Applicability of the minimum entropy generation method for optimizing thermodynamic cycles*

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Entropy generation is often used as a figure of merit in thermodynamic cycle optimizations. In this paper, it is shown that the applicability of the minimum entropy generation method to optimizing output power is conditional. The minimum entropy generation rate and the minimum entropy generation number do not correspond to the maximum output power when the total heat into the system of interest is not prescribed. For the cycles whose working medium is heated or cooled by streams with prescribed inlet temperatures and prescribed heat capacity flow rates, it is theoretically proved that both the minimum entropy generation rate and the minimum entropy generation number correspond to the maximum output power when the virtual entropy generation induced by dumping the used streams into the environment is considered. However, the minimum principle of entropy generation is not tenable in the case that the virtual entropy generation is not included, because the total heat into the system of interest is not fixed. An irreversible Carnot cycle and an irreversible Brayton cycle are analysed. The minimum entropy generation rate and the minimum entropy generation number do not correspond to the maximum output power if the heat into the system of interest is not prescribed.

Keywords: entropy generation, thermodynamic cycles, heat–work conversion, optimization

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1. Introduction

Since the world energy situation is getting more and more serious, the analysis and optimization of heat transfer and heat–work conversion attract increasing attention.[1–8] Optimizing these processes can improve energy utilization and is of great significance for reducing energy consumption. For instance, the heat convection optimization can increase the heat transfer rate,[1] and the thermodynamic cycle optimization can increase the power output.[2]

Entropy generation is one of the most important concepts in optimizing heat transfer and heat–work conversion. Both heat transfer and heat–work conversion are irreversible and non-equilibrium processes from the thermodynamic viewpoint except for the ideal reversible physical processes which could not be achieved in practice.[3] Entropy generation is inevitable in any practical heat transfer process and any practical thermodynamic cycle. Much work has been done on optimizing heat transfer and heat–work conversion with the minimum entropy generation as a figure of merit.[1,2,9–22] For instance, Ahmadi *et al*.[15] optimized a cross-flow plate fin heat exchanger with the minimum entropy generation method. Chen *et al*.[2] showed that the minimum entropy generation always corresponds to the maximum work output in the thermodynamic process optimization with heat exchanger groups. Myat *et al*.[20] demonstrated that the minimization of entropy generation in the absorption cycle leads to the maximization of the coefficient of performance (COP) in their analysis of an absorption chiller.

However, the minimum entropy generation is always related to the best performance. Bejan[21] noticed an entropy generation paradox in a counter flow heat exchanger. The heat exchanger effectiveness does not always increase with decreasing entropy generation number. Instead, it becomes small for some conditions. Bejan explained the paradox. The paradox occurs when the effectiveness is smaller than 0.5, and he thought that an engineering heat exchanger would not exist with an effectiveness close to zero.[21] Shah and Skiepko[23] demonstrated that the heat exchanger effectiveness can reach a maximum, an intermediate value, or a minimum at the maximum entropy generation in their analyses of 18 kinds of heat exchangers. Chakraborty and Ray[24] revealed that the entropy generation minimization might lead to contradictory results for some of the performance evaluation criteria when they optimized the performance of a laminar fully-developed flow through square ducts with rounded corners. Other research on the analyses of heat exchangers and heat exchanger networks also showed that the minimum entropy generation does not always correspond to the best heat transfer.[25–28] In the heat–work conversion optimization, Klein and Reindl[29] showed that minimizing the entropy generation rate of a refrigeration system does not always result in the same design as maximizing the system performance unless the refrigeration capacity is fixed.

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For the queries about the minimum entropy generation method in heat transfer optimization, some researchers considered the entropy generation as a physical quantity that is developed for thermodynamic cycle analyses and is used to describe the loss of the ability to do work.[30,31] However, in problems of pure heat transfer, the concern is not the loss of the ability to do work but the heat transfer rate or the efficiency. Therefore, the optimization object of the minimum entropy generation method is not the target, and then the minimum entropy generation method is not always suitable for heat transfer optimization. The minimum entropy generation has been shown to not always match the best performance of a thermodynamic system[29] even though the entropy generation is a very important concept for thermodynamic cycles. It is worth having further discussion on the applicability of the minimum entropy generation method to the optimization of thermodynamic cycles.

2. Entropy generation in thermodynamic cycles

A common thermodynamic system shown in Fig. 1 is composed of a working medium and heat sources. The rate of energy absorbed from the high temperature heat sources by the working medium is $Q_{\rm H}$, while that released to the low temperature heat sources is $Q_{\rm L}$; the output power is W. For the heat sources and the working medium of the system, the entropy balance equation is[30]

$$\mathrm{d}S = \mathrm{d}S_{\rm f} + \delta S_{\rm g}, \tag{1}$$

where $\mathrm{d}S$ is the entropy change rate, $\mathrm{d}S_{\rm f}$ is the entropy flow rate, and $\delta S_{\rm g}$ is the entropy generation rate. As the system is steady, $\mathrm{d}S$ equals zero. The entropy generation rate could be expressed as

$$\delta S_{\rm g} = -\mathrm{d}S_{\rm f}. \tag{2}$$

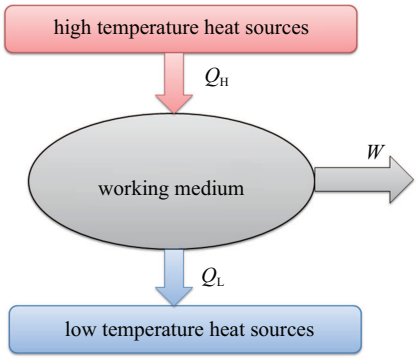


Fig. 1. (color online) Sketch of a thermodynamic cycle.

Entropy is a state quantity. There is no entropy change once the working medium returns to its initial state after a cycle. The entropy generation rate of the system is

$$S_{\rm g} = \int \delta S_{\rm g} = -\int \mathrm{d}S_{\text{f-hs}} = -\int (\delta Q_{\text{f-hs}}/T_{\rm f}), \tag{3}$$

where $\mathrm{d}S_{\text{f-hs}}$ is the entropy flow rate from the heat sources, and $\delta Q_{\text{f-hs}}$ is the heat rate out of the heat source with temperature $T_{\rm f}$.

In practical engineering, the working medium may be heated or cooled by streams through heat exchangers. Bejan[21] considered the entropy generation associated with dumping the used streams into the environment in the discussion of power plant optimization. The environment is one heat source for the thermodynamic process. In the thermodynamic cycle shown in Fig. 2, the environment is also treated as a cold source. Assume that there are n hot streams and m cold streams. The heat capacity flow rates of the i-th hot stream and the j-th cold stream are $C_{\text{H-}i}$ and $C_{\text{L-}j}$, the inlet temperatures are $T_{\text{H-in-}i}$ and $T_{\text{L-in-}j}$, and the outlet temperatures are $T_{\text{H-out-}i}$ and $T_{\text{L-out-}j}$, respectively. In a cycle, the working medium absorbs heat at rate $Q_{\text{H-}i}$ from the i-th hot stream at temperature $T_{\text{H-}i}$, and releases heat at rate $Q_{\text{L-}j}$ to the j-th cold stream, and a power W is produced. The entropy flow rate from the i-th hot stream could be calculated by

$$\begin{aligned} S_{\text{f-H-}i} &= \int \frac{\delta Q_{\text{H-}i}}{T_{\text{H-}i}} \\ &= \int_{T_0}^{T_{\text{H-in-}i}} \frac{C_{\text{H-}i}\,\mathrm{d}T_{\text{H-}i}}{T_{\text{H-}i}} \\ &= \int_{T_0}^{T_{\text{H-in-}i}} C_{\text{H-}i}\,\mathrm{d}\left(\ln T_{\text{H-}i}\right) \\ &= C_{\text{H-}i} \ln \frac{T_{\text{H-in-}i}}{T_0}. \end{aligned} \tag{4}$$

For the j-th cold stream, its temperature would decrease to the temperature of the environment T_0 finally although it receives $Q_{\text{L-}j}$ from the cycle. The entropy flow rate out of the cold stream is

$$\begin{aligned} S_{\text{f-L-}j} &= \int \frac{\delta Q_{\text{L-}j}}{T_{\text{L-}j}} \\ &= \int_{T_0}^{T_{\text{L-in-}j}} \frac{C_{\text{L-}j}\,\mathrm{d}T_{\text{L-}j}}{T_{\text{L-}j}} \\ &= \int_{T_0}^{T_{\text{L-in-}j}} C_{\text{L-}j}\,\mathrm{d}\left(\ln T_{\text{L-}j}\right) \\ &= C_{\text{L-}j} \ln \frac{T_{\text{L-in-}j}}{T_0}. \end{aligned} \tag{5}$$

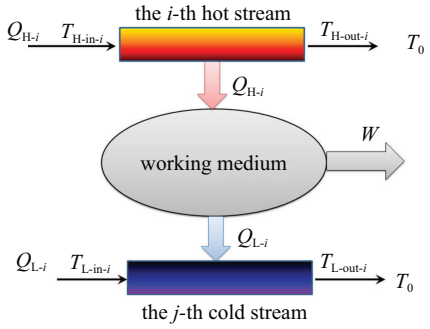


Fig. 2. (color online) Thermodynamic cycle with the working medium heated and cooled by streams.

As all the used streams are finally discharged into the environment, the environment is also one of the low temperature heat sources for the thermodynamic process in Fig. 2. Then, the entropy flow rate into the environment is

$$\begin{aligned} & S_{\text{f-e}} \\ = & -\int \frac{\delta Q_{\text{e}}}{T_0} \\ = & -\frac{\sum_{i=1}^{n} C_{\text{H-}i}\left(T_{\text{H-out-}i}-T_0\right)+\sum_{j=1}^{m} C_{\text{L-}j}\left(T_{\text{L-out-}j}-T_0\right)}{T_0}, \end{aligned} \tag{6}$$

where Q_{e} is the flow rate of heat released to the environment. The entropy generation rate of the whole system is

$$\begin{aligned} S_{\text{g}} &= -\int \mathrm{d}S_{\text{f-hs}} \\ &= -\left(S_{\text{f-e}}+\sum_{i=1}^{n} S_{\text{f-H-}i}+\sum_{j=1}^{m} S_{\text{f-L-}j}\right) \\ &= \frac{\sum_{i=1}^{n} C_{\text{H}}\left(T_{\text{H-out-}i}-T_0\right)+\sum_{j=1}^{m} C_{\text{L}}\left(T_{\text{L-out-}j}-T_0\right)}{T_0} \\ &\quad -\sum_{i=1}^{n} C_{\text{H-}i}\ln\frac{T_{\text{H-in-}i}}{T_0}-\sum_{j=1}^{m} C_{\text{L-}j}\ln\frac{T_{\text{L-in-}j}}{T_0}. \end{aligned} \tag{7}$$

Considering the energy conservation, we have

$$\begin{aligned} W &= Q_{\text{s}}-Q_{\text{e}} \\ &= \left[\sum_{i=1}^{n} C_{\text{H-}i}\left(T_{\text{H-in-}i}-T_0\right)\right. \\ &\quad \left.-\sum_{j=1}^{m} C_{\text{L-}j}\left(T_{\text{L-in-}j}-T_0\right)\right]-Q_{\text{e}}, \end{aligned} \tag{8}$$

where Q_{s} is the total heat absorption rate of the working medium. Substituting Eq. (8) into Eq. (7) leads to

$$\begin{aligned} & S_{\text{g}} \\ = & \frac{\sum_{i=1}^{n} C_{\text{H-}i}\left(T_{\text{H-in-}i}-T_0\right)+\sum_{j=1}^{m} C_{\text{L-}j}\left(T_{\text{L-in-}j}-T_0\right)-W}{T_0} \\ & -\sum_{i=1}^{n} C_{\text{H-}i}\ln\frac{T_{\text{H-in-}i}}{T_0}-\sum_{j=1}^{m} C_{\text{L-}j}\ln\frac{T_{\text{L-in-}j}}{T_0}. \end{aligned} \tag{9}$$

For prescribed $C_{\text{H-}i}$, $C_{\text{L-}j}$, $T_{\text{H-}i}$, and $T_{\text{H-}j}$, the entropy generation rate decreases with increasing output power. The minimum entropy generation rate corresponds to the maximum output power for the thermodynamic cycle shown in Fig. 2. The principle of minimum entropy generation is held in this case.

The concept of entropy generation number is widely used in heat-conversion optimization.[21] The definition of entropy generation number is[21]

$$N_{\text{S}}=\frac{S_{\text{g}}}{C_{\min}}, \tag{10}$$

where $C_{\min}$ is the minimum heat capacity flow rate of the streams and the working media. It is given because $C_{\text{H-}i}$ and $C_{\text{L-}j}$ are prescribed. Therefore, the entropy generation number has the same variation tendency as the entropy generation rate. It means that the minimum entropy generation number also corresponds to the maximum output power for the thermodynamic cycle shown in Fig. 2.

As is well known, entropy generation is the physical quantity used to describe the loss of the ability of heat to do work at a temperature. For the thermodynamic system shown in Fig. 2, including the environment, Eq. (8) indicates that the total heat input into the system is a constant with prescribed $C_{\text{H-}i}$, $C_{\text{L-}j}$, $T_{\text{H-}i}$, and $T_{\text{H-}j}$. For given heat flow rate Q_{s} at a prescribed temperature, its ability to do work can be determined. Therefore, a smaller entropy generation rate means less loss of the ability to do work and a larger output power.

However, the minimum entropy generation method may be not tenable if the total heat absorbed by the working medium in the heat–work conversion process is not prescribed. In Fig. 2, the used streams may not be discharged into the environment directly. The heat in the used streams may be stored or used to heat something or do work in other heat-conversion processes. In such cases, we can only assume that the used streams are discharged into the environment if we want to use the minimum entropy generation method to optimize the thermodynamic cycle. The entropy generation associated with dumping the used streams into the environment is not real but virtual. If we do not consider the virtual entropy generation, the entropy flow rates due to the heat exchange of the hot and cold streams are

$$\begin{aligned} S'_{\text{f-H-}i} &= \int \left(\delta Q_{\text{H-}i}/T_{\text{H-}i}\right) \\ &= \int_{T_{\text{H-out-}i}}^{T_{\text{H-in-}i}} \left(C_{\text{H-}i}\,\mathrm{d}T_{\text{H-}i}/T_{\text{H-}i}\right) \\ &= C_{\text{H-}i}\ln\left(T_{\text{H-in-}i}/T_{\text{H-out-}i}\right), \end{aligned} \tag{11}$$

$$\begin{aligned} S'_{\text{f-L-}j} &= \int \left(\delta Q_{\text{L-}j}/T_{\text{L-}j}\right) \\ &= C_{\text{L-}j}\ln\left(T_{\text{L-in-}j}/T_{\text{L-out-}j}\right). \end{aligned} \tag{12}$$

Then, the entropy generation rate is

$$\begin{aligned} S'_{\text{g}} &= -\int \mathrm{d}S'_{\text{f-hs}} \\ &= -\left(\sum_{i=1}^{n} S'_{\text{f-H-}i}+\sum_{j=1}^{m} S'_{\text{f-L-}j}\right) \\ &= -\sum_{j=1}^{m} C_{\text{L-}j}\ln\left(T_{\text{L-in-}j}/T_{\text{L-out-}j}\right) \\ &\quad -\sum_{i=1}^{n} C_{\text{H-}i}\ln\left(T_{\text{H-in-}i}/T_{\text{H-out-}i}\right) \\ &= \sum_{j=1}^{m} C_{\text{L-}j}\ln\left(T_{\text{L-out-}j}/T_{\text{L-in-}j}\right) \\ &\quad -\sum_{i=1}^{n} C_{\text{H-}i}\ln\left(T_{\text{H-in-}i}/T_{\text{H-out-}i}\right). \end{aligned} \tag{13}$$

From Eq. (13), the monotone function between the entropy generation and the output power cannot be set up with prescribed $C_{\text{H-}i}$, $C_{\text{L-}j}$, $T_{\text{H-}i}$, and $T_{\text{H-}j}$. Chen *et al.*[2] analyzed the optimization of heat-conversion in heat exchanger groups, and

showed that the minimum entropy generation rate does not correspond to the maximum output power when the virtual entropy generation is not considered. This is because the amount of heat into the system which is used for the heat–work conversion is not prescribed, and then the total ability to do work is not known. A smaller entropy generation rate (in other words, less loss of the ability to do work) does not correspond to a larger output power. In addition, as the entropy generation number has the same variation tendency as the entropy generation rate, the minimum entropy generation number does not correspond to the maximum output power in such a case either.

3. Examples of thermodynamic cycle optimization and analyses

3.1. Thermodynamic process optimization of heat exchanger networks

Figure 3 shows a thermodynamic cycle. A hot stream with heat capacity flow rate $C_{\rm H}$ and inlet temperature $T_{\rm H\text{-}in}$ runs into two series-wounded heat exchangers with prescribed wall temperatures T_1 and T_2 respectively. The heat transfer rates between the stream and the heat exchangers are Q_1 and Q_2, respectively. The outlet temperature of the stream is $T_{\rm H\text{-}out}$. Two Carnot engines work between the heat exchangers and the environment at temperature T_0, and the output power is W.

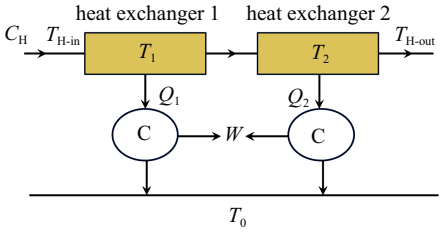


Fig. 3. (color online) Sketch of a thermodynamic process in a heat exchanger network.

The sum of the thermal conductances of the heat exchangers is a constant

$$kA = (kA)_1 + (kA)_2 = \text{const.}, \tag{14}$$

where $(kA)_1$ and $(kA)_2$ are the thermal conductances of the two heat exchangers, respectively. The thermal conductance distribution is to be optimized to increase the output power of the system as much as possible.

The heat transfer rates of the two heat exchangers can be calculated by[2]

$$Q_1 = C_{\rm H}(T_{\rm H\text{-}in} - T_1)\{1 - \exp[-(kA)_1/C_{\rm H}]\}, \tag{15}$$

$$\begin{aligned} Q_2 &= C_{\rm H}(T_{\rm H\text{-}m} - T_2)\{1 - \exp[-(kA)_2/C_{\rm H}]\} \\ &= C_{\rm H}(T_{\rm H\text{-}in} - Q_1/C_{\rm H} - T_2) \\ &\quad \times \{1 - \exp[-(kA)_2/C_{\rm H}]\}, \end{aligned} \tag{16}$$

$$Q = Q_1 + Q_2, \tag{17}$$

where $T_{\rm H\text{-}m}$ is the stream outlet temperature of heat exchanger 1, and Q is the total heat transfer rate of the heat exchangers. The output power can be calculated by

$$W = Q_1(1 - T_0/T_1) + Q_2(1 - T_0/T_2). \tag{18}$$

When the virtual entropy generation rate is considered, from Eq. (9), we have

$$S_{\rm g} = [C_{\rm H}(T_{\rm H\text{-}in} - T_0) - W]/T_0 - C_{\rm H}\ln(T_{\rm H}/T_0). \tag{19}$$

On the other hand, if we do not consider the virtual entropy generation rate, Eq. (3) gives

$$S'_{\rm g} = Q_1/T_1 + Q_2/T_2 - C_{\rm H}\ln(T_{\rm H\text{-}in}/T_{\rm H\text{-}out}). \tag{20}$$

Look at a numerical example with $kA = 2$ W/K, $C_{\rm H} = 3$ W/K, $T_{\rm H\text{-}in} = 450$ K, $T_1 = 400$ K, $T_2 = 350$ K, and $T_0 = 300$ K. The variations of Q, W, $S_{\rm g}$, and $S'_{\rm g}$ with $(kA)_1$ are calculated, and the results are shown in Fig. 4. It can be seen that the minimum entropy generation rate corresponds to the maximum output power when the virtual entropy generation rate associated with dumping the used streams into the environment is considered. This is because that the total heat input into the system, $C_{\rm H}(T_{\rm H\text{-}in} - T_0)$, is fixed.

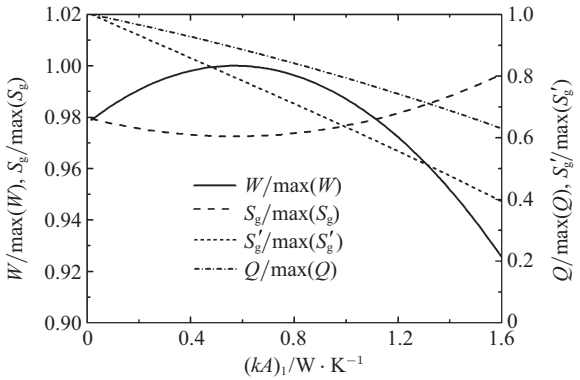


Fig. 4. Variations of Q, W, $S_{\rm g}$, and $S'_{\rm g}$ with $(kA)_1$.

On the other hand, if we do not consider the virtual entropy generation rate, the total absorbed heat by the heat engines is not a constant, but decreases with $(kA)_1$ increasing. Hence, the minimum entropy generation rate does not correspond to the maximum output power. In such a case, the decrease in entropy generation rate $S'_{\rm g}$ cannot describe the change of the output power of the system.

According to the definition of entropy generation number, $C_{\rm min}$ is $C_{\rm H}$ in this problem. The entropy generation number has the same variation tendency as the entropy generation rate. The minimum entropy generation number corresponds to the maximum output power when the virtual entropy generation rate is considered; however, the minimum entropy generation number does not correspond to the maximum output power when the virtual entropy generation rate is not considered.

3.2. Analyses of an irreversible Carnot cycle

An irreversible Carnot cycle is shown in Fig. 5. The working medium absorbs heat at rate $Q_{\rm H}$ from the high temperature heat source whose temperature is $T_{\rm H}$, and releases heat at rate $Q_{\rm L}$ to the low temperature heat source whose temperature is $T_{\rm L}$. The output power of the cycle is W. Assume that the heat transfer conductance between the working medium and the high temperature heat source is kA, then there will be no temperature difference between the working medium and the low temperature heat source when $Q_{\rm L}$ is transferred. The high temperature of the cycle $T_{\rm m}$ is to be optimized to increase the output power.

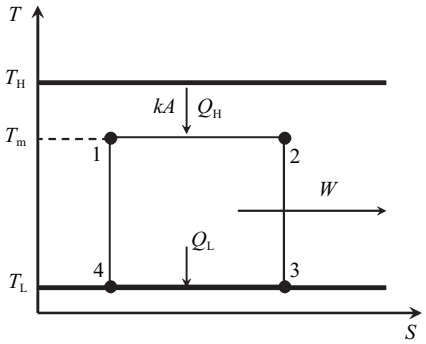


Fig. 5. An irreversible Carnot cycle.

The heat transfer rate $Q_{\rm H}$ can be calculated by

$$Q_{\rm H} = kA\,(T_{\rm H}\text{-}T_{\rm m})\,. \tag{21}$$

The output power is

$$\begin{aligned} W &= Q_{\rm H}\,(1-T_{\rm L}/T_{\rm m}) \\ &= kA\,(T_{\rm H}\text{-}T_{\rm m})\,(1-T_{\rm L}/T_{\rm m}) \\ &= kA\,[T_{\rm H}+T_{\rm L}-(T_{\rm H}T_{\rm L}/T_{\rm m}+T_{\rm m})] \\ &\leq kA\left(\sqrt{T_{\rm H}}-\sqrt{T_{\rm L}}\right)^2. \end{aligned} \tag{22}$$

The equality is tenable when

$$T_{\rm m} = \sqrt{T_{\rm H}T_{\rm L}}. \tag{23}$$

The output power reaches a maximum when Eq. (23) holds.

The entropy generation rate can be simply obtained since the only irreversible process is the heat transfer process between the working medium and the high temperature heat source. There is

$$\begin{aligned} S_{\rm g} &= Q_{\rm H}\,(1/T_{\rm m}-1/T_{\rm H}) \\ &= kA\,(T_{\rm H}\text{-}T_{\rm m})\,(1/T_{\rm m}-1/T_{\rm H}) \\ &= kA\,(T_{\rm H}/T_{\rm m}+T_{\rm m}/T_{\rm H}-2) \\ &\geq 0. \end{aligned} \tag{24}$$

The equality holds only when $T_{\rm H} = T_{\rm m}$. The entropy generation rate will decrease with $T_{\rm m}$ increasing for $T_{\rm m} < T_{\rm H}$. The minimum entropy generation rate does not correspond to the maximum output power.

Figure 6 shows the variations of $Q_{\rm H}$, W, and $S_{\rm g}$ with $T_{\rm m}$ for a numerical example with $T_{\rm H} = 350$ K, $T_{\rm L} = 300$ K, and $kA = 100$ W/K. The output power has a maximum value, while the entropy generation rate decreases with $T_{\rm m}$ increasing. This is because $Q_{\rm H}$ is not a fixed value but decreases with $T_{\rm m}$ increasing, and thus the total ability to do work decreases. In this case, reducing the entropy generation rate is equivalent to reducing the heat exchange between the cycle and the high temperature heat source. The minimum entropy generation method cannot be used for optimizing the output power of the system.

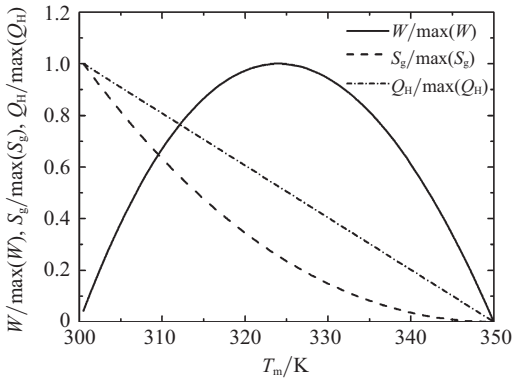


Fig. 6. Variations of $Q_{\rm H}$, W, and $S_{\rm g}$ with $T_{\rm m}$ in an irreversible Carnot cycle.

According to the definition of entropy generation number for this problem, $C_{\rm min}$ is the heat capacity flow rate of the working medium, which is a definite value. Therefore, the entropy generation number has the same variation tendency as the entropy generation rate. The minimum entropy generation number does not correspond to the maximum output power of the system either.

3.3. Analyses of an irreversible Brayton cycle

An irreversible Brayton cycle is shown in Fig. 7. The working medium receives heat at rate $Q_{\rm H}$ from the high temperature heat source with temperature $T_{\rm H}$ through heat exchanger 1, and releases heat at rate $Q_{\rm L}$ to the low temperature heat source with temperature $T_{\rm L}$ through heat exchanger 2. The output power of the cycle is W. The temperature of the working medium is to be optimized for the largest output power.

The heat exchange rates in the heat exchangers are[2]

$$Q_{\rm H} = C_{\rm m}\,(T_{\rm H}\text{-}T_2)\,\{1-\exp\,[-(kA)_1/C_{\rm m}]\}\,, \tag{25}$$

$$Q_{\rm L} = C_{\rm m}\,(T_4-T_{\rm L})\,\{1-\exp\,[-(kA)_2/C_{\rm m}]\}\,, \tag{26}$$

where $C_{\rm m}$ is the heat capacity flow rate of the working medium, $(kA)_1$ and $(kA)_2$ are the thermal conductances of the

two heat exchangers, T_2 and T_4 are the temperatures at states 2 and 4, respectively. For the Brayton cycle,[30]

$$T_2/T_1 = T_3/T_4, \tag{27}$$

where T_1 and T_3 are the temperatures at states 1 and 3, respectively. When T_H, T_L, C_m, $(kA)_1$, and $(kA)_2$ are all prescribed, T_1, T_3, and T_4 can be calculated by Eqs. (25)–(27) if T_2 is given.

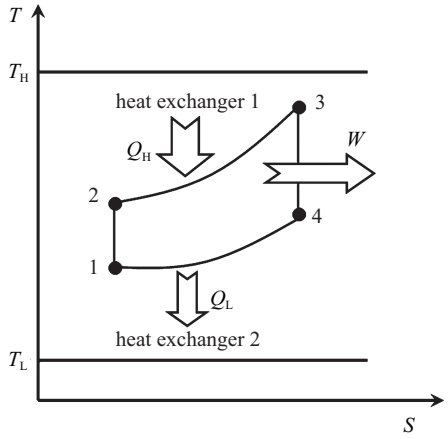


Fig. 7. An irreversible Brayton cycle.

The output power is

$$W = Q_H\text{-}Q_L. \tag{28}$$

The entropy generation rate of the cycle can be calculated by Eq. (3) as

$$S_g = Q_L/T_L - Q_H/T_H. \tag{29}$$

As an example, we assume that $T_H = 400$ K, $T_L = 300$ K, $C_m = 2$ W/K, $(kA)_1 = 2$ W/K, and $(kA)_2 = 3$ W/K. The variations of Q_H, W, and S_g with T_2 are calculated by using Eqs. (25)–(29), and the results are shown in Fig. 8. The minimum entropy generation rate does not correspond to the maximum output power of the cycle. As shown in Fig. 8, this is also because heat Q_H is not a given value but changes with T_2.

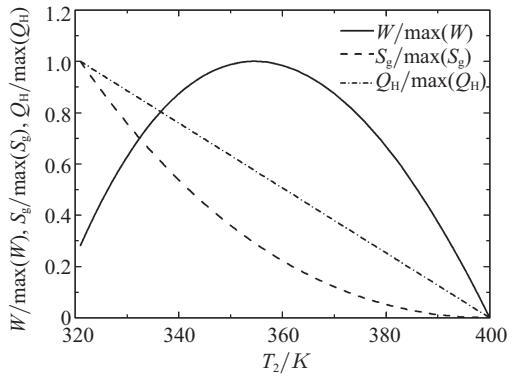


Fig. 8. Variations of Q_H, W, and S_g with T_2 in an irreversible Brayton cycle.

According to the definition of entropy generation number, C_{min} is C_m in this case. The entropy generation number has the same variation tendency as the entropy generation rate, and it does not correspond to the maximum output power of the cycle either.

3.4. Analyses of the Carnot cycle

As shown in Fig. 9, a Carnot engine absorbs heat at rate Q_H from the isothermal heat source with temperature T_H, and releases heat at rate Q_L to the other isothermal heat sink with temperature T_L. The output power is[30]

$$W = Q_H\,(1 - T_L/T_H)\,. \tag{30}$$

The output power increases with Q_H and T_H increasing or T_L decreasing. However, the entropy generation rate is always zero because the cycle is reversible. It is not related to the output power. Therefore, the entropy generation rate cannot describe the change of the output power in the reversible cycle.

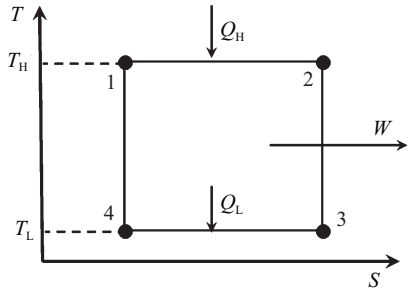


Fig. 9. Sketch of the Carnot cycle.

4. Conclusions

In this paper, we discuss the applicability of the minimum entropy generation method to optimizing thermodynamic cycles. We show that the applicability of the minimum entropy generation method is conditional. When the total heat into the system of interest is not prescribed, the minimum entropy generation method is not justifiable.

For the cycles in which the working medium is heated or cooled by streams with prescribed inlet temperatures and prescribed heat capacity flow rates, it is theoretically proved that both the minimum entropy generation rate and the minimum entropy generation number correspond to the maximum output power when the virtual entropy generation rate induced by dumping the used streams into the environment is considered. This is because the heat into the system of interest is prescribed. However, the minimum entropy generation method is not applicable if the virtual entropy generation rate is not considered. This is because the heat rate absorbed by the working medium as well as the total heat into the system of interest is not fixed.

The analyses of an irreversible Carnot cycle and an irreversible Brayton cycle also prove that the minimum entropy generation rate and the minimum entropy generation number do not correspond to the maximum output powers of the cycles because the heat rate into the system of interest is not prescribed. The analysis of the Carnot cycle reveals that the entropy generation rate and the entropy generation number are not directly related to the output power of the cycle. This is also true for any reversible cycle because of the zero entropy generation.

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