Screw dislocations interacting with two asymmetrical interfacial cracks emanating from an elliptical hole*

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The interaction between screw dislocations and two asymmetrical interfacial cracks emanating from an elliptic hole under loads at infinity is studied. The closed-form solution is derived for complex potentials. The stress intensity factor and the critical applied stress for the dislocation emission are also calculated. In the limiting cases, well-known results can be obtained from the present solutions. Moreover, new exact solutions for a screw dislocation interacting with some complicated cracks are derived. The results show that the shielding effect increases with the increase in the length of the other cracks and the minor semi axis, but decreases with the increase of dislocation azimuth. The repulsion acting on the dislocation from the other phase and the other crack extend in the horizontal direction, which makes the dislocation emission at the crack tip take place more easily, but the minor semi axis of the elliptical hole extending in the vertical direction makes it more difficult.

Keywords: interfacial crack, elliptic hole, shielding effect, dislocation emission

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1. Introduction

The behavior of material near a crack tip forms the basis for all fracture theories. In the literature,[1–16] the interaction between cracks and dislocations was studied in order to build a fracture theory that describes plastic deformation based on the behavior of dislocations near the crack tip. Once dislocations are emitted, they move out of the crack tip area, leaving behind a dislocation-free zone. An internal back stress due to dislocations emitted from the crack tip accommodates the stress intensity due to an applied load, causing an increase of fracture toughness of materials. Rice and Thomson[17] obtained a quantitative criterion for ductile verse brittle behavior. Wang[18] analyzed dislocation nucleation and emission with a slightly modified Rice framework. In addition, a few works reported dislocation emission criteria for a V-shape crack[19] and a blunt crack tip.[20,21]

The behavior of dislocations near an interface plays an important role in determining the mechanical properties of composite materials and polycrystals. The study of dislocations interacting with interfacial cracks in the mechanics and material science is motivated by the need for a better understanding of the mechanism of strengthening and toughening of materials. Head[22] evaluated the image forces on a screw dislocation due to a grain boundary or a free surface with an oxide film. Pacheco and Mura[23] investigated a screw dislocation near an interface. Chu[24] studied the stress field of a screw dislocation in a two-phase isotropic thin film by using the conformal mapping technique. Shiue *et al.*[25] investigated the elastic interaction between screw dislocation and an interfacial surface crack. Lee and Huang[26] studied the elastic interaction between screw dislocation and an interfacial surface crack emanating from a semi-elliptic hole. Lee[27] obtained a general solution of screw dislocations near an interfacial crack. Ouyang and Lee[28] investigated screw dislocations and a crack penetrating through an interface. Zhang and Qian[29] analyzed the emission of a screw dislocation dipole from an interface. Wu *et al.*[30] studied interface and interfacial cracks interacting with screw dislocations in piezoelectric bimaterials. Li *et al.*[31] investigated an uniformly moving screw dislocation interacting with interface cracks in anisotropic bimaterials. Liu *et al.*[32] analyzed the interaction between screw dislocations and an interfacial blunt crack with a viscoelastic interface.

The elastic interactions mentioned above for different boundaries have been investigated, but attention has only been focused on simple shapes of cracks such as semi-infinite cracks, internal cracks, and surface cracks. Sometimes cracks can grow from an elliptic hole or a circular hole under the action of an applied stress. It is very interesting and meaningful to present the explicit and exact solutions in closed-form for these complicated crack problems, since these solutions can provide a theoretical analysis for fracture problems, and can

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also serve as a benchmark for judging the accuracy and efficiency of various numerical and approximate methods.[33–38] Sih[39] investigated two symmetrical edge cracks originating from a circular hole using the mapping function method, and presented the exact solution of the stress intensity factors. Yokobori *et al.*[40] addressed the exact solution of a single crack at the edge of an elliptical hole by using the stress function method. Shiue and Lee[41] studied the elastic interaction between screw dislocations and cracks emanating from an elliptic hole. Lam *et al.*[42] obtained the Dugdale solution for cracks at the edge of an elliptical hole in an infinite or finite plate. Guo *et al.*[43–45] investigated cracks emanating from an elliptical hole in a piezoelectric material and in a one-dimensional hexagonal quasi-crystal. Rogowski and Bogdan[46] studied the mode III cracks emanating from an elliptical hole in the piezo-electro-magneto-elastic materials.

However, no work has been done involving the interaction between interfacial cracks emanating from an elliptic hole and screw dislocations. Hence, it stimulates us to study the elastic interaction of a screw dislocation and two asymmetrical interfacial cracks emanating from an elliptical hole under remotely anti-plane mechanical loading. The complex potential, stress field, image force on the dislocation, stress intensity factor, and the critical condition for the dislocation emission are derived. This generalized investigation can be reduced to several special cases. This paper focuses on the anti-plane problem of screw dislocation and crack propagation in a blunt interfacial crack tip. Because the plane problem of edge dislocation and crack propagation in a blunt interfacial crack tip is more complex, it will be investigated in the future.

2. Solution

Consider a two-phase material with shear modulus μ_1 for phase 1 (S^+) and shear modulus μ_2 for phase 2 (S^-). The interface (real axis) contains two asymmetrical edge interfacial cracks emanating from an elliptical hole, as shown in Fig. 1. Here a and b are the lengths of the major and the minor semi axes of the elliptical hole, respectively, and l_1 and l_2 are the lengths of the right and the left edge interfacial cracks, respectively. A screw dislocation of Burgers vector b_z is located at z_0 in phase 1. The dislocation line is perpendicular to the z plane.

The problem under consideration is an anti-plane elastic strain one. The anti-plane displacement ω_j $(j=1,\,2)$, shear stresses σ_{xzj} and σ_{yzj} $(j=1,\,2)$ can be written in terms of an analytical function $f_j(z)$ of the complex variable $z=x+\mathrm{i}y$[47] as

$$\omega_j = \left[f_j(z)+\overline{f_j(z)}\right]/2, \tag{1}$$

$$\sigma_{xzj}-\mathrm{i}\sigma_{yzj} = \mu_j f_j'(z). \tag{2}$$

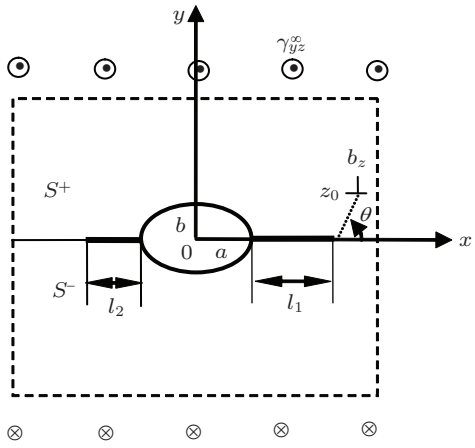


Fig. 1. Screw dislocation and two asymmetrical interfacial cracks emanating from an elliptical hole.

Now, we introduce the transform function

$$\zeta = G(z) = \frac{z}{a+b} + \frac{b}{z+\left(z^2-a^2+b^2\right)^{1/2}}, \tag{3}$$

which transforms a linear interfacial crack in the ζ plane $(a_1 \le \zeta \le b_1)$ into two asymmetrical edge interfacial cracks emanating from an elliptical hole. Here a_1 and b_1 are the left and the right tips, respectively, and

$$a_1 = -\frac{a+l_2}{a+b} - \frac{b}{a+l_2+\left(b^2+2al_2+l_2^2\right)^{1/2}},$$

$$b_1 = \frac{a+l_1}{a+b} + \frac{b}{a+l_1+\left(b^2+2al_1+l_1^2\right)^{1/2}}.$$

Under the action of a mode III stress $\Gamma' = -\mathrm{i}\sigma_{yz}^{\infty}$, the complex potential in the ζ plane is given as[31]

$$\begin{aligned}\frac{\mathrm{d}f_1(\zeta)}{\mathrm{d}\zeta} = {} & \frac{b_z\mu_2}{2\pi\mathrm{i}(\mu_1+\mu_2)}\left[\frac{\sqrt{(\zeta_0-a_1)(\zeta_0-b_1)}}{\zeta-\zeta_0}\right. \\ & \left.+\frac{\sqrt{(\bar{\zeta}_0-a_1)(\bar{\zeta}_0-b_1)}}{\zeta-\bar{\zeta}_0}+2\right]\frac{1}{\sqrt{(\zeta-a_1)(\zeta-b_1)}} \\ & +\frac{b_z\mu_1}{2\pi\mathrm{i}(\mu_1+\mu_2)}\left(\frac{1}{\zeta-\zeta_0}-\frac{1}{\zeta-\bar{\zeta}_0}\right) \\ & +\frac{(\mu_2\Gamma'-\mu_1\bar{\Gamma}')(2\zeta-a_1-b_1)}{2\mu_1(\mu_1+\mu_2)\sqrt{(\zeta-a_1)(\zeta-b_1)}},\end{aligned} \tag{4}$$

where $\bar{\zeta}_0$ is the complex conjugate to ζ_0.

From Eq. (3) and expression (4), the complex potential of a screw dislocation interacting with two asymmetrical edge cracks emanating from an elliptical hole can be derived as

$$\begin{aligned}\frac{\mathrm{d}f_1(z)}{\mathrm{d}\zeta} = {} & \frac{b_z}{2\pi\mathrm{i}(\mu_1+\mu_2)}\left\{\frac{\mu_1}{G(z)-G(z_0)}\right. \\ & -\frac{\mu_1}{G(z)-\overline{G(z_0)}}+\frac{\mu_2}{\sqrt{(G(z)-a_1)(G(z)-b_1)}} \\ & \times\left[\frac{\sqrt{(G(z_0)-a_1)(G(z_0)-b_1)}}{G(z)-G(z_0)}\right. \\ & \left.\left.+\frac{\sqrt{\left(\overline{G(z_0)}-a_1\right)\left(\overline{G(z_0)}-b_1\right)}}{G(z)-\overline{G(z_0)}}+2\right]\right\}\end{aligned}$$

$$+\frac{(a+b)\left(\mu_2\Gamma-\mu_1\bar{\Gamma}\right)[2G(z)-a_1-b_1]}{2\mu_1(\mu_1+\mu_2)\sqrt{(G(z)-a_1)(G(z)-b_1)}}, \quad (5)$$

where $\Gamma=-\mathrm{i}\sigma_{yz}^{\infty}$, and $\overline{G(z_0)}$ is the complex conjugate to $G(z_0)$.

The stress field produced by the screw dislocation and the mode III load is

$$\begin{aligned}
&\sigma_{xz1}-\mathrm{i}\sigma_{yz1}\\
&=\frac{\mu_1 \mathrm{d}f_1(z)}{\mathrm{d}z}=\frac{\mu_1 \mathrm{d}f_1(z)}{\mathrm{d}\zeta}\frac{\mathrm{d}\zeta}{\mathrm{d}z}\\
&=\left\{\frac{1}{a+b}-\frac{b}{(z^2-a^2+b^2)^{1/2}\left[z+(z^2-a^2+b^2)^{1/2}\right]}\right\}\\
&\times\left\{\frac{\mu_1\mu_2 b_z}{2\pi\mathrm{i}(\mu_1+\mu_2)\sqrt{(G(z)-a_1)(G(z)-b_1)}}\right.\\
&\times\left[\frac{\sqrt{(G(z_0)-a_1)(G(z_0)-b_1)}}{G(z)-G(z_0)}\right.\\
&\left.+\frac{\sqrt{\left(\overline{G(z_0)}-a_1\right)\left(\overline{G(z_0)}-b_1\right)}}{G(z)-\overline{G(z_0)}}+2\right]\\
&+\frac{\mu_1^2 b_z}{2\pi\mathrm{i}(\mu_1+\mu_2)}\left[\frac{1}{G(z)-G(z_0)}-\frac{1}{G(z)-\overline{G(z_0)}}\right]\\
&\left.-\mathrm{i}\sigma_{yz}^{\infty}\frac{(a+b)[2G(z)-a_1-b_1]}{2\sqrt{(G(z)-a_1)(G(z)-b_1)}}\right\}. \quad (6)
\end{aligned}$$

If $\mu_1=\mu_2$, the elastic interaction between a screw dislocation and two asymmetrical edge cracks emanating from an elliptical hole can be calculated. With Eq. (6), we have

$$\sigma_{xz}-\mathrm{i}\sigma_{yz}=\sigma_{xz}^{b}-\mathrm{i}\sigma_{yz}^{b}+\sigma_{xz}^{\Gamma}-\mathrm{i}\sigma_{yz}^{\Gamma}, \quad (7)$$

where $\sigma_{xz}^{b}-\mathrm{i}\sigma_{yz}^{b}$ and $\sigma_{xz}^{\Gamma}-\mathrm{i}\sigma_{yz}^{\Gamma}$ are the stress fields produced by the screw dislocation and the mode III load, respectively. In addition,

$$\begin{aligned}
&\sigma_{xz}^{b}-\mathrm{i}\sigma_{yz}^{b}\\
&=\left\{\frac{1}{a+b}-\frac{b}{(z^2-a^2+b^2)^{1/2}\left[z+(z^2-a^2+b^2)^{1/2}\right]}\right\}\\
&\times\left\{\frac{\mu_1 b_z}{4\pi\mathrm{i}}\left[\frac{1}{G(z)-G(z_0)}-\frac{1}{G(z)-\overline{G(z_0)}}\right]\right.\\
&+\frac{\mu_1 b_z}{4\pi\mathrm{i}\sqrt{(G(z)-a_1)(G(z)-b_1)}}\\
&\times\left[\frac{\sqrt{(G(z_0)-a_1)(G(z_0)-b_1)}}{G(z)-G(z_0)}\right.\\
&\left.\left.+\frac{\sqrt{\left(\overline{G(z_0)}-a_1\right)\left(\overline{G(z_0)}-b_1\right)}}{G(z)-\overline{G(z_0)}}+2\right]\right\}
\end{aligned}$$

is identical to the results of Shiue and Lee,[41]

$$\begin{aligned}
&\sigma_{xz}^{\Gamma}-\mathrm{i}\sigma_{yz}^{\Gamma}\\
&=-\mathrm{i}\sigma_{yz}^{\infty}\frac{(a+b)[2G(z)-a_1-b_1]}{2\sqrt{(G(z)-a_1)(G(z)-b_1)}}\\
&\times\left\{\frac{1}{a+b}-\frac{b}{(z^2-a^2+b^2)^{1/2}\left[z+(z^2-a^2+b^2)^{1/2}\right]}\right\}
\end{aligned}$$

and is in agreement with the results of Guo *et al.*[44]

3. Discussion

In some limiting special cases, new exact solutions of some crack configurations can be derived from the present results, as shown in Fig. 2, a discussion in detail is given as follows.

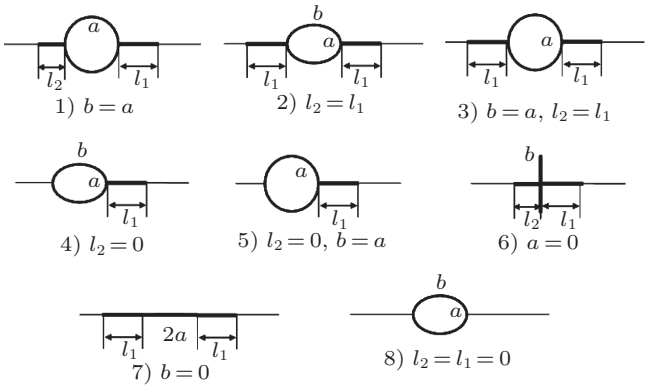


Fig. 2. Some special cases.

1) If $b=a$, equation (6) reduces to the case of a screw dislocation interacting with two asymmetrical edge cracks emanating from an circular hole

$$\begin{aligned}
&\sigma_{xz1}-\mathrm{i}\sigma_{yz1}\\
&=\left(\frac{1}{2a}-\frac{a}{2z^2}\right)\left\{\frac{\mu_1^2 b_z}{2\pi\mathrm{i}(\mu_1+\mu_2)}\right.\\
&\times\left[\frac{1}{G_1(z)-G_1(z_0)}-\frac{1}{G_1(z)-\overline{G_1(z_0)}}\right]\\
&+\frac{\mu_1\mu_2 b_z}{2\pi\mathrm{i}(\mu_1+\mu_2)\sqrt{(G_1(z)-a_2)(G_1(z)-b_2)}}\\
&\times\left[\frac{\sqrt{(G_1(z_0)-a_2)(G_1(z_0)-b_2)}}{G_1(z)-G_1(z_0)}\right.\\
&+\frac{\sqrt{\left(\overline{G_1(z_0)}-a_2\right)\left(\overline{G_1(z_0)}-b_2\right)}}{G_1(z)-\overline{G_1(z_0)}}+2\Bigg]\\
&\left.-\mathrm{i}\sigma_{yz}^{\infty}\frac{a[2G_1(z)-a_2-b_2]}{\sqrt{(G_1(z)-a_2)(G_1(z)-b_2)}}\right\}, \quad (8)
\end{aligned}$$

where $a_2=-\frac{1}{2}\left(\frac{a+l_2}{a}+\frac{a}{a+l_2}\right)$, $b_2=\frac{1}{2}\left(\frac{a+l_1}{a}+\frac{a}{a+l_1}\right)$, and $G_1(z)=\frac{1}{2}\left(\frac{z}{a}+\frac{a}{z}\right)$.

2) If the crack length l_2 approaches to l_1, equation (6) becomes

$$\begin{aligned}
&\sigma_{xz1}-\mathrm{i}\sigma_{yz1}\\
&=\left\{\frac{1}{a+b}-\frac{b}{(z^2-a^2+b^2)^{1/2}\left[z+(z^2-a^2+b^2)^{1/2}\right]}\right\}\\
&\times\left\{\left[\frac{\sqrt{(G^2(z_0)-b_1^2)}}{G(z)-G(z_0)}+\frac{\sqrt{\left(\overline{G^2(z_0)}-b_1^2\right)}}{G(z)-\overline{G(z_0)}}+2\right]\right.\\
&\times\frac{\mu_1\mu_2 b_z}{2\pi\mathrm{i}(\mu_1+\mu_2)\sqrt{(G^2(z)-b_1^2)}}+\frac{\mu_1^2 b_z}{2\pi\mathrm{i}(\mu_1+\mu_2)}\\
&\times\left[\frac{1}{G(z)-G(z_0)}-\frac{1}{G(z)-\overline{G(z_0)}}\right]\\
&\left.-\mathrm{i}\sigma_{yz}^{\infty}\frac{(a+b)G(z)}{\sqrt{(G^2(z)-b_1^2)}}\right\}. \qquad (9)
\end{aligned}$$

Equation (9) gives the stress field of the elastic interaction between a screw dislocation and two symmetrical edge interfacial cracks emanating from an elliptical hole.

3) When $b=a$ and $l_2=l_1$, it can be further reduced to the case of a screw dislocation interacting with two symmetrical edge cracks emanating from an circular hole, then equation (9) can be simplified as

$$\begin{aligned}
&\sigma_{xz1}-\mathrm{i}\sigma_{yz1}\\
&=\left(\frac{1}{2a}-\frac{a}{2z^2}\right)\left\{\frac{\mu_1\mu_2 b_z}{2\pi\mathrm{i}(\mu_1+\mu_2)\sqrt{(G_1^2(z)-b_2^2)}}\right.\\
&\times\left[\frac{\sqrt{(G_1^2(z_0)-b_2^2)}}{G_1(z)-G_1(z_0)}+\frac{\sqrt{\left(\overline{G_1^2(z_0)}-b_2^2\right)}}{G_1(z)-\overline{G_1(z_0)}}+2\right]\\
&+\frac{\mu_1^2 b_z}{2\pi\mathrm{i}(\mu_1+\mu_2)}\left[\frac{1}{G_1(z)-G_1(z_0)}-\frac{1}{G_1(z)-\overline{G_1(z_0)}}\right]\\
&\left.-\mathrm{i}\sigma_{yz}^{\infty}\frac{2aG_1(z)}{\sqrt{(G_1^2(z)-b_2^2)}}\right\}. \qquad (10)
\end{aligned}$$

4) If the crack length l_2 tends to zero, i.e., $l_2=0$, equation (6) leads to

$$\begin{aligned}
&\sigma_{xz1}-\mathrm{i}\sigma_{yz1}\\
&=\left\{\frac{1}{a+b}-\frac{b}{(z^2-a^2+b^2)^{1/2}\left[z+(z^2-a^2+b^2)^{1/2}\right]}\right\}\\
&\times\left\{\frac{\mu_1\mu_2 b_z}{2\pi\mathrm{i}(\mu_1+\mu_2)\sqrt{(G(z)+1)(G(z)-b_1)}}\right.\\
&\times\left[\frac{\sqrt{(G(z_0)+1)(G(z_0)-b_1)}}{G(z)-G(z_0)}\right.\\
&+\frac{\sqrt{\left(\overline{G(z_0)}+1\right)\left(\overline{G(z_0)}-b_1\right)}}{G(z)-\overline{G(z_0)}}+2\Big]\\
&+\frac{\mu_1^2 b_z}{2\pi\mathrm{i}(\mu_1+\mu_2)}\left[\frac{1}{G(z)-G(z_0)}-\frac{1}{G(z)-\overline{G(z_0)}}\right]\\
&\left.-\mathrm{i}\sigma_{yz}^{\infty}\frac{(a+b)[2G(z)+1-b_1]}{2\sqrt{(G(z)+1)(G(z)-b_1)}}\right\}. \qquad (11)
\end{aligned}$$

This is the stress field of the elastic interaction between a screw dislocation and an edge interfacial crack emanating from an elliptical hole.

5) When $b=a$ and $l_2=0$, it can yield the result of a screw dislocation interacting with an edge crack emanating from an circular hole, then equation (11) becomes

$$\begin{aligned}
&\sigma_{xz1}-\mathrm{i}\sigma_{yz1}\\
&=\left(\frac{1}{2a}-\frac{a}{2z^2}\right)\left\{\frac{\mu_1^2 b_z}{2\pi\mathrm{i}(\mu_1+\mu_2)}\right.\\
&\times\left[\frac{1}{G_1(z)-G_1(z_0)}-\frac{1}{G_1(z)-\overline{G_1(z_0)}}\right]\\
&+\frac{\mu_1\mu_2 b_z}{2\pi\mathrm{i}(\mu_1+\mu_2)\sqrt{(G_1(z)+1)(G_1(z)-b_2)}}\\
&\times\left[\frac{\sqrt{(G_1(z_0)+1)(G_1(z_0)-b_2)}}{G_1(z)-G_1(z_0)}\right.\\
&+\frac{\sqrt{\left(\overline{G_1(z_0)}+1\right)\left(\overline{G_1(z_0)}-b_2\right)}}{G_1(z)-\overline{G_1(z_0)}}+2\Big]\\
&\left.-\mathrm{i}\sigma_{yz}^{\infty}\frac{(a+b)[2G_1(z)+1-b_2]}{2\sqrt{(G_1(z)+1)(G_1(z)-b_2)}}\right\}. \qquad (12)
\end{aligned}$$

6) If the major semi axis tends to zero, i.e., $a=0$, equation (6) can give the solution of a screw dislocation interacting with an interfacial crisscross crack.[48]

7) If the minor semi axis tends to zero, i.e., $b=0$, equation (6) can be reduced to the solution of a screw dislocation interacting with an interfacial crack.[27]

8) With $l_1=l_2=0$, we have $b_1=-a_1=1$, and equation (6) can be reduced to the case of the interaction between a screw dislocation and an interfacial blunt crack.[49] When $\mu_1=\mu_2$, equation (6) can be reduced to the case of a screw dislocation interacting with a blunt crack, which is in accordance with the results of Li *et al.*[50]

4. Shielding effect

The stress intensity factor at the right tip can be obtained from a stress field defined as

$$K_{\mathrm{III}} = \lim_{z\to a+l_1}\sqrt{2\pi\,(z-a-l_1)}\sigma_{yz1} = \sqrt{\frac{1}{a+b}-\frac{b}{\left(l_1^2+2al_1+b^2\right)^{1/2}\left[a+l_1+\left(l_1^2+2al_1+b^2\right)^{1/2}\right]}}$$
$$\times\left\{\frac{\mu_1\mu_2 b_z}{(\mu_1+\mu_2)\sqrt{2\pi\,(b_1-a_1)}}\left[\frac{\sqrt{(G(z_0)-a_1)\,(G(z_0)-b_1)}}{b_1-G(z_0)}+\frac{\sqrt{\left(\overline{G(z_0)}-a_1\right)\left(\overline{G(z_0)}-b_1\right)}}{b_1-\overline{G(z_0)}}+2\right]\right.$$
$$\left.+\sqrt{\frac{\pi}{2}}\,(a+b)\sqrt{b_1-a_1}\sigma_{yz}^{\infty}\right\}. \tag{13}$$

It is found that the stress intensity factor induced by the screw dislocation is just the same as that in the single-phase case if the shear modulus of the single-phase medium μ is replaced by the effective shear modulus $2\mu_1\mu_2/(\mu_1+\mu_2)$.

Figure 3 shows the normalized stress intensity factor (SIF) $K_{\mathrm{III}}/\mu_1\sqrt{b_z}$ versus $|z_0-a-l_1|/b_z$ for different values of μ_2/μ_1 at $\sigma_{yz}^{\infty}=0$, $\theta=1$, $a/b_z=10^4$, $b/a=0.5$, and $l_1=l_2=a$. It can be seen that $K_{\mathrm{III}}/\mu_1\sqrt{b_z}$ is always negative, which means that positive screw dislocations can reduce the SIF (shielding effect). The shielding effect increases with the value of μ_2/μ_1 increasing, but it quickly decreases as the value of $|z_0-a-l_1|/b_z$ increases.

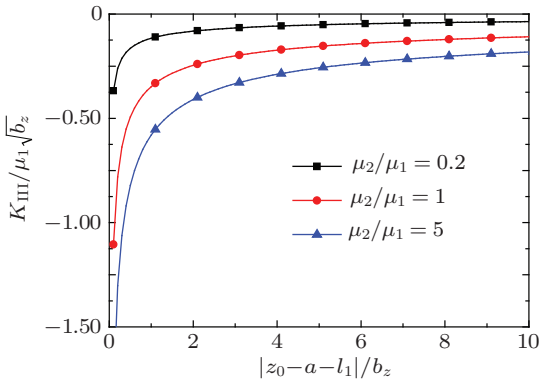


Fig. 3. (color online) $K_{\mathrm{III}}/\mu_1\sqrt{b_z}$ versus $|z_0-a-l_1|/b_z$ for different values of μ_2/μ_1.

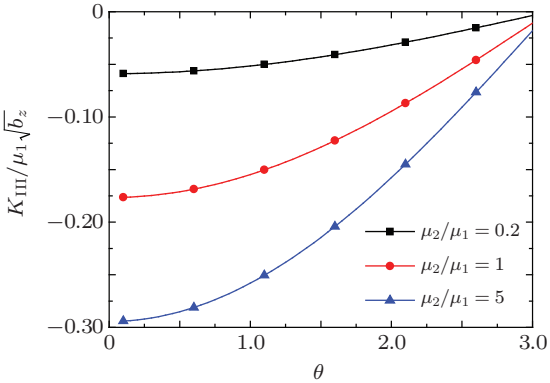


Fig. 4. (color online) $K_{\mathrm{III}}/\mu_1\sqrt{b_z}$ versus θ for different values of μ_2/μ_1.

The variations of $K_{\mathrm{III}}/\mu_1\sqrt{b_z}$ with θ for different values of μ_2/μ_1 are shown in Fig. 4 for the case of $\sigma_{yz}^{\infty}=0$, $a/b_z=10^4$, $|z_0-a-l_1|/b_z=5$, $b/a=0.5$, and $l_1=l_2=a$. It can be seen that the shielding effect increases as the value of μ_2/μ_1 increases, but decreases as the value of θ increases.

Figure 5 shows $K_{\mathrm{III}}/\mu_1\sqrt{b_z}$ versus l_2/l_1 for different values of b/a at $\sigma_{yz}^{\infty}=0$, $\theta=1$, $|z_0-a-l_1|/b_z=5$, $a/b_z=10^4$, $\mu_2/\mu_1=0.2$, and $l_1=a$. It can be seen that the shielding effect increases as the value of b/a increases, and increases as the value of l_2/l_1 increases.

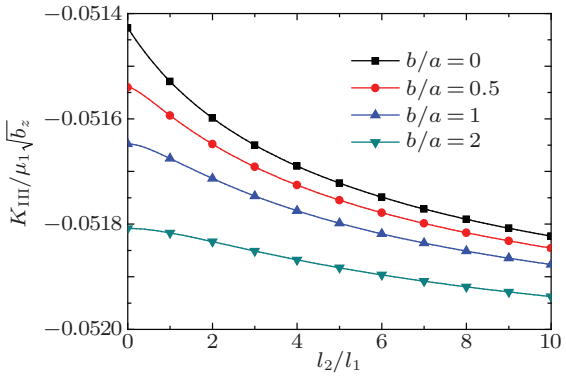


Fig. 5. (color online) $K_{\mathrm{III}}/\mu_1\sqrt{b_z}$ versus l_2/l_1 for different values of b/a.

5. Dislocation emission

According to the work of Pak,[51] the image force acting on the screw dislocation can be written as

$$F=F_x-iF_y=ib_z\left(\hat{\sigma}_{xz}-i\hat{\sigma}_{yz}\right), \tag{14}$$

where $\hat{\sigma}_{xz}$ and $\hat{\sigma}_{yz}$ are the components of the perturbation stress in point z_0.

Substituting expression (9) into expression (14) yields

$$F=F_x-\mathrm{i}F_y=F_{\mathrm{imag}}+F_{\Gamma}, \tag{15}$$

where

$$F_{\mathrm{imag}} = \Omega\left(z_0\right) = \frac{\mu_1 b_z^2 b\,(a+b)}{4\pi\left[\left(z_0^2-a^2+b^2\right)^{3/2}-\dfrac{b\,(a+b)\left(z_0^2-a^2+b^2\right)}{z_0+\left(z_0^2-a^2+b^2\right)^{1/2}}\right]}$$

$$+\frac{\mu_1 b_z^2 \Lambda}{2\pi(\mu_1+\mu_2)}$$
$$\times\left\{\frac{-\mu_1}{G(z_0)-\overline{G(z_0)}}+\frac{\mu_2}{\sqrt{(G(z_0)-a_1)(G(z_0)-b_1)}}\right.$$
$$\times\left[\frac{\sqrt{\left(\overline{G(z_0)}-a_1\right)\left(\overline{G(z_0)}-b_1\right)}}{G(z_0)-\overline{G(z_0)}}+2\right]$$
$$\left.-\frac{\mu_2\left[2G(z_0)-a_1-b_1\right]}{2(G(z_0)-a_1)(G(z_0)-b_1)}\right\},$$

$$\Lambda = \frac{1}{a+b}-\frac{b}{\left(z_0^2-a^2+b^2\right)^{1/2}\left[z_0+\left(z_0^2-a^2+b^2\right)^{1/2}\right]},$$

$$F_\Gamma = b_z\sigma_{yz}^{\infty}\frac{\Lambda(a+b)\left[2G(z_0)-a_1-b_1\right]}{2\sqrt{(G(z_0)-a_1)(G(z_0)-b_1)}}.$$

For a screw dislocation slip on a plane denoted by angle θ, the slip force acting on the screw dislocation can be written as

$$F_{\text{emit}} = \text{Re}[F]\cos\theta - \text{Im}[F]\sin\theta. \tag{16}$$

A commonly accepted criterion[14] for the emission of dislocation monopoles from a crack tip is that the force acting on them is equal to zero. Moreover, the dislocation distance to the crack surface must be equal to, or larger than, the dislocation core radius. In the present work, a similar assumption is held, and $r = r_d + a + l_1$ is set to be the condition for the emission of screw dislocation. By combining expressions (15) and (16), the critical condition for the dislocation emission at the right tip of the right crack yields

$$\sigma_{yz}^{\infty\text{D}} = \frac{\sin\theta\,\text{Im}\left[\Omega(z_1)\right]-\cos\theta\,\text{Re}\left[\Omega(z_1)\right]}{b_z\left(\cos\theta\,\text{Re}[H]-\sin\theta\,\text{Im}[H]\right)}, \tag{17}$$

$$K_{\text{III}}^{\text{D}} = \sqrt{\frac{\pi}{2}}(a+b)\sqrt{b_1-a_1}\,\sigma_{yz}^{\infty\text{D}}\sqrt{\frac{1}{a+b}-\frac{b}{\left(l_1^2+2al_1+b^2\right)^{1/2}\left[a+l_1+\left(l_1^2+2al_1+b^2\right)^{1/2}\right]}}, \tag{18}$$

where $z_1 = a+l_1+b_z(\cos\theta+\text{i}\sin\theta)$,

$$H = \left\{\frac{1}{a+b}-\frac{b}{\left(z_1^2-a^2+b^2\right)^{1/2}\left[z_1+\left(z_1^2-a^2+b^2\right)^{1/2}\right]}\right\}\frac{(a+b)\left[2G(z_1)-a_1-b_1\right]}{2\sqrt{(G(z_1)-a_1)(G(z_1)-b_1)}}.$$

Figure 6 shows the normalized critical load at infinity $\sigma_{yz}^{\infty\text{D}}/\mu_1$ versus θ for different values of μ_2/μ_1 at $b_z = 10^{-9}$ m, $b = 0.5\times10^{-5}$ m, and $l_2 = l_1 = a = 10^{-5}$ m. It can be found that the critical load at infinity $\sigma_{yz}^{\infty\text{D}}/\mu_1$ increases with the increase of the emission angle θ. So the most probable angle for positive screw dislocation emission is always zero. When emission angle θ is fixed at a certain value, the value of $\sigma_{yz}^{\infty\text{D}}/\mu_1$ decreases with the value of μ_2/μ_1 increasing, which means that the repulsion acting on the dislocation from phase 2 makes the dislocation emission take place more easily.

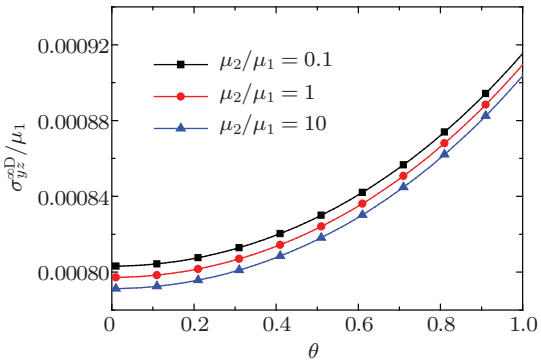


Fig. 6. (color online) The $\sigma_{yz}^{\infty\text{D}}/\mu_1$ versus θ for different values of μ_2/μ_1.

The variations of $\sigma_{yz}^{\infty\text{D}}/\mu_1$ with θ for different values of b/a are shown in Fig. 7 for the case of $b_z = 10^{-9}$ m, $\mu_2/\mu_1 = 0.2$, and $l_2 = l_1 = a = 10^{-5}$ m. It can also be seen that the critical load at infinity $\sigma_{yz}^{\infty\text{D}}/\mu_1$ increases with the increase of the emission angle θ. In addition, when emission angle θ is fixed at a certain value, the value of $\sigma_{yz}^{\infty\text{D}}/\mu_1$ increases with the value of b/a increasing, which means that the minor semi axis of the elliptical hole extending in the vertical direction makes it more difficult for the dislocation emission to take place.

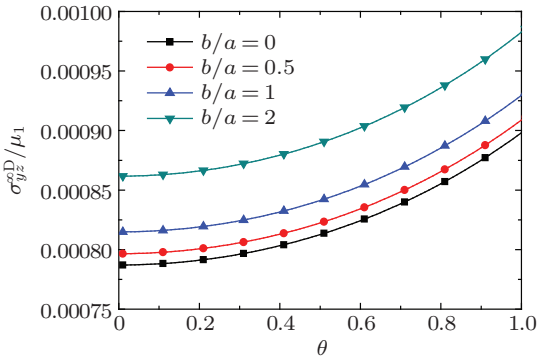


Fig. 7. (color online) The $\sigma_{yz}^{\infty\text{D}}/\mu_1$ versus θ for different values of b/a.

Figure 8 shows the $\sigma_{yz}^{\infty\text{D}}/\mu_1$ versus θ for different values of l_2/l_1 at $b_z = 10^{-9}$ m, $b = 0.5\times10^{-5}$ m, $\mu_2/\mu_1 = 0.2$, and $l_1 = a = 10^{-5}$ m. It can also be seen that the critical load at

infinity $\sigma_{yz}^{\infty D}/\mu_1$ increases with the increase of the emission angle θ. When emission angle θ is fixed at a certain value, the value of $\sigma_{yz}^{\infty D}/\mu_1$ decreases with the value of l_2/l_1 increasing, which means that the left crack extending in the horizontal direction makes the dislocation emission at the right tip of the right crack take place more easily.

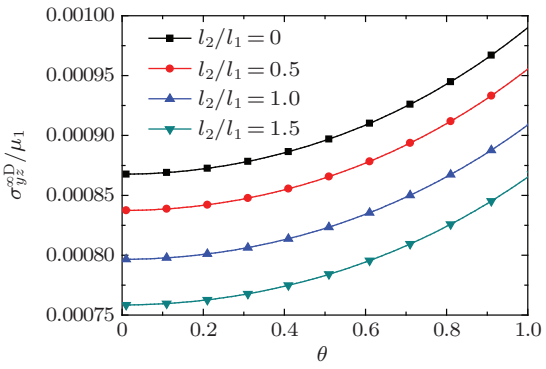


Fig. 8. (color online) The $\sigma_{yz}^{\infty D}/\mu_1$ versus θ for different values of l_2/l_1.

6. Conclusions

The interaction between screw dislocations and two asymmetrical interfacial cracks emanating from an elliptic hole under loads at infinity is studied. By utilizing the conformal mapping technique, the closed-form solutions are derived for complex potentials and stress fields. In the limiting cases, well-known results can be obtained from the present solutions. Moreover, new exact solutions for a screw dislocation interacting with some complicated defects including two asymmetrical edge cracks emanating from a circular hole, two symmetrical edge interfacial cracks emanating from an elliptical/circular hole, and an edge interfacial crack emanating from an elliptical/circular hole are derived. The stress intensity factor due to a screw dislocation located near the right interfacial crack and the critical applied stress for a dislocation emission at the right tip are also calculated.

The results show that the shielding effect in a two-phase material is just the same as that in the single-phase case with the effective shear modulus, which is the harmonic mean of the shear modulus of the two phases. The positive screw dislocations can reduce the SIF of the crack emanating from an elliptic hole (shielding effect), and the shielding effect increases with the increase of the lengths of the other crack and the minor semi axis, but decreases with the increase of the dislocation azimuth.

The most probable angle for positive screw dislocation emission is zero. The repulsion acting on the dislocation from phase 2 and the other cracks extend in the horizontal direction make the dislocation emission at the crack tip take place more easily, but the minor semi axis of the elliptical hole extend in the vertical direction makes it more difficult for it to take place.

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