&= -2\eta\frac{\partial v_y}{\partial y},
\end{aligned} \tag{3}$$

where $v_x = u$ and $v_y = v$ are the flowing velocities of the lubricant in Fig. 2.

The mass conservation equation and the motion equations are also adopted according to the tribopairs in Fig. 2. Next, based on a non-dimensional process (non-dimensional parameters were defined in Tichy's study[1]),

$$\begin{aligned}
&x^* = \frac{x}{L},\ y^* = \frac{y}{H_0},\ u^* = \frac{u}{U},\ v^* = \frac{v}{UH_0/L},\\
&\tau_{xx}^* = \frac{\tau_{xx}}{\eta UL/H_0^2},\ \tau_{xy}^* = \frac{\tau_{xy}}{\eta U/H_0},\ \tau_{yy}^* = \frac{\tau_{yy}}{\eta U/L},\\
&p^* = \frac{p-p_\mathrm{a}}{\eta UL/H_0^2},\ De = \frac{\lambda U}{L},\ h^*(x^*) = \frac{h(x)}{H_0},
\end{aligned}$$

where $h(x) = H_0$ at $x = 0$, and $p = p_\mathrm{a}$ at $x = 0, L$. The superscript $*$ indicates the non-dimensional parameters. Then the non-dimensional lubrication equation is obtained as

$$\begin{aligned}
&\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0,\\
&\tau_{xx}^* + De\left(u^*\frac{\partial\tau_{xx}^*}{\partial x^*} + v^*\frac{\partial\tau_{xx}^*}{\partial y^*} - 2\frac{\partial u^*}{\partial y^*}\tau_{xy}^* - 2\frac{\partial u^*}{\partial x^*}\tau_{xx}^*\right) = 0,\\
&\tau_{xy}^* + De\left(u^*\frac{\partial\tau_{xy}^*}{\partial x^*} + v^*\frac{\partial\tau_{xy}^*}{\partial y^*} - \frac{\partial u^*}{\partial y^*}\tau_{yy}^* - \frac{\partial v^*}{\partial x^*}\tau_{xx}^*\right) = -\frac{\partial u^*}{\partial y^*},\\
&\tau_{yy}^* + De\left(u^*\frac{\partial\tau_{yy}^*}{\partial x^*} + v^*\frac{\partial\tau_{yy}^*}{\partial y^*} - 2\frac{\partial v^*}{\partial y^*}\tau_{yy}^* - 2\frac{\partial v^*}{\partial x^*}\tau_{xy}^*\right)\\
&= -2\frac{\partial v^*}{\partial y^*},
\end{aligned}$$

$$\frac{\partial\tau_{xx}^*}{\partial x^*} + \frac{\partial\tau_{xy}^*}{\partial y^*} + \frac{\partial p^*}{\partial x^*} = 0, \quad \frac{\partial p^*}{\partial y^*} = 0, \tag{4}$$

where subscripts x and y indicate the x and y directions labeled in Fig. 2, respectively.

Equation (4) describes the mass conservation (the first equation), the constitutive relationship of the lubricant liquid unit (the following three equations), and the force balance of the sliding pairs (the last two equations). The De number is defined as $De{=}\lambda U/L$, where U is the sliding velocity and L is the width of the sliding area, as shown in Fig. 2. The relaxation time λ and the lubricant viscosity η describe the lubricant properties and are functions of the local shear rate, pressure, and temperature.[10] In this work, the normal load of tribopairs and the environmental temperature are assumed to be constant, therefore, λ and η are functions of the lubricant film thickness. The De number indicates that the response of the confined lubricant film during lubrication depends on the material's relaxation time λ and the velocity.

To solve Eq. (4), Tichy considered a regular perturbation expansion. Thus, the solutions are divided into two parts: the leading term (with superscript [l]) of the conventional lubrication solution and the higher-order correction (with superscript [D]) of the viscoelastic effect of the lubricant[1]

$$u = u^{[\mathrm{l}]} + Deu^{[\mathrm{D}]} + \cdots, \quad \tau_{xx} = \tau_{xx}^{[\mathrm{l}]} + De\tau_{xx}^{[\mathrm{D}]} + \cdots. \tag{5}$$

Assuming that the leading terms labeled by superscript [l] and higher-order terms labeled by superscript [D] are uncorrelated, the solutions of the leading and higher-order parts can be obtained separately and then combined to express Eq. (4) in two similar parts

$$\begin{aligned}
&\frac{\partial u^{*[\mathrm{l}]}}{\partial x^*} + \frac{\partial v^{*[\mathrm{l}]}}{\partial y^*} = 0, \quad \tau_{xx}^{*[\mathrm{l}]} = 0, \quad \frac{\partial p^{*[\mathrm{l}]}}{\partial y^*} = 0,\\
&\tau_{xy}^{*[\mathrm{l}]} = -\frac{\partial u^{*[\mathrm{l}]}}{\partial y^*}, \quad \tau_{yy}^{*[\mathrm{l}]} = -2\frac{\partial v^{*[\mathrm{l}]}}{\partial y^*}, \quad \frac{\partial\tau_{xy}^{*[\mathrm{l}]}}{\partial y^*} + \frac{\partial p^{*[\mathrm{l}]}}{\partial x^*} = 0,\\
&\frac{\partial u^{*[\mathrm{D}]}}{\partial x^*} + \frac{\partial v^{*[\mathrm{D}]}}{\partial y^*} = 0,\\
&\tau_{xx}^{*[\mathrm{D}]} = 2\frac{\partial u^{*[\mathrm{l}]}}{\partial y^*}\tau_{xy}^{*[\mathrm{l}]} = -2\left(\frac{\partial u^{*[\mathrm{l}]}}{\partial y^*}\right)^2,\\
&\tau_{xy}^{*[\mathrm{D}]} + \frac{\partial u^{*[\mathrm{D}]}}{\partial y^*}\\
&= -u^{*[\mathrm{l}]}\frac{\partial\tau_{xy}^{*[\mathrm{l}]}}{\partial x^*} - v^{*[\mathrm{l}]}\frac{\partial\tau_{xy}^{*[\mathrm{l}]}}{\partial y^*} + \frac{\partial u^{*[\mathrm{l}]}}{\partial y^*}\tau_{yy}^{*[\mathrm{l}]},\\
&\tau_{yy}^{*[\mathrm{D}]} + 2\frac{\partial v^{*[\mathrm{D}]}}{\partial y^*}\\
&= -u^{*[\mathrm{l}]}\frac{\partial\tau_{yy}^{*[\mathrm{l}]}}{\partial x^*} - v^{*[\mathrm{l}]}\frac{\partial\tau_{yy}^{*[\mathrm{l}]}}{\partial y^*} + 2\frac{\partial v^{*[\mathrm{l}]}}{\partial y^*}\tau_{yy}^{*[\mathrm{l}]} + 2\frac{\partial v^{*[\mathrm{l}]}}{\partial x^*}\tau_{xy}^{*[\mathrm{l}]},\\
&\frac{\partial\tau_{xx}^{*[\mathrm{D}]}}{\partial x^*} + \frac{\partial\tau_{xy}^{*[\mathrm{D}]}}{\partial y^*} + \frac{\partial p^{*[\mathrm{D}]}}{\partial x^*} = 0.
\end{aligned}$$

2.2. Dynamic rheological model of the thin-film lubrication

The following assumptions of the rheology of the thin-film lubricant, including the increased relaxation time, development of yielding stress, and power-law viscosity behavior, are based on the experimental results of previous studies.[2,8–10,13,14]

2.2.1. Relaxation time

The relaxation time of a liquid or a lubricant is the ratio of the viscosity to the elastic modulus of the liquid. For water, the relaxation time is typically at the order of 10^{-12} s, while for the lubricant oils confined in a pair of meshed gears, the fluid relaxation times are typically at the order of 10^{-6} s.[15] In addition, the response of the lubricant during the lubrication process is typically strongly time-dependent if the shearing time is of the same order.

In the previous thin-film lubrication simulations,[1,10] the relaxation time was adopted as a constant and similar to the characteristic value of the bulk fluid, despite the fact that the lubricant generally contains polymeric additive and may exhibit solid-like properties as the film thickness decreases to the nanometer level,[8,9] which causes the relaxation time of lubricant to vary in a large amplitude. In this study, the relaxation time of the lubricant is described using a power-law distribution based on the previous experimental results, which is dependent on the dynamic film thickness of the lubricant h

$$\lambda = \lambda_0 \mathrm{e}^{c/h}, \tag{6}$$

where h is the film thickness, which varies with the sliding velocity under a constant normal load, and c is a constant, which is adopted as 250 nm in this study since the film thickness of thin-film lubrication is generally reported to be at the scale of nanometers to micrometers, and the confined polymer film is reported to present a viscoelastic property when the thickness decreases below 250 nm in the previous experiments.[16] The model includes a constant lubricant relaxation time λ_0, which is adopted as 10^{-6} s in this study according to the typical relaxation time of lubricant reported in meshed gears.[15] The relaxation time of lubricant film may increase in a power-law distribution as the lubricant film thickness h decreases. These trends suggest that when the film is sufficiently thick, the relaxation time λ approaches the characteristic relaxation time of bulk liquid λ_0. As the film thickness decreases to the micrometer/nanometer scale, the relaxation time λ becomes much longer than that of the bulk liquid λ_0, indicating that the confined lubricant may exhibit solid-like properties.

2.2.2. Shear stress of the adsorbed thin-film layer

When a lubricant is confined within the contact area, a near-surface lubricant layer may stick onto the solid surface by chemical or physical adsorption to protect the surface, resulting in a high-viscosity layer adjacent to the solid surface.[2,14] This high-viscosity layer has two effects. Firstly, the high-viscosity layer mainly results from the polar additives in the oil, which is generally immobile on the substrate during shearing. Thus, the actual shearing plane is located at a distance b away from the solid wall, as shown in Fig. 3. Secondly, the shear stress at the shearing plane may increase (see Eq. (7)). In this paper, the thickness of this strongly adsorbed layer is defined as b, where a lower b value indicates that the lubricant can be thoroughly adsorbed or spread onto the substrate to form a thin layer.

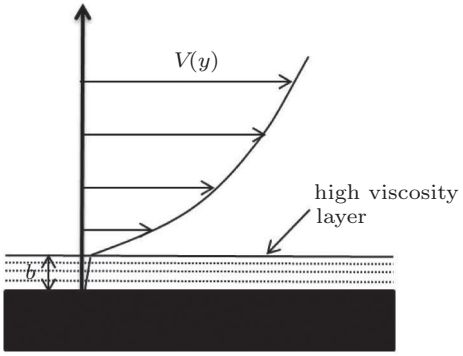


Fig. 3. Schematic diagram of the adsorbed high-viscosity layer and shearing plane during sliding, where b indicates the layer thickness and $V(y)$ is the velocity distribution of the lubricant.

Note that the existence of a high-viscosity layer has been demonstrated in many previous studies on the boundary properties of viscous liquids.[2,14] The immobile layer may be several molecules thick according to a molecular dynamics simulation,[17] because the liquid–solid interaction is stronger than the liquid–liquid interaction. In this paper, the layer thickness b is an essential parameter for describing the interaction between the lubricant and the solid substrate. Since we assume that this boundary layer is a high-viscosity layer, we estimate the thickness according to experimental results in the literature. It has been shown that the viscosity of the confined liquids begins to increase when the film thickness is less than several tens of nanometers or several hundred nanometers depending on the kind of liquid and the liquid–solid interaction. This parameter is similar to the boundary slip length.[14] However, in this study, b indicates a negative slip length because the adsorbed boundary layer is assumed to be immobile during shearing. The immobile film will cause the increase of the local shear rate. Then the shear stress can be shown as follows:

$$\tau = \eta \frac{U}{(h-2b)} = \tau_0 \frac{1}{(1-2b/h)}, \tag{7}$$

where τ_0 is the local shear stress when $b = 0$ and is calculated directly from Eq. (4); b is the negative slip length (the adsorbed film thickness), which exists on the surfaces of the sliding tribopair; and h is the dynamic lubricant film thickness, which is also calculated from Eq. (4). This equation shows that b

is dependent on the interfacial interaction. As the dynamic film thickness decreases to approximately 2b, the shear stress increases sharply, indicating that the lubricant is closely confined.

From an engineering perspective, the negative slipping length may indicate the interfacial bonding between the lubricant and substrate or the disjoining pressure within the adsorbed boundary layer.[14] On the confining substrate surfaces, the adsorbed thin-film layers are assumed to be uniform and unbroken (integrated) during the sliding process. A small negative slipping length b indicates that the disjoining pressure in the lubricant thin film can be much higher, according to a b^{-3} dependence.[14] Thus, the thin film can be closely confined and uniformly spread onto the substrate surface. Clearly, if a certain lubricant has a stronger interfacial interaction with the substrate, e.g., it contains a polar additive. It is not typically broken during the shearing action and can be kept until a very thin, uniform, and unbroken layer forms to protect the substrate surfaces. In this situation of high disjoining pressure, the negative slipping length b is very small. Thus, the negative slipping length b indicates the potential unbroken protection film thickness of the lubricant. The smaller the b, the higher the disjoining pressure that exists in the adsorbed layer. In contrast, a larger negative slipping length implies a smaller disjoining pressure within the adsorbed thin film. In this situation, the lubricant film can be easily broken during the shearing process.

Combining Eqs. (1), (2), (6), and (7), we can obtain a unified dynamic rheological model of the lubricant while transforming from the hydrodynamic regime to the boundary regime. The proposed model is referred to as a dynamic rheological model because two new parameters are adopted to describe the thin-film lubrication: λ describes the solid-like transition behavior and b describes the interfacial interaction with the substrate. Furthermore, both λ and the shear stress related with b are assumed to depend on the lubricant film thickness h, while h varies dynamically and significantly with the sliding velocity. It is expected that the unified model might capture the state transitions of the lubricant in a wide range of lubrication regimes, from hydrodynamic lubrication through thin-film lubrication to boundary lubrication.

2.3. Calculation process

To focus on the lubricant rheology in all of the lubrication regimes, we investigated the lubrication behavior of the convergent wedge geometry shown in Fig. 2 under a constant normal load. The aforementioned non-dimensional equations were solved using the finite difference method and an iterative process. A diagram of the calculation process is shown in Fig. 4. The calculation was performed using Matlab software.

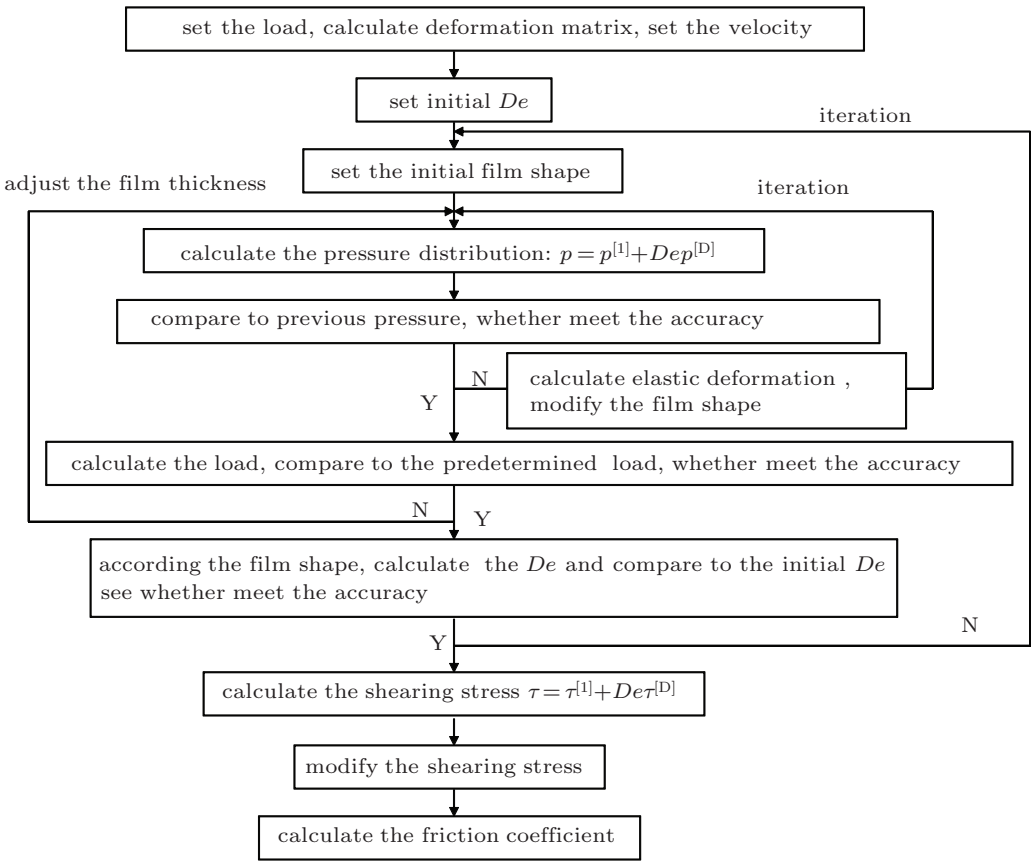


Fig. 4. Calculation diagram of Eq. (4) with the dynamic rheological model of the thin-film layer.

3. Results and discussion

In this study, we calculated the COF of the sliding pairs in Fig. 2 using the lubrication Eq. (4) for a wide range of sliding velocities. We obtained a series of lubrication Stribeck curves using different λ_0 and b values to develop a qualitative picture of the lubrication behavior emerging from our dynamic rheological model. We expected that the adopted rheological parameters λ_0 and b will allow us to calibrate our model to the experimental results (as shown in Fig. 1). The adopted normal load was 50 N.

3.1. Stribeck curve and lubrication transition parameter

The COF simulation results over a velocity range from 0.0001 m/s to 10 m/s are shown in Fig. 5. First, a typical Stribeck curve is observed, where the minimum COF region indicates the transition from dynamic lubrication through thin-film lubrication to boundary lubrication. Second, the minimum film thickness of the lubricant decreases as the velocity decreases. For velocities below 0.1 m/s, the film thickness decreases to several hundred nanometers. This lubricant film thickness on a smooth surface indicates a thin-film lubrication regime. The decreased film thickness may result in a power-law increase of the relaxation time λ according to Eq. (6). Thus, the *De* number increases as the sliding velocity decreases, even though the definition of the *De* number suggests that it is proportional to the sliding velocity. In Fig. 5(c), when the sliding velocity is greater than 10 m/s, the *De* number approaches zero. This result indicates that the lubricant film acts as a Newtonian liquid when the Stribeck curve is in the hydrodynamic lubrication regime. The *De* number increases with the decrease of the sliding velocity. At a velocity of approximately 0.01 m/s, the *De* number approaches 1.0, corresponding to the minimum COF in the Stribeck curve.

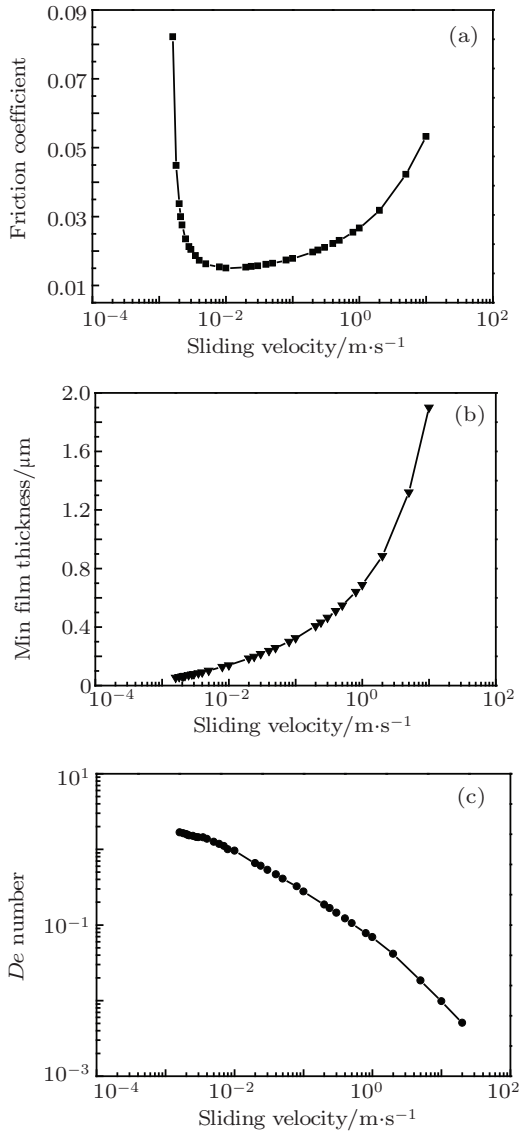


Fig. 5. (a) Stribeck curves, variations of (b) the film thickness and (c) *De* number for different sliding velocities. The minimum COF in the Stribeck curve appears at the transition from hydrodynamic lubrication to boundary lubrication and corresponds to a critical *De* of 1.0 and a minimum film thickness of approximately 100 nm.

Several mechanisms could be invoked to explain the minimum COF and low-COF region in the Stribeck curve. According to the definition of the *De* number, variations in the *De* number in Fig. 5 indicate that the lubrication state is closely related to the rheological state of the lubricant. In the hydrodynamic lubrication regime, the *De* number approaches zero and the lubricant exhibits mainly liquid properties. Thus, the lubricant viscosity accounts for the velocity-increased friction laws. For very low velocities, the *De* number is greater than 1.0, which indicates that the confined thin film has become a solid-like layer. As a result, the COF increases sharply. In the transient regime, the *De* number approaches 1.0, this indicates that the lubricant may transform from the liquid state to a solid-like or ordering state. Therefore, a low friction may result from the ordered structure of the lubricant induced by the confinement. Recently, low COFs resulting from the ordering of confined molecules have been experimentally established on single-asperity contacts at critical load, velocity, and temperature.[10,17] SFA experimental results[18] also indicate a critical transition parameter of the boundary layer when the *De* number is 1.0, which is also attributed to the increased relaxation time of the boundary layer.

3.2. Effect of the rheological parameters

To determine the effects of the rheological parameters on the thin-film lubrication properties, a series of Stribeck curves were calculated for different negative slip lengths b and characteristic relaxation time λ_0.

3.2.1. Effect of the negative slip length

Figure 6 shows a series of Stribeck curves calculated for different negative slip lengths b. The variation of $2b/h$ versus the sliding velocity is also presented. The following conclusions are drawn from the figure.

(i) When the sliding velocity is greater than 10 m/s, the COFs for different b values converge. The ratio $2b/h$ is also very small. These results suggest that higher sliding velocities result in a larger film thickness, the negative slip length b becomes less significant as part of the total gap, this also suggests the minor role of the high-viscosity adsorbed layer in the hydrodynamic lubrication regime.

(ii) As the sliding velocity decreases, the film thickness h decreases and the ratio $2b/h$ approaches 1.0. These results suggest that the sliding resistance may increase sharply according to Eq. (7). Therefore, the COF increases sharply.

(iii) The negative slip length b plays a significant role in the COF curves of the thin-film lubrication regime. When b is quite large, at least 200 nm, $2b/h$ approaches 1.0 quickly as the sliding velocity decreases to less than 1.0 m/s. Thus, the COF increases sharply, as indicated by the triangle points. The minimum COF occurs at a velocity of 1.0 m/s. As the negative slip length decreases further to 25 nm, the entire Stribeck curve shifts left on the velocity axis, the minimum COF decreases from approximately 0.05 to 0.015, and the low-COF regime widens. When b is less than 25 nm, the lubricant is closely adsorbed onto the substrate surface, resulting in a rather stable low-COF regime over a wide sliding velocity range.

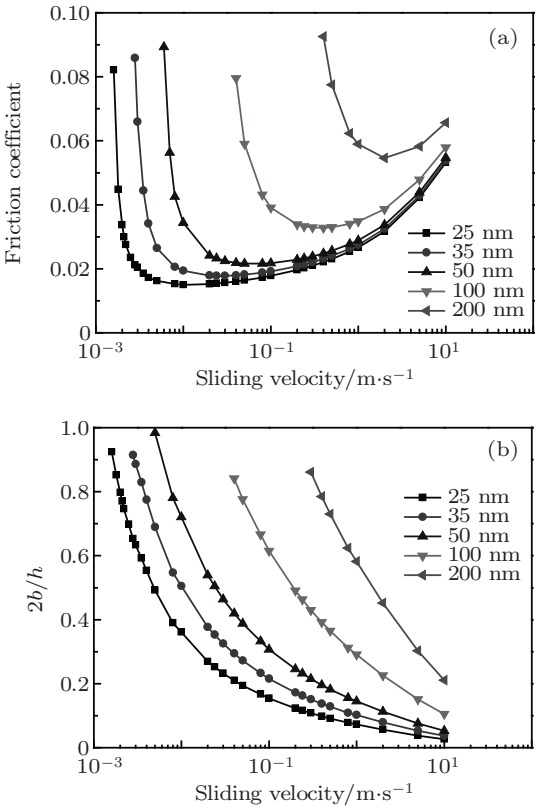


Fig. 6. (a) Stribeck curves with negative slip lengths b of $b = 25$ nm, 35 nm, 50 nm, 100 nm, and 200 nm. (b)The ratio of $2b$ to the film thickness h increases as the sliding velocity decreases and approaches 1.0 at low velocities. As the negative slip length decreases, the entire Stribeck curve shifts left along the velocity axis and the minimum COF decreases. Lower b values result in a wider low-COF region and a lower minimum COF. When $2b/h$ approaches 1.0, the COF increases sharply.

The Stribeck curve denoted by solid triangle points in Fig. 6(a) is most attractive for engineering applications, such as sealing and stable lubrication, because the minimum COF and low-COF region can be maintained within a rather wide sliding velocity range. This velocity range clarifies the transition regime from hydrodynamic lubrication to boundary lubrication. In this regime, the negative slipping length b can be a quantitative indicator to describe the effect of the lubricant-substrate interaction on the lubrication properties.

3.2.2. Effect of the relaxation time

Figure 7 shows the Stribeck curves for different characteristic relaxation time λ_0. Obviously, the characteristic relaxation time plays a significant role in determining the width of the low-COF region. As λ_0 increases, the entire Stribeck curve shifts left and has a widened low-COF region.

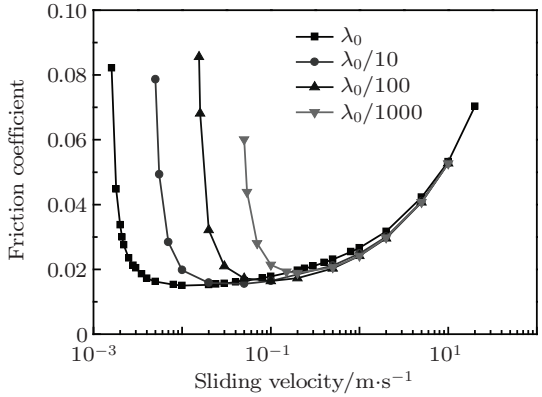


Fig. 7. Stribeck curves with different relaxation time of λ_0, $\lambda_0/10$, $\lambda_0/100$, and $\lambda_0/1000$ for a negative slip length of $b = 25$ nm. As the characteristic relaxation time increases, the entire Stribeck curve shifts left on the velocity axis, the low-COF region widens, and the minimum COF decreases slightly.

As an indicator of the liquid or solid nature of the lubricant, the increased characteristic relaxation time λ_0 may indicate a confinement-induced ordering/glass transition of ultra-thin films.[9,10] The confined solid-like or ordered thin film can then result in a velocity-independent low COF within a certain velocity region. The velocity-independent low COF can be considered as a primary characteristic of the thin-film layer when the layer becomes solid-like.[19] A lower friction is caused by the confinement of the adsorbed polymer layer, as has been proven by experimental studies.[17] The results in Fig. 7 suggest that a lubricant with a higher λ_0 may widen the low-COF region in the Stribeck curve; wider low-COF regions are advantageous for engineering applications.

By now, with regard to the thin-film lubricant layer, two important rheological parameters have been presented in addition to the conventional bulk viscosity: the negative slip length b and the relaxation time of the boundary layer λ. The former is an indicator of the interfacial effect of the boundary layer,

while the latter is an indicator of the viscoelasticity or solid-like phase of the confined thin-film layer (also related to the *De* number). The proposed rheological parameters dominate the transition locations of the thin-film lubricant and the width of the low-COF region. More importantly, the two adopted parameters dominate the properties of the thin-film lubrication. Based on our calculation results, a lubricant with a low b is preferred for thin-film lubrication because a low b indicates a strong interfacial interaction to maintain the uniformity of the thin film and sustain the shear flow. For a given b, the thin-film layer with a higher relaxation time λ_0 may become more solid-like or ordered, resulting in a widened low-COF range. Considering the molecular structure and interaction, this result may be due to the fact that when a lubricant layer has a strong interfacial interaction with the substrate to result in a small negative slip length b, and also has a strong intermolecular interaction to result in a large relaxation time, the thin-film lubrication state will be promoted and maintained over a rather wide velocity range.

The transition properties of the Stribeck curve play a significant role in lubrication design.[5,20] Previous studies have investigated the manner in which the Stribeck curve changes or the minimum COF shifts by tailoring the surface wetting property[5] or texturing.[20] In this paper, the effects of lubricant rheology on the Stribeck curve and thin-film lubrication behaviors are investigated based on the dynamic rheology model. Our work indicates that the thin-film lubrication properties in the Stribeck curve are strongly determined by the lubricant rheological properties, such as the thickness of the high-viscosity layer and the relaxation time of the thin film.

4. Conclusions

When the lubricant is confined to a thin-film layer, its rheological properties differ a lot from the bulk rheology, exhibiting increased yielding stress, a solid-like transition, and increased relaxation time. A dynamic rheological model of a lubricant was proposed to describe the lubrication state transitions from the hydrodynamic lubrication regime through thin-film lubrication to the boundary lubrication regime. The dynamic rheological model was developed with two parameters for the lubricant thin film: the negative slipping length b, which describes the lubricant and substrate interfacial interaction, and the relaxation time parameter λ, which describes the solid-like transition of the lubricant film.

Based on Tichy's extended lubrication equations with a perturbation of the *De* number, a series of lubrication Stribeck curves were calculated with different rheological parameters. The results show that the minimum COF is within the thin-film lubrication regime when the lubrication state transforms from the hydrodynamic regime into the thin-film regime. At the minimum COF, the confined lubricant layer has a critical *De* number of approximately 1.0, which corresponds to the confinement-induced liquid-to-solid transition of the lubricant film. The proposed rheological parameters play a significant role in the thin-film lubrication regime. A smaller negative slipping length b results in a lower minimum COF and a wider low-COF region, while a larger characteristic relaxation time λ_0 results in a widened velocity-independent low-COF region. Both the lower-COF region and the widened velocity-independent COF region are of benefit for low energy dissipation applications. Thus the dynamic rheological model has practical significance for investigating the mechanisms of thin film lubrication and evaluating interfacial and molecular properties of lubricants in thin-film lubrication.

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