Prediction of heat transfer of nanofluid on critical heat flux based on fractal geometry*

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Analytical expressions for nucleate pool boiling heat transfer of nanofluid in the critical heat flux (CHF) region are derived taking into account the effect of nanoparticles moving in liquid based on the fractal geometry theory. The proposed fractal model for the CHF of nanofluid is explicitly related to the average diameter of the nanoparticles, the volumetric nanoparticle concentration, the thermal conductivity of nanoparticles, the fractal dimension of nanoparticles, the fractal dimension of active cavities on the heated surfaces, the temperature, and the properties of the fluid. It is found that the CHF of nanofluid decreases with the increase of the average diameter of nanoparticles. Each parameter of the proposed formulas on CHF has a clear physical meaning. The model predictions are compared with the existing experimental data, and a good agreement between the model predictions and experimental data is found. The validity of the present model is thus verified. The proposed fractal model can reveal the mechanism of heat transfer in nanofluid.

Keywords: nanofluid, fractal geometry, heat transfer

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1. Introduction

Nanofluid technology has emerged as a new heat transfer technique in recent years.[1–11] Applications of superconducting magnets and most importantly superfast computing are posing a tremendous challenge to thermal management. These devices not only have respective applications with high precision but also need to occupy the minimum space. Critical heat flux (CHF) in pool boiling is an undesirable phenomenon causing an excessive increase of the temperature at the boiling surface. Thus in the CHF region, the mechanisms of nucleate pool boiling heat transfer of nanofluid have been extensively discussed, and their physical nature is still far from being well understood because of its complexity and diversity. The mechanism of CHF of a nanofluid is currently one of the most controversial subjects of heat transfer in pool boiling. There are some empirical correlations and models for CHF of nanofluid in the literature, each of which is applicable to a restricted range of experimental conditions.[1–3,8,11] From a mechanistic viewpoint, although the influences of some parameters such as the average diameter of nanoparticles, the volumetric nanoparticle concentration, and the thermal conductivity of nanoparticles have been discussed, an overall mechanistic description is still unavailable. In addition, each model/correlation has its disadvantages because of the limitations of experimental conditions. It is a challenge to predict the CHF of nanofluid analytically due to the extremely complicated mechanism of heat transfer as well as the interrelationship between CHF and the size of nanoparticles and volumetric nanoparticle concentration.

As universally received, the fractal technique is now used to predict the transport properties of fluid,[6,8,9,11–39] such as the permeability of porous medium,[12,13,16,21,23,27,30,34,39] heat transfer of fluid,[6,8,9,11,14,15,17–20,22,24,32,35,37,38] and spontaneous imbibition in porous medium.[25,26,28,29,31,33,36] In the present work, we derive analytical expressions for the CHF of nanofluid taking into account the effect of nanoparticles moving in liquid based on the fractal geometry theory.

The rest of this paper is organized as follows. In Section 2, we give the fractal model description of the pool boiling heat transfer of nanofluid. In Section 3, the results and discussion are presented. Some concluding remarks are given in Section 4.

2. Formulation of the proposed model

It was shown experimentally[1] that the addition of nanoparticles enhances the convective heat transfer of nanofluid. The nanoparticles moving in fluids carry energy, and heat exchange may occur between hot and cold regions. In a base fluid, it is generally recognized that the main mechanism contributing to nucleate boiling heat transfer is the bubble generation and departure from the active cavity on the super-heated surface in a CHF region. Thus there are two main mechanisms contributing to nucleate pool boiling heat transfer of nanofluid in the CHF region: one is the heat ($Q_{\rm n}$) trans-

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ferred by the heat convection caused by the Brownian motion of nanoparticles, and the other is the heat ($Q_{b,CHF}$) transferred by the bubble generation and departure from the base fluid in the CHF region. In the CHF region, the total heat flux of nucleate pool boiling heat transfer of nanofluid can be expressed as

$$Q_{N,CHF} = Q_n + Q_{b,CHF}. \tag{1}$$

The quantity of heat caused by a single nanoparticle moving in liquid can be expressed as

$$\varphi_n = h_{n,eff} A_n \Delta T, \tag{2}$$

where $h_{n,eff}$ is the heat transfer coefficient related to the Brownian motion of nanoparticles in fluid, ΔT is the wall superheat, and A_n is the surface area of a nanoparticle whose diameter is D_n. The surface area of a nanoparticle can be given by $A_n = \pi D_n^2$ if the shape of the nanoparticle is considered as a sphere.

The cumulative number N_p of pores in a porous medium whose sizes are greater than or equal to D_p can be expressed according to the fractal scaling law as[13]

$$N_p(D_l \geq D_p) = (D_{p,max}/D_p)^{d_p}, \quad D_{p,min} \leq D_p \leq D_{p,max}, \tag{3}$$

where D_p is the diameter of a pore, $D_{p,max}$ and $D_{p,min}$ are the maximum and the minimum diameters of pores in the porous medium, respectively, and d_p is the fractal dimension of the pores. It is shown that the distribution of nanoparticles in a suspension exhibits the fractal property.[15] Analogous to pores in a porous medium, the cumulative number of nanoparticles in fluid with diameters larger than and equal to D_n is also described by Eq. (3) with D_p replaced by D_n

$$N_n(D_l \geq D_n) = (D_{n,max}/D_n)^{d_N}, \quad D_{n,min} \leq D_n \leq D_{n,max}, \tag{4}$$

where d_N is the fractal dimension of nanoparticles, D_n is the diameter of nanoparticles, and $D_{n,max}$ and $D_{n,min}$ are the maximum and the minimum diameters of nanoparticles, respectively. The number of nanoparticles of sizes lying between D_n and $D_n + dD_n$ can be obtained from Eq. (4) as

$$-dN_n = d_N D_{n,max}^{d_N} D_n^{-d_N-1} dD_n, \tag{5}$$

where $dD_n > 0$ and $-dN_n > 0$. Equation (5) indicates that the number of nanoparticles decreases with the increase of nanoparticle diameter.

The fractal dimension d_N of the nanoparticle in Eqs. (4) and (5) is given by[13]

$$d_N = d - \ln\phi / \ln\lambda_n, \tag{6}$$

where ϕ is the volumetric nanoparticle concentration, $\lambda_n = D_{n,min}/D_{n,max}$, and d refers to 2 for two dimensions. Equation (6) shows that the fractal dimension of nanoparticles is a function of the nanoparticle volume fraction in nanofluid. In general, the size distribution of particles in fluid or pores in porous medium follows the fractal power law if $\lambda_n = D_{n,min}/D_{n,max} \leq 10^{-2}$, thus equations (3)–(6) approximately hold. Actually, it is found that the ratio $\lambda_n = 10^{-3}$ best fits the experimental data for different particles in fluid or pores in a porous medium.[16] Then the ratio $\lambda_n = 10^{-3}$ is used for nanoparticles in this work.

According to Eqs. (2) and (5), the total heat caused by all nanoparticles moving in liquid from $D_{n,min}$ to $D_{n,max}$ can be obtained as

$$\phi_{n,tot} = \Delta T \int_{D_{n,min}}^{D_{n,max}} h_{n,eff} A_n (-dN_n). \tag{7}$$

Equation (7) can be integrated if both $h_{n,eff}$ and A_n are expressed in terms of D_n.

The heat transfer coefficient $h_{n,eff}$ is usually defined as

$$h_{n,eff} = (Nu \cdot k_{eff})/D_n, \tag{8}$$

where Nu is the Nusselt number, k_{eff} is the effective thermal conductivity of nanofluids, which can be obtained as[40]

$$k_{eff} = \frac{[k_n(1+2\phi) + 2k_l(1-\phi)]}{[k_n(1-\phi) + k_l(2+\phi)]} k_l, \tag{9}$$

where k_l and k_n are the thermal conductivities of liquid and nanoparticles, respectively.

According to the theory by Tomotika *et al.*,[41] the Nusselt number for liquid flowing around a sphere can be written as

$$Nu = 2 + Re \cdot Pr/2. \tag{10}$$

Here Pr is the Prandtl number of fluid defined as

$$Pr = \upsilon/\alpha = \mu/\rho_l\alpha = \mu c_{pl}/k_l, \tag{11}$$

where υ, α, μ, ρ_l, and c_{pl} are the kinematical viscosity, thermal diffusion coefficient, absolute viscosity, density, and specific heat of a fluid under a constant pressure, respectively.

The Reynolds number is usually defined as

$$Re = D_n V_p/\upsilon, \tag{12}$$

where V_p is the velocity of the nanoparticle, and $V_p = D_d/l_m$.[42] Here l_m is the mean free path of the liquid molecule and is assumed to be the same as the diameter of the liquid molecule ($l_m = 1.7 \times 10^{-10}$ m[43]), and D_d is the nanoparticle diffusion coefficient, which is determined by $D_d = k_B T/(3\pi\mu D_n)$, with T being the temperature and k_B being the Boltzmann constant ($k_B = 1.38 \times 10^{-23}$ J/K). So the values of Re and Pr can be calculated to be 4×10^{-3} and 6, respectively. It can be seen that the Reynolds number $Re << 1$. So equation (10) can be further reduced to

$$Nu = 2 + Re \cdot Pr/2 \approx 2. \tag{13}$$

By inserting Eqs. (9) and (13) into Eq. (8), the heat transfer coefficient $h_{\rm nf}$ can be further modified as

$$h_{\rm n,eff} = \frac{2k_{\rm l}[k_{\rm n}(1+2\phi)+2k_{\rm l}(1-\phi)]}{[k_{\rm n}(1-\phi)+k_{\rm l}(2+\phi)]}D_{\rm n}^{-1}. \tag{14}$$

It is seen that both $h_{\rm n,eff}$ and $A_{\rm n}$ $(=\pi D_{\rm n}^2)$ are related to $D_{\rm n}$. Equation (14) indicates that the smaller nanoparticles exhibit a larger heat transfer coefficient due to the convection of nanoparticles moving in fluid. This can be explained by the fact that the small nanoparticles cause the increase of nanoparticles moving in fluid, leading to the increase in heat transfer of the nanofluid.

Inserting Eq. (14) and $A_{\rm n}=\pi D_{\rm n}^2$ into Eq. (7), we can obtain the heat flux

$$\begin{aligned} Q_{\rm n} &= \frac{\varphi_{\rm n,tot}}{A_{\rm n,tot}} = \frac{\Delta T\int_{D_{\rm n,min}}^{D_{n,\max}} h_{\rm n,eff}A_{\rm n}(-{\rm d}N_{\rm n})}{\int_{D_{\rm n,min}}^{D_{n,\max}} A_{\rm n}(-{\rm d}N)} \\ &= \frac{\Delta T\int_{D_{\rm n,min}}^{D_{\rm n,max}} \frac{2k_{\rm l}[k_{\rm n}(1+2\phi)+2k_{\rm l}(1-\phi)]}{[k_{\rm n}(1-\phi)+k_{\rm l}(2+\phi)]}D_{\rm n}^{-1}\pi D_{\rm n}^2(d_{\rm N}D_{n,\max}^{d_{\rm N}}D_{\rm n}^{-d_{\rm N}-1}{\rm d}D_{\rm n})}{\int_{D_{\rm n,min}}^{D_{\rm n,max}}\pi D_{\rm n}^2(d_{\rm N}D_{n,\max}^{d_{\rm N}}D_{\rm n}^{-d_{\rm N}-1}{\rm d}D_{\rm n})} \\ &= \frac{[k_{\rm n}(1+2\phi)+2k_{\rm l}(1-\phi)](\lambda_{\rm n}^{-d_{\rm N}+1}-1)(4-2d_{\rm N})k_{\rm l}}{[k_{\rm n}(1-\phi)+k_{\rm l}(2+\phi)](1-\lambda_{\rm n}^{-d_{\rm N}+2})(d_{\rm N}-1)D_{n,\max}}\Delta T. \end{aligned} \tag{15}$$

Next we will obtain the relationship between the average diameter $D_{\rm av}$ and the maximum diameter $D_{\rm n,max}$ of nanoparticles. The average diameter of particles can be obtained as[21]

$$D_{\rm av} = 2\left(\sum_{i=1}^{N_{\rm n}} N_i r_i^4\right)^{1/4}, \tag{16}$$

where N_i $(i=1,\ 2,\ \cdots,\ N_{\rm n})$ is the number of nanoparticles with radius r_i in fluid. Usually, N_i is unknown, and it is difficult to calculate directly with Eq. (15). Because the number of nanoparticles is numerous in fluid, the average diameter of nanoparticles can be obtained with the aid of Eq. (5) as follows:

$$\begin{aligned} D_{\rm av} &= 2\left[\int_{D_{\rm n,min}}^{D_{n,\max}}\left(\frac{D_{\rm n}}{2}\right)^4(-{\rm d}N_{\rm n})\right]^{1/4} \\ &= \left(\frac{d_{\rm N}}{4-d_{\rm N}}\right)^{1/4}(1-\lambda_{\rm n}^{4-d_{\rm N}})^{1/4}D_{\rm n,max}. \end{aligned} \tag{17}$$

Since $0<d_{\rm N}<2$, $4-d_{\rm N}>2$, $\lambda_{\rm n}=10^{-3}$, and $\lambda_{\rm n}^{4-d_{\rm N}}<<1$, equation (17) can be further reduced to

$$D_{\rm av} \cong \left(\frac{d_{\rm N}}{4-d_{\rm N}}\right)^{1/4}D_{\rm n,max}. \tag{18}$$

Equation (18) shows a fractal expression of the average diameter of nanoparticles.

By inserting Eq. (18) into Eq. (15), the heat flux $Q_{\rm n}$ can be further modified as

$$\begin{aligned} Q_{\rm n} &= d_{\rm N}^{1/4}(4-2d_{\rm N})k_{\rm l}[k_{\rm n}(1+2\phi)+2k_{\rm l}(1-\phi)] \\ &\times(\lambda_{\rm n}^{-d_{\rm N}+1}-1)\Delta T/\{(d_{\rm N}-1)(4-d_{\rm N})^{1/4} \\ &\times[k_{\rm n}(1-\phi)+k_{\rm l}(2+\phi)](1-\lambda_{\rm n}^{-d_{\rm N}+2})D_{\rm av}\}, \end{aligned} \tag{19}$$

where $d_{\rm N}$ is obtained from Eq. (6) and is related to the nanoparticle volume fraction of suspension ϕ. Equation (19) is the fractal analytical expression for the heat transferred by nanoparticles moving in nanofluid. Equation (19) shows that the heat flux $Q_{\rm n}$ increases as the average diameter of the nanoparticles $D_{\rm av}$ decreases. This is expected because the small diameter of a nanoparticle may cause the increase of the velocity of the nanoparticle, leading to more heat being transferred by nanoparticles moving in nanofluid. Equation (19) depicts that the heat flux $Q_{\rm n}$ is a function of the average diameter of nanoparticles $D_{\rm av}$, the volumetric nanoparticle concentration ϕ, the thermal conductivity of nanoparticles $k_{\rm n}$, the fractal dimension of nanoparticles $d_{\rm N}$, and the thermal conductivity of liquid $k_{\rm l}$. No additional/new empirical constant is introduced in this model. It is expected that equation (19) has less empirical constants than the conventional models, and each parameter in Eq. (19) has a clear physical meaning.

Several investigators have found that the surface characteristics affect the pool boiling performance and mechanisms when a pure liquid is boiled over heating surfaces.[8,9,14,17,18,20,44–48] The density of active sites on the heater surface is affected by the interaction among several parameters on the heater and the liquid sides, as well as the liquid–solid contact angle.[8,9,14,17,18,20,44–48] For example, the distribution of available cavities on the heater surface and the liquid–solid contact angle determine which cavities could potentially be active. At the same time, the transport properties of the heater affect the extent of the thermal interaction among the cavities, causing the activation and deactivation of individual cavities.[49–55,58,59]

Xiao and Yu[18] have obtained the fractal analytical ex-

pressions of nucleate boiling heat transfer $Q_{l,c}$ from the base fluid in the CHF region. The CHF of pool boiling can be expressed as

$$Q_{b,CHF} = c_q \frac{d_{fc}}{d_{fc}+2} \frac{4\pi\alpha}{3} \left(\frac{\Delta T}{T_w - T_l}\right)^2 \times D_{c,max}^{-2} \left[\left(\frac{D_{c,max}}{D_{c,min}}\right)^{d_{fc}+2} - 1\right], \quad (20)$$

where $c_q = \pi h_{fg} \rho_g D_b^3/6$ is the heat flux removed by a single bubble, h_{fg} is the latent heat of vaporization, ρ_g is the vapor density, T_l is the bulk temperature of liquid, $\Delta T = T_w - T_s$, T_w and T_s are the wall temperature and the saturation temperature of liquids, respectively, D_b is the bubble departure diameter, $D_{c,max}$ and $D_{c,min}$ are respectively the maximum and the minimum diameters of active cavity, and d_{fc} is the fractal dimension of active cavity on the heated surface.

The bubble departure diameter D_b can be obtained as[56]

$$D_b = c \left[\frac{\sigma}{g(\rho_l - \rho_g)}\right]^{1/2} J^{*5/4}, \quad (21)$$

with $c = 1.5 \times 10^{-4}$ for water, and $c = 4.65 \times 10^{-4}$ for the other liquid, σ is the surface tension of the liquid, g is the acceleration due to gravity, and J^* is the Jakob number which is given by

$$J^* = \rho_l c_{pl} T_s/(\rho_g h_{fg}). \quad (22)$$

It is shown by Eqs. (20) and (21) that the boiling heat transfer from the base fluid in the CHF region increases with increasing bubble departure diameter.

According to the fractal power law,[13,14,17,18] $D_{c,max}/D_{c,min} \geq 10^2$, thus equations (3)–(6) approximately hold. Since $0 < d_{fc} < 2$, $0 < 2 + d_{fc} < 4$, and $(D_{c,max}/D_{c,min})^{d_{fc}+2} << 1$, equation (20) can be further reduced to

$$Q_{b,CHF} = c_q \frac{d_{fc}}{d_{fc}+2} \frac{4\pi\alpha}{3} \left(\frac{\Delta T}{T_w - T_l}\right)^2 \times D_{c,max}^{-2} \left(\frac{D_{c,max}}{D_{c,min}}\right)^{d_{fc}+2}. \quad (23)$$

The minimum active cavity diameter $D_{c,min}$ and the maximum active cavity diameter $D_{c,max}$ can be predicted by Hsu's model as[57]

$$D_{c,min} = \frac{2\delta}{C_1}\left[1 - \frac{\theta_s}{\theta_w} - \sqrt{\left(1 - \frac{\theta_s}{\theta_w}\right)^2 - \frac{4\zeta C_2}{\delta\theta_w}}\right], \quad (24a)$$

$$D_{c,max} = \frac{2\delta}{C_1}\left[1 - \frac{\theta_s}{\theta_w} + \sqrt{\left(1 - \frac{\theta_s}{\theta_w}\right)^2 - \frac{4\zeta C_2}{\delta\theta_w}}\right], \quad (24b)$$

where $\zeta = 2\sigma T_s/(\rho_g h_{fg})$; $C_1 = (1 + \cos\theta)/\sin\theta$ and $C_2 = 1 + \cos\theta$, with θ being the contact angle of the fluid and the heater material; $\theta_s = T_s - T_l$; $\theta_w = T_w - T_l$; and δ is the thermal boundary layer thickness in the nanofluid which can be expressed as

$$\delta = k_l/h_{n,eff}, \quad (25)$$

with $h_{n,eff}$ being the heat transfer coefficient obtained from Eq. (14).

In the nucleate pool boiling of the base fluid, the fractal dimension d_{fc} of the active cavity on the heated surface is given by Yu and Cheng as[14]

$$d_{fc} = \ln[\bar{D}_{c,max}/(\sqrt{2}D_{c,min})]^2/\ln(\lambda_c^{-1}). \quad (26)$$

Here $\bar{D}_{c,max}$ is the averaged value over all the maximum active cavities

$$\begin{aligned}\bar{D}_{c,max} &= \frac{1}{(T_w - T_s)} \int_{T_s}^{T_w} D_{c,max}(T_w)\,dT_w \\ &= \frac{1}{\Delta T} \sum_{j=1}^{m} D_{c,max}(T_{w_j})\delta T_w \\ &= \frac{1}{m} \sum_{j=1}^{m} D_{c,max}(T_{w_j}), \end{aligned} \quad (27)$$

where $m = \Delta T/\delta T_w$, and δT_w is assumed to be a constant. In the above equation, $T_{w_j} = T_s + j(\delta T_w)$ with $j = 1, 2, \ldots, m$. For example, if we choose $\delta T_w = 0.2\ ^\circ$C, then $m = 5$ for $\Delta T = 1\ ^\circ$C, and $m = 50$ for $\Delta T = 10\ ^\circ$C.

Inserting Eqs. (19) and (23) into Eq. (1), we can obtain a fractal model for the heat transfer of nanofluid in the CHF region as

$$\begin{aligned} Q_{N,CHF} &= Q_n + Q_{b,CHF} \\ &= d_N^{1/4}(4 - 2d_N)k_l[k_n(1 + 2\phi) + 2k_l(1 - \phi)] \\ &\quad \times (\lambda_n^{-d_N+1} - 1)\Delta T/\{(d_N - 1)(4 - d_N)^{1/4} \\ &\quad \times [k_n(1 - \phi) + k_l(2 + \phi)](1 - \lambda_n^{-d_N+2})D_{av}\} \\ &\quad + c_q \frac{d_{fc}}{d_{fc}+2} \frac{4\pi\alpha}{3} D_{c,max}^{-2} \left(\frac{D_{c,max}}{D_{c,min}}\right)^{d_{fc}+2} \\ &\quad \times \left(\frac{\Delta T}{T_w - T_l}\right)^2, \end{aligned} \quad (28)$$

where d_N and d_{fc} are obtained from Eqs. (6) and (26), respectively. In Eq. (28) ΔT is a variable. Equation (28) is the fractal analytical expression of CHF for pool boiling heat transfer in a nanofluid and indicates that the CHF of pool boiling heat transfer in nanofluid is explicitly related to the average diameter of nanoparticles, the volumetric nanoparticle concentration, the thermal conductivity of nanoparticles, the fractal dimension of nanoparticles, the fractal dimension of an active cavity on the heated surface, the temperature, and the properties of fluid. It is expected that equation (28) has less empirical constants and each parameter in Eq. (28) has a clear physical meaning. The proposed fractal model can reveal the mechanism of pool boiling heat transfer on CHF in nanofluid.

3. Results and discussion

In the CHF region, the pool boiling characteristics of a water-based nanofluid with titania ($D_{av} = 45$ nm, $\phi = 0.1\%$) and alumina ($D_{av} = 47$ nm, $\phi = 0.1\%$) nanoparticles are investigated on a thermally heated disk heater at saturated temperature and atmospheric pressure.[7] The water-based nanofluid is prepared by dispersing the dry powders into de-ionized, distilled water with 3 h ultrasonic vibration.[7] Figure 1 shows a comparison between the CHF of pool boiling heat transfer from the present model calculated using Eq. (28) and that from reported experiments[7] at $D_{av} = 45$ nm and $\phi = 0.1\%$ for TiO_2 nanofluids. There is a good agreement between the model prediction and the experimental data. The solid line in Fig. 1 represents the predictions by the present model with d_N and d_{fc} computed from Eqs. (6) and (26), respectively. Figure 2 shows a comparison between the CHF predicted by the present model (Eq. (28)) and that from reported experiments[7] at $D_{av} = 47$ nm and $\phi = 0.1\%$ for Al_2O_3 nanofluids. It is seen that an excellent agreement between the model predictions and the experimental data is again found. Liu and Liao[5] conducted experiments on CHF in pool boiling under atmospheric pressure at $D_{av} = 35$ nm and $\phi = 0.5\%$ for SiO_2 nanofluid. The pool nucleate boiling heat transfer experiments of water based nanoparticle suspensions on a plain heated copper surface are carried out. Figure 3 shows a comparison between the model predictions and the experimental data[5] at $D_{av} = 35$ nm and $\phi = 0.5\%$ for SiO_2 nanofluid. A reasonable agreement is also observed between the experimental and the predicted values. To investigate the CHF characteristics of nanofluid, pool boiling experiments of nanofluid with various concentrations of Al_2O_3 nanoparticles ($D_{av} = 47$ nm) are carried out using a 0.2 mm diameter cylindrical Ni–Cr wire under atmospheric pressure.[3] In Fig. 4, the results from Eq. (28) are compared with the present prediction results and the data collected by others[2,3] over a wide range of volumetric nanoparticle concentrations at $D_{av} = 47$ nm for Al_2O_3 nanofluids. As can be seen from Fig. 4, the results from the model agree fairly well with the experimental data in the literature. The data presented by Bang and Chang[2] are higher than the others because the CHF of pure water in their experiment is higher than that in the other studies shown in Fig. 4. As a result, the maximum CHF increments in all work on nanofluids are about 900 kW/m^2. This shows that the result of our study is consistent with that of other work under atmospheric pressure. Figure 5 shows the CHF of pool boiling heat transfer versus the average diameter of nanoparticles at $\phi = 0.1\%$ in a CuO nanofluid. It can be seen from Fig. 5 that the CHF decreases with the increase in the average diameter of nanoparticles. This can be explained by the fact that the small nanoparticles cause the increase of nanoparticles moving in liquid, leading to the increase of heat transfer of nanofluids. These are all expected and consistent with the practical physical phenomena Therefore, the present fractal model can reveal the mechanism of heat transfer of nanofluid in the CHF region.

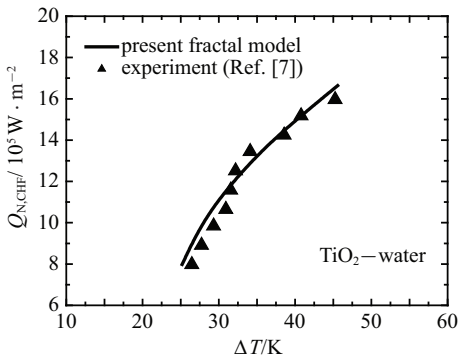


Fig. 1. Comparison between the present model predictions and the experimental data at $D_{av} = 45$ nm and $\phi = 0.1\%$ for a TiO_2 nanofluid.

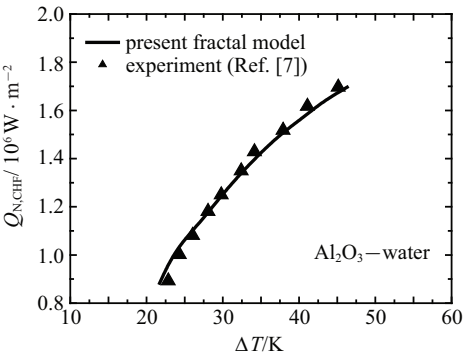


Fig. 2. Comparison between the present model predictions and the experimental data at $D_{av} = 47$ nm and $\phi = 0.1\%$ for an Al_2O_3 nanofluid.

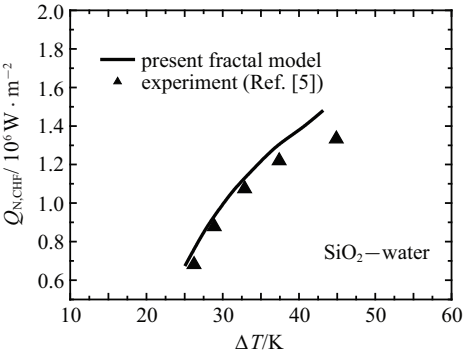


Fig. 3. Comparison between the model prediction and experimental data at $D_{av} = 35$ nm and $\phi = 0.5\%$ for a SiO_2 nanofluid.

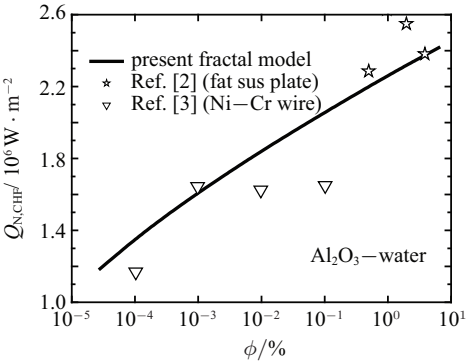


Fig. 4. Comparison of predicted fractal models and existing experimental data at $D_{av} = 47$ nm for an Al_2O_3 nanofluid.

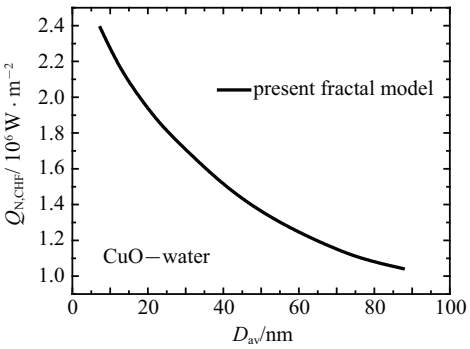


Fig. 5. CHF of pool boiling heat transfer versus the average diameter of nanoparticles at $\phi = 0.1\%$ in CuO nanofluid.

4. Conclusions

With the consideration of nanoparticles moving in liquid, in this paper we derive the analytical expressions for pool boiling heat transfer of nanofluid in the CHF region based on the fractal geometry theory. In the present approach, the CHF of nanofluid is found to be a function of the average diameter of the nanoparticles, the volumetric nanoparticle concentration, the thermal conductivity of nanoparticle, the fractal dimension of nanoparticles, the fractal dimension of active cavity on the heated surface, the temperature, and the properties of the fluid. From the parametric effect analysis, we conclude that the CHF of nanofluid is negatively correlated with the average diameter of nanoparticles but positively correlated with the volumetric nanoparticle concentration. So the proposed fractal model can reveal the mechanism of pool boiling heat transfer on CHF in nanofluid. The analytical solution CHF model of nanofluid does not have an empirical constant, and each parameter in this model has a specific physical meaning. The model predictions are compared with the existing experimental data, and a good agreement is found. The validity of the present fractal model is thus verified.

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