Interaction of a bubble and a bubble cluster in an ultrasonic field*

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Using an appropriate approximation, we have formulated the interacting equation of multi-bubble motion for a system of a single bubble and a spherical bubble cluster. The behavior of the bubbles is observed in coupled and uncoupled states. The oscillation of bubbles inside the cluster is in a coupled state. The numerical simulation demonstrates that the secondary Bjerknes force can be influenced by the number density, initial radius, distance, driving frequency, and amplitude of ultrasound. However, if a bubble approaches a bubble cluster of the same initial radii, coupled oscillation would be induced and a repulsive force is evoked, which may be the reason why the bubble cluster can exist steadily. With the increment of the number density of the bubble cluster, a secondary Bjerknes force acting on the bubbles inside the cluster decreases due to the strong suppression of the coupled bubbles. It is shown that there may be an optimal number density for a bubble cluster which can generate an optimal cavitation effect in liquid for a stable driving ultrasound.

Keywords: bubble cluster, secondary Bjerknes force, multi-bubble motion, nonlinear oscillation

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1. Introduction

High-intensity ultrasound has been widely used in liquid not only for sonochemical reactions but also for emulsification, cell disruption, extraction, mixing, decreasing the liquid viscosity, etc.[1–4] It is well known that cavitation bubbles play a major role in those processes. When a liquid in that gas is dissolved and irradiated by strong ultrasound, many tiny bubbles are generated.[5–8] Bubbles may undergo a number of different motions, such as a breathing mode (or "pulsation"), translational motion, shape oscillations, and fragmentation or coalescence,[9–11] which are likely to be determined by the local acoustic environment and the interactions of bubbles.

The interaction of bubbles has been extensively investigated by many researchers to obtain an understanding of the physical features of a multibubble system.[12–16] The phase of the acoustic field, the phase difference of bubbles, the dimensionless separation distance, and their size ratios are proposed to characterize the behavior of bubbles.[17] Furthermore, the secondary radiation of a pressure wave related to the neighbouring bubbles and the secondary Bjerknes force between bubbles influence the dynamics of every bubble.[18–21] Therefore, the distribution of the near field around the bubbles varies with their nonlinear oscillations, which can be monitored by light emission or acoustic emission. In other words, cavitation noise could be a powerful method to characterize the dynamic processes of a multibubble system.[21,22]

It is extremely difficult to carry out an accurate description of the dynamics of cavitation bubbles in a very simple way. Because of the bubble–bubble interactions, the bubbles in some specific regions may oscillate simultaneously in a collective mode. A recent experiment revealed that the clouds of bubbles collapse simultaneously to emit a light that is synchronized with the applied ultrasound.[23] Brikin *et al*.[5] have observed that a large bubble cluster exists at the tip of ultrasonic horn collapse periodically every three or four cycles of the fundamental frequency of the horn. In the inertial cavitation region, a high amplitude of pressure is also recorded, which correlates temporally with the cluster collapse. In the non-inertial cavitation region, bubble clusters may shield the sound field, set up microstreaming currents that affect the transport of bubble nuclei and dissolved gases, etc.[6] Using the software for the analysis of fluid motion with parker particles, the motion of bubbles recorded in the video frames was analyzed by Yasui *et al*.[15] The results show that many bubbles move toward the horn tip, where there are many bubbles. Occasionally, we have observed many microbubbles moving toward the region below the ultrasonic horn tip, where many bubbles pulsate synchronously.

The interaction of bubbles in clouds or clusters makes the ultrasonic field complicated, and configuration of bubble clusters varies with the acoustic surrounding. It is virtually impossible to directly simulate the cavitation process for a multibubble system. A small spherical cluster of bubbles is introduced by An[14] to simulate the multibubble sonoluminescence. The interaction of two homogeneous bubble clouds of

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different coupling strength is simulated numerically by Yasui *et al*.[15] They found that the secondary Bjerknes force acting on the bubbles near the horn tip makes a contribution to the translational motion of bubbles. In the present study, numerical simulations of the secondary Bjerknes force on the bubbles have been performed in two states: coupled and uncoupled bubbles. The interaction between a single bubble and a bubble cluster under a uniform driving pressure is introduced, which may help us understand the formation and stability of a bubble cluster in an ultrasonic field.

2. Model

If a bubble is oscillating in a liquid with a stationary sound field, an additional pressure field would be generated. The additional pressure can be expressed as follows:[18]

$$p_{\rm ad} = \frac{\rho}{r}\frac{\rm d}{{\rm d}t}\left(R^2\dot{R}\right), \tag{1}$$

where R is the instantaneous radius of the bubble, r is the distance measured from the center of the bubble, and ρ is the density of the liquid. For a multibubble system, the motion of bubbles would be influenced by the additional pressure caused by the neighboring bubbles. As a result, the mutual interaction between bubbles cannot be neglected. In order to explain the coupled dynamics of bubbles in liquid, the modified Rayleigh–Plesset equations taking into account the effect of the bubble–bubble interactions have been proposed by many authors.[14–20] Based on these theories, the radial motion of a single bubble of instantaneous radius a_1 beyond a bubble cluster can be described as

$$\begin{aligned}&\left(1-\frac{\dot{a}_1}{c}\right)a_1\ddot{a}_1+\left(\frac{3}{2}-\frac{\dot{a}_1}{2c}\right)\dot{a}_1^2\\ &=\frac{p_{1\rm b}}{\rho}\left(1+\frac{\dot{a}_1}{c}\right)+\frac{a_1}{\rho c}\frac{{\rm d}p_{1\rm b}}{dt}-\frac{p_{1int}}{\rho},\end{aligned} \tag{2}$$

and the motion for a bubble of instantaneous radius a_2 inside the cluster is described as follows:

$$\begin{aligned}&\left(1-\frac{\dot{a}_2}{c}+M_0\right)a_2\ddot{a}_2+\left(\frac{3}{2}-\frac{\dot{a}_2}{2c}+2M_0\right)\dot{a}_2^2\\ &=\left(1+\frac{\dot{a}_2}{c}\right)\frac{p_{2\rm b}}{\rho}+\frac{a_2}{\rho c}\frac{{\rm d}p_{2\rm b}}{{\rm d}t}-\frac{1}{L}\frac{d}{{\rm d}t}\left(a_1^2\dot{a}_1\right),\end{aligned} \tag{3}$$

where c is the sound speed of the liquid,

$$M_0 = N\frac{a_2}{r_{\rm clust}}\left(1-\frac{r_0^2}{3r_{\rm clust}^2}\right),$$

r_0 is the relative position of the bubble in the spherical cluster of a radius $r_{\rm clust}$ and N bubbles.[14] The pressure of the liquid at the bubble boundaries $p_{j\rm b}$ can be expressed as

$$\begin{aligned}p_{j\rm b} =& \left(p_0+\frac{2\sigma}{a_{j0}}\right)\left(\frac{a_{j0}^3-h_j^3}{a_j^3-h_j^3}\right)^{\kappa}-\frac{2\sigma}{a_j}-\frac{4\eta\dot{a}_j}{a_j}\\ &-p_0-p_a\sin\omega\left(t+\frac{a_j}{c}\right),\ \ (j=1,2),\end{aligned} \tag{4}$$

where p_0 is the hydrostatic pressure, a_{j0} is the ambient radius, and h_j is the hard-core radius ($a_{j0}/8.54$ for air).[20] The polytropic exponent of the gas κ, is assumed to be equal to its specific heat ratio (1.4 for air). The σ and η are the surface tension and viscosity of the liquid, respectively. The $p_{\rm a}$ and $\omega = 2\pi f$ are the pressure amplitude and the frequency of the external acoustic field, respectively. From Eq. (3), we can see that the bubble motion in different regions of the cluster varies. However, based on the numerical estimation for a small bubble cluster by An,[14] the difference between the bubble motions is slight. Therefore, for the case that the distance L between the single bubble and the center of the cluster is much larger than the cluster radius, the last term in Eq. (2), i.e., the additional pressure generated by the bubble cluster, can be approximately simplified into

$$\frac{p_{1\rm int}}{\rho} = \frac{N}{L}\frac{d}{{\rm d}t}\left(a_2^2\dot{a}_2\right), \tag{5}$$

where the bubble motion at the center of the cluster would be used.

The intensity of mutual interaction between bubbles is usually estimated based on the magnitude of the secondary Bjerknes force, which is caused by the gradient of the additional pressure of the oscillating bubbles. Therefore, a single bubble under the action of additional pressure Eq. (5) experiences a force[18]

$$\boldsymbol{F}_{1\rm int} = -\frac{N\rho}{4\pi L^2}V_1\frac{{\rm d}^2V_2}{{\rm d}t^2}\boldsymbol{e}_{\rm L}, \tag{6}$$

where $V_j = 4\pi a_j^3/3$ $(j=1,2)$ is the volume of the bubbles, and $\boldsymbol{e}_{\rm L}$ is the unit vector directed from the center of the cluster toward the center of the single bubble. The second Bjerknes force, which is the net radiation force exerted on the single bubble, is obtained by integrating Eq. (6) over a period of the volume oscillation

$$\boldsymbol{F}_{1\rm B} = -\frac{N\rho}{4\pi L^2}\left\langle\dot{V}_2\dot{V}_1\right\rangle\boldsymbol{e}_{\rm L}, \tag{7}$$

where $\langle\ \rangle$ denotes the time average.

For the bubbles inside the cluster, the additional pressure is derived by An,[14] and its expression is approximated to be

$$p_{2\rm int} = \frac{3\rho N}{8\pi}\left(1-\frac{r_0^2}{3r_{\rm clust}^2}\right)\frac{1}{r_{\rm clust}}\frac{{\rm d}^2V_2}{{\rm d}t^2}. \tag{8}$$

Similarly, we can obtain the secondary Bjerknes force exerted on the bubbles inside the cluster

$$\boldsymbol{F}_{2\rm B} = \boldsymbol{F}_{2\rm B}^{(1)} + \boldsymbol{F}_{2B}^{(2)}, \tag{9}$$

where

$$\boldsymbol{F}_{2\rm B}^{(1)} = \frac{\rho}{4\pi L^2}\left\langle\dot{V}_2\dot{V}_1\right\rangle\boldsymbol{e}_{\rm L}, \tag{10}$$

$$\boldsymbol{F}_{2\mathrm{B}}^{(2)} = \frac{N\rho}{4\pi}\frac{r_0}{r_{\mathrm{clust}}^3}\left\langle V_2\ddot{V}_2\right\rangle \boldsymbol{e}_{r_0}. \tag{11}$$

Here, $\boldsymbol{e}_{r_0}$ is the radial unit vector in the spherical coordination at the center of the cluster. The first term in Eq. (9) is the net force exerted on the single bubble, and the second term is the internal force due to the internal interactions inside the cluster. It is obvious that $\boldsymbol{F}_{2\mathrm{B}}^{(2)}$ varies with the position of the bubbles, and determines the relative drift of the bubble within the cluster. Therefore, we can analyze the stability of the cluster by estimating the change of $\boldsymbol{F}_{2\mathrm{B}}^{(2)}$.

3. Numerical results

In the present study, numerical calculations of the secondary Bjerknes force are carried out for the system of a single bubble and a bubble cluster when it is driven by external uniform ultrasonic fields with a driving frequency of 20 kHz in water. The other parameters are set as follows: density $\rho = 1000$ kg/m^3, viscosity $\eta = 0.001$ Pa·s, surface tension $\sigma = 0.0728$ N/m, sound speed $c = 1500$ m/s, and static pressure $p_0 = 1$ bar. It is assumed that the equilibrium bubble radii are in the range from 2 to 10 μm, which covers the typical values for each ultrasonic frequency.[22] Effects of the interactions between bubbles are embodied in Eqs. (2) and (3). In addition, the same calculations are made for the case of an isolated oscillating bubble.

In order to investigate effects of additional pressure on each bubble's behavior, we have calculated the oscillating radii for the single bubble and the bubble at the center and the surface of the cluster. As shown in Fig. 1, the total bubble number varies from 100 to 10^4 for a cluster with a radius of 1 mm. The evolution of the single bubble varies very slightly with the increase of the total number of bubbles when the distance between the single bubble and the center of the cluster is 1 cm, which may be an uncoupled distance. However, the evolution of bubbles in the cluster of different number density obviously varies because the couple strength of bubbles increases with the increment of number density. By comparing the bubble motion in different regions, we find that the maximum radius of a bubble in the cluster is suppressed, and the suppression is enhanced as the bubble density increases. It should be noted that for a relatively small bubble density (e.g. $N = 100$), bubbles in the cluster collapse simultaneously. However, with the increment of number density, the delay of collapse time between bubbles at the center and the surface increases slightly.

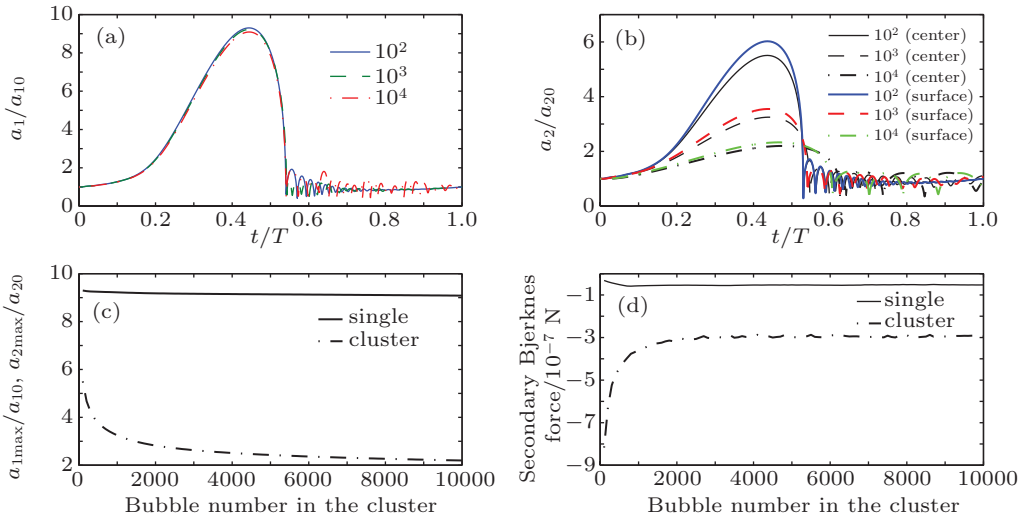


Fig. 1. (color online) Effects of bubble number inside the cluster. Calculations are performed during one driving period at a pressure amplitude of 1.32 atm for $a_{10} = a_{20} = 5$μm and $L = 1$ cm. (a) Single bubble radius a_1/a_{10} versus normalized t (t/T) for three different total bubble numbers: 10^2, 10^3, and 10^4. (b) Bubble radius inside the cluster a_2/a_{20} versus normalized t (t/T) for the same condition as case (a) when a bubble is at the center or surface of the cluster. (c) The maximum radii for the single bubble and a bubble inside the cluster versus the total bubble number of the cluster. (d) The secondary Bjerknes force versus the total bubble number of the cluster.

Equation (11) shows that the value of the secondary Bjerknes force for a bubble inside the cluster varies from 0 ($r_0 = 0$) to the maximum ($r_0 = r_{\mathrm{clust}}$) which is the force acting on a bubble at the surface of the cluster. Figure 1(d) shows the secondary Bjerknes force for different number densities for the uncoupled bubble ($L = 1$ cm) and a coupled bubble at the surface of the cluster. Bubbles are supposed to be in the same size (5 μm). In this case, a single bubble is attracted by the bubble cluster, and the attracting force increases slightly when the total bubble number exceeds about 700. Similarly, the bubbles at the surface of the cluster have a tendency of moving to the center. Therefore, a cluster of identical bubbles is relatively stable. However, with the increment of bubble density, the secondary Bjerknes force decreases and tends to

be close to a fixed value after the total bubble number exceeds about 2000. This phenomenon indicates that the restricting ability of the bubble cluster decreases with the increment of bubble density, and the bubbles at the surface of the cluster would more easily escape from the high density bubble cluster under the disturbance of the surrounding conditions. Although the couple strength of bubbles increases with the increment of number density, the effects on the secondary Bjerknes force would be canceled by the suppression of bubble motions.

If the amplitude of driving pressure increases, the maximum radii of all bubbles will increase. However, the increment of the uncoupled bubble is much greater than that of the coupled bubble in the cluster, as shown in Fig. 2(a). Because the oscillation amplitude is enhanced, the secondary Bjerknes force acting on the bubbles increases, especially for the uncoupled bubble, as shown in Fig. 2(b). It should be noted that the possibility of bubble coalescence would be enhanced by the large amplitude oscillation of the coupled bubble in the cluster, which makes the bubble–bubble interaction in the cluster complex. Our present model in this paper cannot settle down this kind of problem.

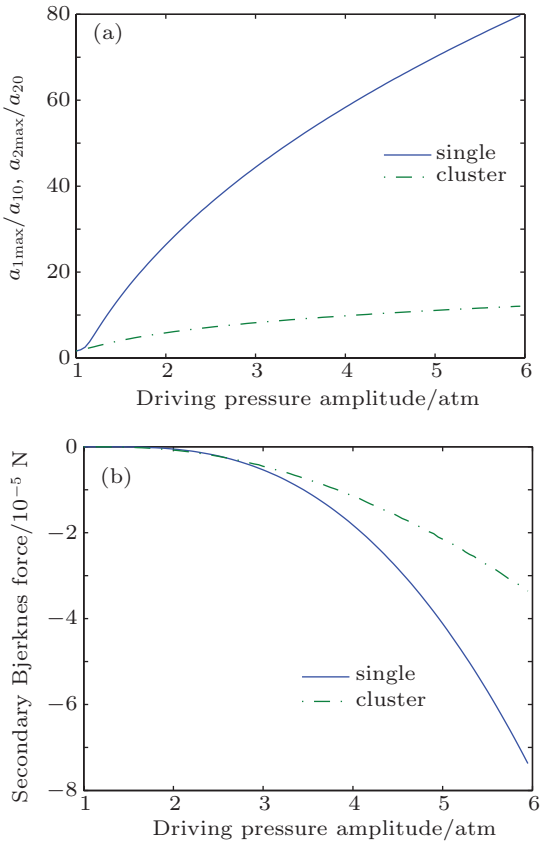


Fig. 2. (color online) Effects of the driving amplitude. Here the other parameters are the same as those in Fig. 1 except for $N = 10^3$. (a) The maximum radii for the single bubble and a bubble inside a cluster versus the driving amplitude. (b) The secondary Bjerknes force versus the driving amplitude.

Chen *et al.*[24] reported that cavitation bubble clusters caused by a focused ultrasonic transducer first emerged in the focal region, then appeared in the post-focal region, and finally forms in the pre-focal region. Larger cavitation bubbles generated because of the violent collision and coalescence of bubbles in the focal region with a higher number density of bubbles. Cao *et al.*[25] reported that the sonochemiluminescence (SCL) emissions move along specific paths and cause branch-like streamers in the post-focal region of a focused ultrasonic transducer, which is related to the translational motion of cavitation bubbles. These results demonstrate that the emersion of cavitation bubbles and structural feature of clusters are determined by the driving pressure field. The number density of bubbles may be proportional to the amplitude of driving pressure. When the number density of bubbles exceeds a specific value, the coalescence of bubbles would arise and some bubbles may escape from the cluster along specific paths, which agrees with the results we obtained in Figs. 1 and 2. In the cavitation field, many factors may influence the interaction of bubbles, and the efficiency of energy transition would be dependent on the motion of every cavitation bubble. Thus, further studies are required on this topic.

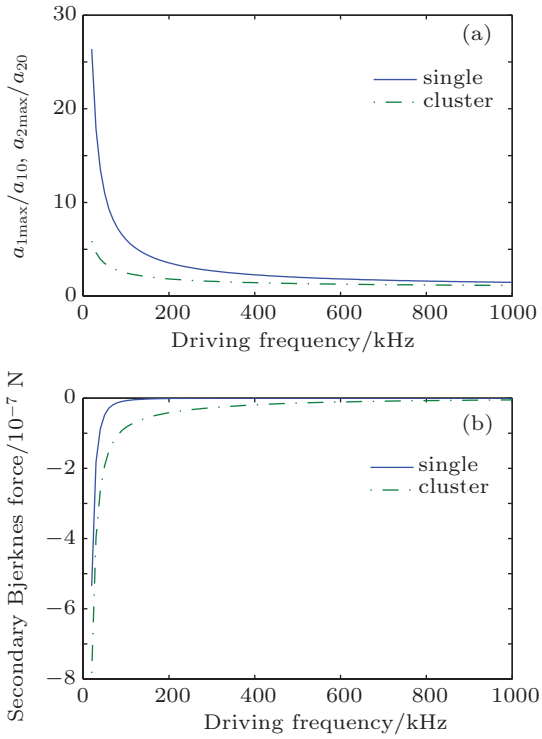


Fig. 3. (color online) Effects of the driving frequency. Here the other parameters are the same as that in Fig. 1 except for $N = 10^3$. (a) The maximum radii for the single bubble and a bubble inside the cluster versus driving frequency. (b) The secondary Bjerknes force versus driving frequency.

In Fig. 3(a), it is observed that the maximum radii of bubbles decrease with the increment of driving frequency of sound waves, although their amplitudes of pressure are the same. When the driving frequency is larger than 200 kHz for the driving pressure amplitude of 1.32 atm, the oscillation of bubbles

is nearly linear. Therefore, the interaction of bubbles is much weaker than that of nonlinear oscillating bubbles, as shown in Fig. 3(b). The secondary Bjerknes force decreases quickly with the increment of driving frequency. As a result, we can infer that the oscillating bubbles are relatively independent of each other as they are driven by high-frequency ultrasound.

As the single bubble is close to the bubble cluster, a transition from the uncoupled state to the coupled state occurs when the distance between the single bubble and the center of the cluster is about 2 mm, as shown in Fig. 4. When the distance is about 1.5 mm, the maximum radius of the single bubble reaches a minimum value, and the secondary Bjerknes force is very close to zero. The direction of the secondary Bjerknes force is reversed when the single bubble nearly reaches the surface of the cluster, which means that when the bubbles are close to each other, a repulsive potential would be induced naturally. However, bubbles in the cluster are hardly affected by the single bubble because their motions are mainly determined by the coupled strength of the cluster.

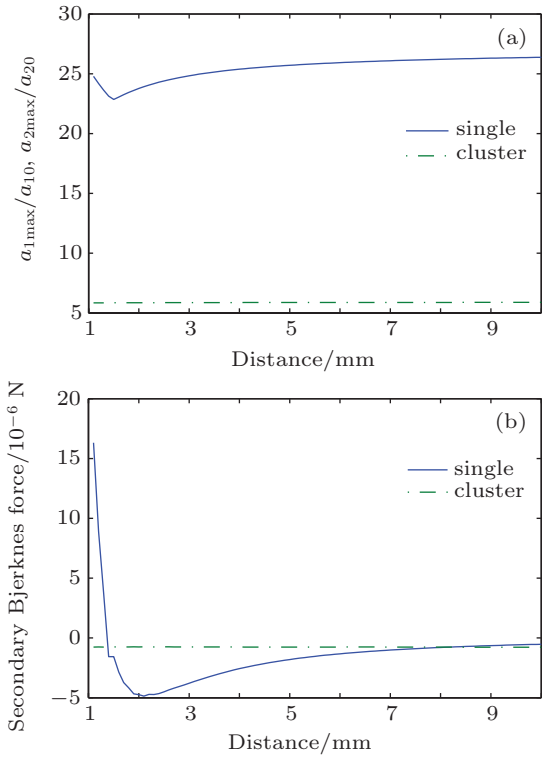


Fig. 4. (color online) Effects of the distance between the single bubble and the center of the cluster. Here the other parameters are the same as that in Fig. 1 except for $N = 10^3$. (a) The maximum radii for the single bubble and a bubble inside the cluster versus the distance. (b) The secondary Bjerknes force versus the distance.

From Fig. 5, we can see that the relative maximum radius of the single bubble reaches its peak as the initial radius is about 3 μm. In this case, the secondary Bjerknes force acting on the single bubble is maximal. The value of the secondary Bjerknes force is proportional to the volume of the single bubble variations, and the bubble volume derivation is proportional to the bubble wall velocity. Because the bubbles in the cluster are hardly affected by the change of the initial radius of the single bubble, the secondary Bjerknes force acting on the bubble inside the cluster are almost invariable. For a larger relative maximum radius, the subsequent collapse would be more violent, and the radiation pressure pulse could be stronger. Therefore, a strong interaction is obtained. Compared with the bubbles whose equilibrium radii range from 2 to 10 μm, the effect of bubble cluster on larger bubbles ($\geq$ 10 μm) is weaker. Therefore, in the high-intensity ultrasound, the translational motion of a single larger bubble may be mainly influenced by the primary Bjerknes force and the buoyant force.

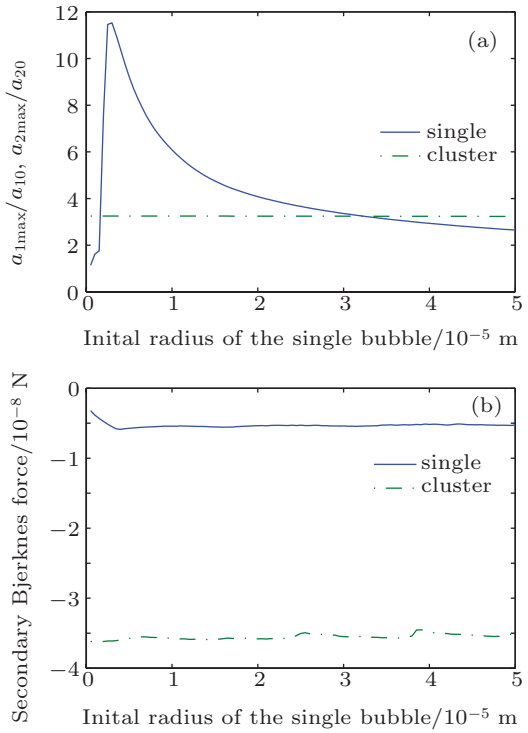


Fig. 5. (color online) Effects of the initial radius of the single bubble. Here the other parameters are the same as that in Fig. 1 except for $N = 10^3$. (a) The maximum radii for the single bubble and a bubble inside the cluster versus the initial radius. (b) The secondary Bjerknes force versus the initial radius.

By comparing the above-mentioned figures, it can be confirmed that the secondary Bjerknes forces for the single bubble and a bubble inside the cluster are of the same order of magnitude when the bubbles are driven by a homogeneous external sound field. The secondary Bjerknes forces are sensitive to the initial radius of nonlinear oscillating bubbles. It should be noted, however, that the acoustic amplitude may considerably change as the acoustic radiation resistance could be strongly reduced by cavitation.[15] The suppression of bubbles close to the center of the cluster may be underestimated. In spite of this, we believe that our model is efficient in dealing with this type of problem because the oscillation of the bubbles only affects the magnitude of the secondary Bjerknes force. The

distance and initial radius difference determines the motion tendency of repulsion or attraction.

4. Conclusions

The distribution of cavitation bubbles may be assembled to many kinds of structures under the drive of high-intensity ultrasound. The interaction of cavitation bubbles is complicated. If many coupled bubbles oscillate almost simultaneously in a spherical-like region, a spherical bubble cluster model can be employed. Based on the assumption of bubble structure, the interaction between a single bubble and a bubble cluster is analyzed in coupled and uncoupled states. The numerical simulation has demonstrated that the secondary Bjerknes force can be influenced by the number density, initial radius, distance, driving frequency, and amplitude of ultrasound. However, if a bubble approaches a bubble cluster of the same initial radii, coupled oscillation will be induced and a repulsive force is evoked, which may be the reason why the bubble cluster can exist steadily. With an increment in the number density of the bubble cluster, the secondary Bjerknes force acting on the bubbles inside the cluster decreases due to the strong suppression of coupled bubbles. We also find that there may be an optimal number density for a bubble cluster, which can generate an optimal cavitation effects in liquid for a stable driving ultrasound. The interaction is stronger for the equilibrium bubble radii ranging from 2 to 10 μm, and the oscillation of bubbles inside clusters is hardly affected by other single bubbles beyond the clusters.

References

[1] Cai M, Zhao S and Liang H 2010 *Desalination* **263** 133
[2] Jagannathan T K, Nagarajan R and Ramamurthi K 2011 *Chem. Eng. Prog.* **50** 305
[3] Collis J, Manasseh R, Liovic P, Tho P, Ooi A, Petkovic-Duran K and Zhu Y 2010 *Ultrason.* **50** 273
[4] Ashokkumar M 2011 *Ultrason. Sonochem.* **18** 864
[5] Birkin P R, Offin D G, Vian C J B and Leighton T G 2011 *J. Acoust. Soc. Am.* **130** 3379
[6] Birkin P R, Offin D G, Vian C J B, Leighton T G and Maksimov A O 2011 *J. Acoust. Soc. Am.* **130** 3297
[7] Ida M, Naoe T and Futakawa M 2007 *Phys. Rev.* E **75** 046304
[8] van der Kroon I, Quinto-Su P A, Li F and Ohl C 2010 *Phys. Rev.* E **82** 066311
[9] Brujan E A, Ikeda T, Yoshinaka K and Matsumoto Y 2011 *Ultrason. Sonochem.* **18** 59
[10] Chen Y and Lin I 2008 *Phys. Rev.* E **77** 026304
[11] Quinto-Su P A, Lim K Y and Ohl C 2009 *Phys. Rev.* E **80** 047301
[12] An Y 2012 *Phys. Rev.* E **85** 016305
[13] Toytman I, Silbergleit A, Simanovski D and Palanker D 2010 *Phys. Rev.* E **82** 046313
[14] An Y 2011 *Phys. Rev.* E **84** 066313
[15] Yasui K, Iida Y, Tuziuti T, Kozuka T and Towata A 2008 *Phys. Rev.* E **77** 016609
[16] Yasui K, Towata A, Tuziuti T, Kozuka T and Kato K 2011 *J. Acoust. Soc. Am.* **130** 3233
[17] Chew L W, Klaseboer E, Ohl S and Khoo B C 2011 *Phys. Rev.* E **83** 066313
[18] Sadighi-Bonabi R, Rezaee N, Ebrahimi H and Mirheydari M 2010 *Phys. Rev.* E **82** 016316
[19] Rezaee N, Sadighi-Bonabi R, Mirheydari M and Ebrahimi H 2011 *Chin. Phys.* B **20** 087804
[20] Ida M 2009 *Phys. Rev.* E **79** 016037
[21] Zhang Y L, Zheng H R, Tang M X and Zhang D 2011 *Chin. Phys.* B **20** 114302
[22] Jiang L, Liu F, Chen H S, Wang J D and Chen D R 2012 *Phys. Rev.* E **85** 036312
[23] Ko I and Kwak H 2010 *J. Phys. Soc. Jpn.* **79** 124401
[24] Chen H, Li X J, Wan M X and Wang X P 2006 *Acta Acustica* **31** 532 (in Chinese)
[25] Cao H, Wan M X, Qiao Y Z, Zhang S and Li R X 2012 *Ultrason. Sonochem.* **19** 257