Beam evolutions of solitons in strongly nonlocal media with fading optical lattices*

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We address the impact of imprinted fading optical lattices on the beam evolution of solitons in strongly nonlocal nonlinear media. The results show that the width of the soliton experiences a change with the increasing propagation distance, the critical power for the soliton varies with the lattice fading away, and the soliton breathing is affected by the initial lattice depth and the nonlocality degree.

Keywords: strong nonlocality, optical lattice, optical soliton

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1. Introduction

Solitons in optical lattices, which might be used for all-optical signal processing and routing, have recently attracted a great deal of interest.[1–3] Surface solitons in one-dimensional composite Bessel optical lattices,[4] self-similar spatiotemporal solitary waves in Bessel–Hermite optical lattices,[5] and surface defect gap solitons in two-dimensional optical lattices[6] were analyzed, and a number of unique properties were revealed. Variations of the lattice shape in the longitudinal direction offer a wealth of opportunities for soliton control. Periodic waveguide arrays built of several tilted segments support breathing discrete diffraction managed solitons.[7] Longitudinal periodic modulations of the coupling strength results in soliton oscillations and decays.[8] A harmonic longitudinal modulation leads to a parametric amplification of transverse soliton swinging,[9] while a strong periodic longitudinal modulation can be used to suppress the diffraction of linear beams.[10]

However, when the spatial scale of a beam is comparable to the characteristic response length of a medium, the nonlinear response of the medium might be nonlocal, which drastically affects the characteristics of light.[11–13] In particular, nonlocality in nematic liquid crystals (NLC) with optical lattices supports a family of chirp-imprinted discrete breathers which can not exist in a purely local medium.[14] All-optical switching and beam steering were demonstrated experimentally in a voltage-controlled array of channel waveguides in NLC.[15] Since the nonlocality drastically reduces the Peierls–Nabarro (PN) potential barrier,[16] the radiative losses of solitons across the lattices are very small in strongly nonlocal media,[17,18] unlike the local case.[3]

2. Variational approach and beam evolution

In this paper, we study the beam evolutions of solitons in a strongly nonlocal nonlinear media with an optical lattice that fades away exponentially along the propagation direction. We consider the following dimensionless nonlocal nonlinear Schrödinger equation (NNLSE)[19] with a fading optical lattice:[20]

$$\mathrm{i}\frac{\partial \widetilde{U}}{\partial z}+\frac{1}{2}\frac{\partial^2 \widetilde{U}}{\partial^2 x}+\widetilde{U}\int R(x-\xi)|\widetilde{U}(\xi,z)|^2\mathrm{d}\xi +\widetilde{U}pQ(x,z)=0, \tag{1}$$

where $\widetilde{U}(x,z)$ is the complex amplitude envelop of the light beam, x and z are transverse and longitudinal coordinates scaled to the beam width and the diffraction length, respectively, $R(x)$ is the real symmetric nonlocal response function, and $Q(x,z)=\cos^2(\Omega x)\exp(-\delta z)$ describes the profile of the optical lattice (Ω is the lattice frequency and δ is the decay rate) (Fig. 1). In practice, such a lattice can be induced optically. Specifically, the refractive index modulation in the transverse direction can be induced with two interfering plane waves.[21,22] The longitudinal modulation can be created by varying intensities,[23] intersection angles, or carrying wavelengths[24] of the lattice-forming plane waves.

In the strongly nonlocal case, if $R(x)$ is twice differentiable at $x=0$, equation (1) is reduced to the following equation:[18,19]

$$\mathrm{i}\frac{\partial U}{\partial z}+\frac{1}{2}\frac{\partial^2 U}{\partial x^2}-\frac{U}{2}\gamma P x^2+Up\cos^2(\Omega x)\exp(-\delta z)=0, \tag{2}$$

where $\gamma=-R''(0)>0$, and $P=\int|U(x,z)|^2\mathrm{d}x$ is the beam power. It is easily shown that equation (2) can be restated as a

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variational problem with the Lagrangian density given by[25]

$$\begin{aligned} L_{\rm g} &= \frac{\rm i}{2}\Big(U\frac{\partial U^*}{\partial z}-U^*\frac{\partial U}{\partial z}\Big)+\frac{1}{2}\Big|\frac{\partial U}{\partial x}\Big|^2+\frac{1}{4}\gamma P x^2|U|^2 \\ &\quad -p\cos^2(\Omega x)\exp(-\delta z)|U|^2 \\ &= L_1+L_2, \end{aligned} \tag{3}$$

where

$$L_1 = \frac{\rm i}{2}\Big(U\frac{\partial U^*}{\partial z}-U^*\frac{\partial U}{\partial z}\Big)+\frac{1}{2}\Big|\frac{\partial U}{\partial x}\Big|^2+\frac{1}{4}\gamma P x^2|U|^2, \tag{4}$$

$$L_2 = -p\cos^2(\Omega x)\exp(-\delta z)|U|^2. \tag{5}$$

This implies that equation (2) results from the variational equation

$$\frac{\delta L_{\rm g}}{\delta U^*}=\frac{\partial}{\partial z}\frac{\partial L_{\rm g}}{\partial(\frac{\partial U^*}{\partial z})}+\frac{\partial}{\partial x}\frac{\partial L_{\rm g}}{\partial(\frac{\partial U^*}{\partial x})}-\frac{\partial L_{\rm g}}{\partial U^*}=0, \tag{6}$$

corresponding to the variational principle

$$\delta\iint L_{\rm g}\Big(U,U^*,\frac{\partial U}{\partial z},\frac{\partial U^*}{\partial z},\frac{\partial U}{\partial x},\frac{\partial U^*}{\partial x}\Big)\,{\rm d}x{\rm d}z=0. \tag{7}$$

Since we focus on the strongly nonlocal case, we employ the following Gaussian-type trial solution:

$$U(x,z)=A(z)\exp[{\rm i}\varphi(z)]\exp\Big[-\frac{x^2}{2w(z)^2}+{\rm i}c(z)x^2\Big], \tag{8}$$

where A and φ are the amplitude and phase of the complex amplitude of the solution, respectively, w is the beam width, and c is the phase-front curvature. This choice of ansatz is justified since in the limit $p\to 0$, equation (8) describes solitons in uniform strongly nonlocal nonlinear media.[19]

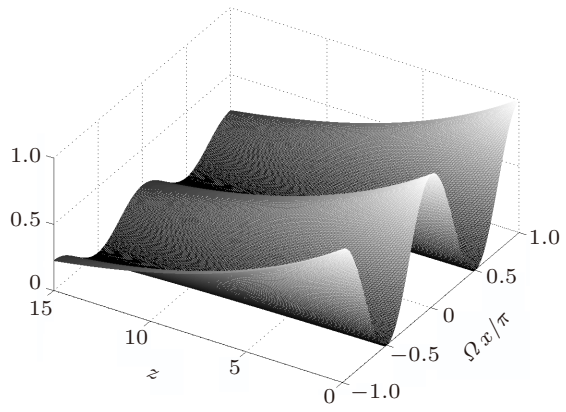


Fig. 1. Profile of a fading lattice with $\delta=0.1$ and $p=1$.

Following the standard procedures of the variational approach,[25] we obtain the following variational equations:

$$\frac{\delta L_{\rm r}}{\delta\varphi} = 0\Rightarrow\frac{{\rm d}(A^2w)}{{\rm d}z}=0, \tag{9}$$

$$\frac{\delta L_{\rm r}}{\delta c} = 0\Rightarrow 3A\frac{{\rm d}w}{{\rm d}z}+2w\frac{{\rm d}A}{{\rm d}z}-4Awc=0, \tag{10}$$

$$\frac{\delta L_{\rm r}}{\delta w} = 0\Rightarrow\frac{3w^2}{2}\frac{{\rm d}c}{{\rm d}z}+\frac{{\rm d}\varphi}{{\rm d}z}=\frac{1}{4w^2}-3w^2c^2+\sqrt{\pi}wA^2(R_0-\gamma w^2)+\frac{p}{2}\exp(-\delta z)\big[1+\exp(-\Omega^2w^2)(1-2\Omega^2w^2)\big], \tag{11}$$

$$\frac{\delta L_{\rm r}}{\delta A} = 0\Rightarrow\frac{{\rm d}c}{{\rm d}z}+\frac{2}{w^2}\frac{{\rm d}\varphi}{{\rm d}z}=-\frac{1}{2w^4}-2c^2+\frac{\sqrt{\pi}A^2}{w}(2R_0-\gamma w^2)+\frac{p}{w^2}\exp(-\delta z)[1+\exp(-\Omega^2w^2)], \tag{12}$$

where $L_{\rm r}=\int L_{\rm g}\,{\rm d}x=\int L_1\,{\rm d}x+\int L_2\,{\rm d}x$.

Equations (9)–(12) describe the evolutions of the nontrivial variational variables. Equation (9) is nothing but the energy conservation law, i.e.,

$$P=P_0=\sqrt{\pi}wA^2. \tag{13}$$

Based on Eqs. (9)–(12), we obtain the following differential equation for the soliton width:

$$\frac{{\rm d}^2w}{{\rm d}z^2}=\frac{1}{w^3}-P\gamma w-2\Omega^2p\exp(-\delta z)\exp(-\Omega^2w^2)w. \tag{14}$$

Equation (14) is equivalent to Newton's second law in classical mechanics describing the motion of a one-dimensional particle acted by an equivalent force $F=1/w^3-P\gamma w-2\Omega^2p\exp(-\delta z)\exp(-\Omega^2w^2)w$. The first term of F is the so-called diffractive force which makes the beam expand, while the last two terms are a refractive force which compresses the beam. Compare the force here with the one discussed in Ref. [19], we find in the former an additional term which fades away with the increasing propagation distance. This indicates that the equilibrium beam width varies with the lattice fading away. Letting $F=0$, we obtain the critical power for the solitons

$$P_{\rm c}=\frac{1}{\gamma}\Big[\frac{1}{w_0^4}-2\Omega^2p\exp(-\delta z)\exp(-\Omega^2w_0^2)\Big]. \tag{15}$$

Equation (15) shows that $P_{\rm c}$ varies to a saturation value $1/(\gamma w_0^4)$ with the propagation distance, which suggests that the soliton can maintain its initial width w_0 only if the power varies according to Eq. (15). However, the soliton power is conservative (Eq. (9)), so the soliton width experiences a change with the propagation distance. In brief, this analysis implies that the soliton width, accompanied with breathing, tends to broaden if it is launched into the medium at the power $P_0=P_{\rm c}|_{z=0}$. At a certain distance where $P_{\rm c}$ approaches its saturation value, the broadening ceases while the breathing remains.

3. Numerical simulation

To confirm the above results, we numerically simulate the propagation of a Gaussian-type beam in a strongly nonlocal nonlinear medium with a fading optical lattice. We take Eq. (1) as the evolution equation and $U(x,z)|_{z=0}$ in Eq. (8) as the initial condition. The numerical arithmetic used here

is the split-step Fourier method.[26] The material response is assumed to be a Gaussian function,[13,18,19] i.e., $R(x) = \exp(-x^2/2w_m^2)/(\sqrt{2\pi}w_m)$, and we introduce a nonlocal parameter α to define the degree of nonlocality, where α is the ratio of the beam width w to the characteristic nonlocal response length w_m of the medium. The smaller the nonlocal parameter, the stronger the degree of nonlocality.

The numerical results for the width evolutions of solitons are summarized in Figs. 2 and 3. It is found in Fig. 2 that the distance needed for the soliton to achieve a stationary regime decreases with the increase of the lattice decay rate, while the soliton breathing amplitude grows with it. This trend is reasonable since in the limit $\delta \to 0$, the soliton becomes a z-invariant lattice soliton which does not oscillate in the propagation direction. Figure 3 shows the width evolutions of solitons for different initial lattice depths (Fig. 3(a)) and different nonlocality degrees (Fig. 3(b)). Figure 3(a) suggests that the soliton breathing amplitude grows with the increase of the lattice depth, and figure 3(b)) shows that the equilibrium width of the soliton grows with the decrease of the nonlocality degree (i.e., the increase of α) as the nonlocality changes the critical power of the system.

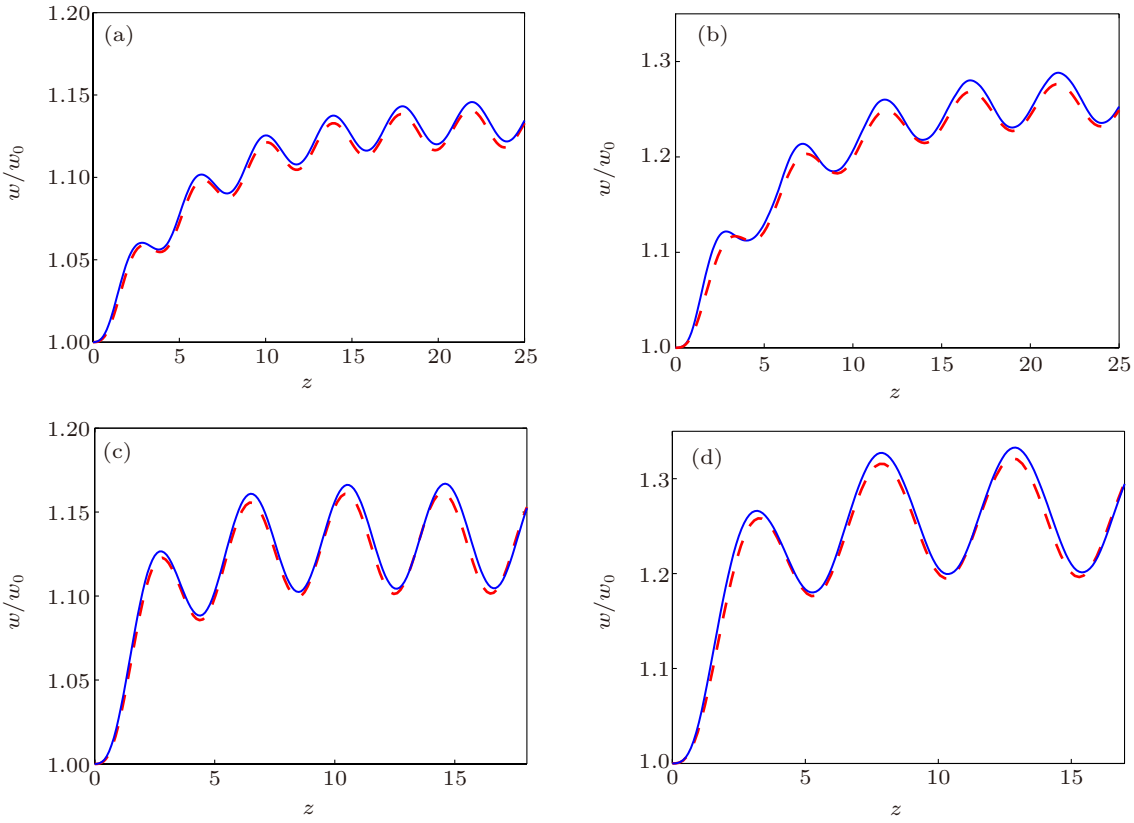


Fig. 2. (color online) Comparison between the numerical simulations (solid lines) and variational results (dashed lines) (Eq. (14)) of beam width evolution when $\alpha = 0.1$ (the strongly nonlocal condition), $p = 1$, $P_0 = P_c|_{z=0}$, $w_0 = 1$, $\frac{dw}{dz}|_{z=0} = 0$, and (a) $\Omega = 0.5$, $\delta = 0.2$; (b) $\Omega = 0.7$, $\delta = 0.2$; (c) $\Omega = 0.5$, $\delta = 0.5$; (d) $\Omega = 0.7$, $\delta = 0.5$.

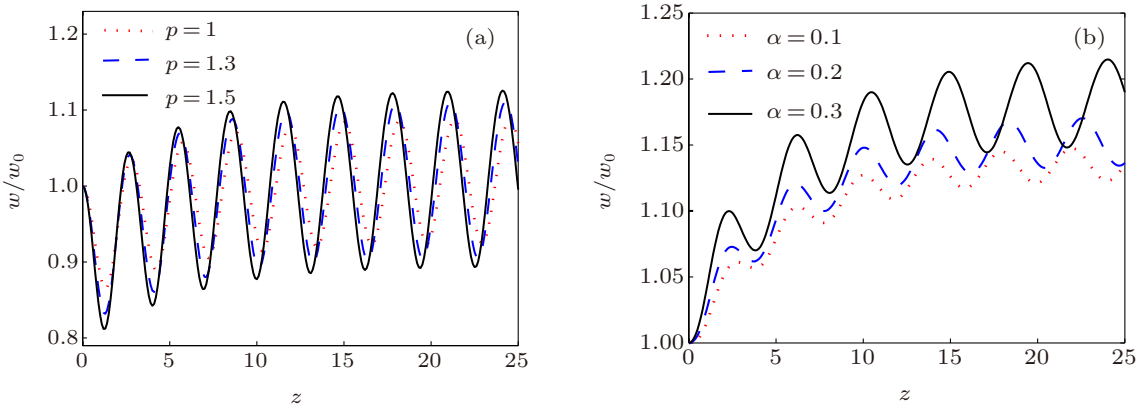


Fig. 3. (color online) Numerical simulations of beam width evolution for (a) different initial lattice depths and (b) different degrees of nonlocality, where (a) $P_0 = P_c|_{z=\infty}$, $\Omega = 0.5$, $\delta = 0.2$, $\alpha = 0.1$; (b) $P_0 = P_c|_{z=0}$, $\Omega = 0.5$, $\delta = 0.2$, $p = 1$.

4. Conclusion

In conclusion, we have studied the impact of imprinted fading optical lattices on the beam evolutions of solitons in strongly nonlocal nonlinear media. Based on the variational approach, we obtain a differential equation for the soliton width and find that the width experiences a change with the propagation distance. It is also found that the critical power for the solitons varies with the lattice fading away, and that the soliton breathing is affected by the initial lattice depth and the nonlocality degree.

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