Chaos detection and control in a typical power system

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In this paper, a new chaotic system is introduced. The proposed system is a conventional power network that demonstrates a chaotic behavior under special operating conditions. Some features such as Lyapunov exponents and a strange attractor show the chaotic behavior of the system, which decreases the system performance. Two different controllers are proposed to control the chaotic system. The first one is a nonlinear conventional controller that is simple and easy to construct, but the second one is developed based on the finite time control theory and optimized for faster control. A MATLAB-based simulation verifies the results.

Keywords: chaos detection, chaos control, finite time control theory, synchronous generator

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1. Introduction

Nowadays, chaotic systems attract great attention in different applications such as information processing, secure communication, and chemical and biological engineering.[1–3] On the other hand, chaotic behavior is undesirable in many engineering systems. For example, the permanent magnet synchronous motor (PMSM) experiences chaos under some conditions, which may lead to speed oscillation, torque ripple, and even instability.[4] Thus, chaos control in PMSM is of great importance and has already been studied.[5] Some power electronic converters, linear motors, and multi-machine power systems also experience chaotic behaviors when their systemic parameters fall into a certain area.[6–9] In recent years, the secure and stable operation of power systems, which is an essential requirement of a reliable energy source, has been actively researched. Some chaotic conditions in power systems have been detected and a large number of control methods for chaos have been developed. The possibility of other chaotic conditions in power systems is an open area of research.

In this paper, we report that chaotic behavior in a common power system is detected and controlled for the first time. The chaotic system (CSGIB: chaotic single-generator infinite-bus) consists of a synchronous generator parallel to an infinite bus. The chaotic behavior appears for real set values of the system parameters. The chaos is suppressed by an ordinary controller and is brought under control within a shorter time frame by an optimized controller.

2. System definition

The schematic of the CSGIB is shown in Fig. 1. This scheme is the most popular way to parallel the generators within a power network, which highlights the importance of its chaotic behavior study.

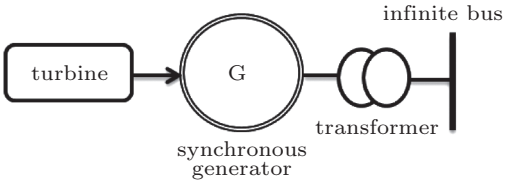


Fig. 1. The case-study system.

The dynamic equations of a synchronous generator are[10]

$$\dot{\delta} = \omega, \quad T_{\mathrm{m}} - T_{\mathrm{e}} = J\frac{\mathrm{d}\omega}{\mathrm{d}t} + D\omega, \tag{1}$$

where δ is the rotor angle, ω is the difference between rotor speed and synchronous speed, T_{e} is the electromagnetic torque, T_{m} is the effective mechanical torque, J is the inertia constant, and D is the friction coefficient. The electromagnetic torque is

$$T_{\mathrm{e}} = \frac{EV_{\mathrm{B}}}{X}\sin\delta, \tag{2}$$

where E is the inner voltage of the generator, V_{B} is the infinite bus voltage, and X is the addition of generator and transformer reactance.

The turbine produces the mechanical torque. Although a robust and steady torque is a desired characteristic for a turbine, the output ripple is unavoidable and can be represented as

$$T_{\mathrm{m}} = T_0 + T\cos(\omega_{\mathrm{T}}t). \tag{3}$$

Replacing Eq. (3) into Eq. (1) leads to the equation below after a shift of the zero point:

$$\begin{cases} \dot{\omega} = \dfrac{1}{J}\left[T\cos(\varphi) - \dfrac{EV_{\mathrm{B}}}{X}\sin\delta - D\omega\right], \\ \dot{\delta} = \omega, \end{cases} \tag{4}$$

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where

$$\varphi = \omega_{\mathrm{T}} t, \quad \hat{\delta} = \delta - \delta_0,$$

and δ_0 shows the rotor angle when constant torque $T_{\mathrm{m}} = T_0$ is applied to the generator. So $\hat{\delta}$ shows the rotor angle oscillation when the turbine output torque oscillates, and is shown simply by δ.

Although the CSGIB is a two-order system, it can be chaotic because of having one time-dependent variable.[11] In fact, the CSGIB is very similar to a chaotic pendulum. When the systemic parameters of the CSGIB fall within certain ranges, the chaotic behavior appears. Table 1 shows a set of chaotic parameters.

Table 1. Chaotic parameters of the system.

Parameter	Value	Unit
J	218	kg·m^2
D	9.61	N·m·s
ω_{T}	1.24	rad/s
T	329	N·m
V_{B}	220	V
E	228	V
X	74.4	Ω

Since the inertia constant, friction coefficient, and transmission reactance are usually constants, the amplitude and frequency of turbine disturbance are the effective parameters that can direct the system to chaos. The existence of at least one positive Lyapunov exponent is the index of chaotic behavior. Table 2 shows the Lyapunov exponents of the system.

Table 2. Lyapunov exponents of the chaotic system.

λ_1	λ_2	λ_3
+0.248	0	–0.292

Figure 2 shows the strange attractor of the system, which is a usual specification of chaotic systems. The cessations in the strange attractor arise from the periodic variable of the system. By ignoring this variable, the resultant 2D image of the strange attractor is incessant, as shown in Fig. 3. Figure 4 shows the oscillation of one of the system variables versus time.

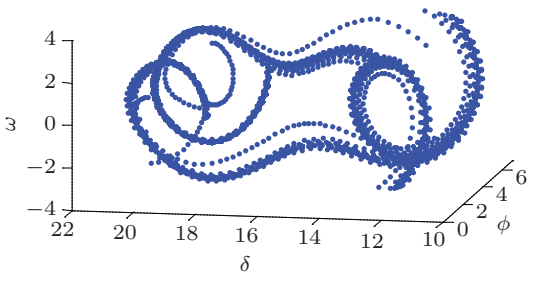


Fig. 2. (color online) The 3D strange attractor of the chaotic system.

This small oscillation has a great influence on the system performance. There are many frequency-sensitive loads in the network such as induction motors which are affected by frequency oscillation. This phenomenon decreases the system stability.[10] Sometimes, the frequency oscillation may lead to a control system action that destroys the balance between generation and demand in the network. Therefore, it is necessary to suppress and control the chaos in the CSGIB.

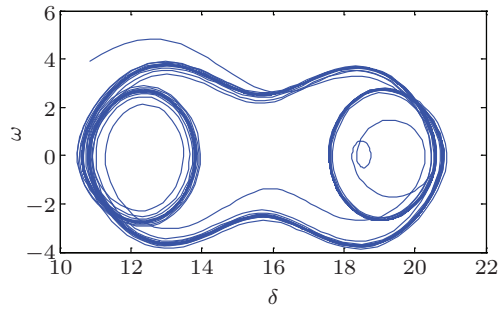


Fig. 3. (color online) The 2D strange attractor of the chaotic system.

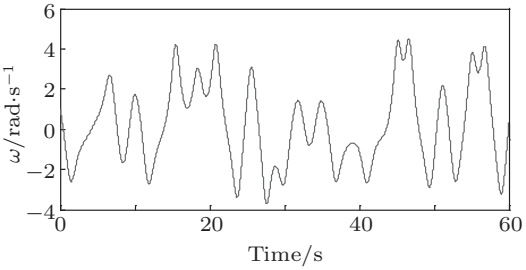


Fig. 4. Speed oscillation in chaotic state.

Different concepts and techniques have been presented up to now. Some methods are constructed based on the classical control.[12–14] Using a control law $u(x)$ to transform an unstable subcritical bifurcation point into a stable supercritical one is another chaos control method.[15] This method has already been used to prevent the voltage collapse in electrical power systems[16] and to prevent sub-synchronous resonance in power systems.[17] A recursive back stepping nonlinear controller has also been proposed to suppress the chaos.[18] In this paper, a control signal $u(x)$ is applied to stabilize an unstable equilibrium point. The stability of the equilibrium point is investigated by a candidate Lyapunov function. Then, the amount of time required to stabilize the system is optimized based on the finite time control theory, using two control signals u_1 and u_2.

3. Chaos control in the CSGIB

By adding the control signal u to the CSGIB dynamic equations, the unstable equilibrium point (0,0) can be stabilized in a finite time

$$\begin{cases} \dot{\omega} = \dfrac{1}{J}\left[T\cos(\varphi) - \dfrac{EV_{\mathrm{B}}}{X}\sin\delta - D\omega\right] + u, \\ \dot{\delta} = \omega. \end{cases} \tag{5}$$

The controller is chosen as

$$u = -\frac{T}{J}\cos(\varphi) - \delta - k\omega, \tag{6}$$

where k is the control parameter. The closed loop system using Eq. (6) is

$$\begin{cases} \dot{\omega} = -\dfrac{EV_{\mathrm{B}}}{JX}\sin\delta - \delta - \dfrac{D}{J}\omega - k\omega, \\ \dot{\delta} = \omega. \end{cases} \tag{7}$$

The candidate Lyapunov function V is used to investigate the stability

$$V = \frac{1}{2}\left(\omega^2 + \delta^2\right) + \frac{EV_{\mathrm{B}}}{JX}\left(1 - \cos\delta\right). \tag{8}$$

It is clear from Eq. (8) that V is a positive scalar function and $V(0) = 0$. The time-derivative of V is

$$\dot{V} = \omega\dot{\omega} + \delta\dot{\delta} + \frac{\omega EV_{\mathrm{B}}}{JX}\sin\delta = -\frac{D}{J}\omega^2 - k\omega^2 \ < \ 0. \tag{9}$$

Therefore, the closed loop dynamic system with the proposed controller is stable. The controller performance is investigated by simulation, and the result will be presented below.

4. Finite time chaos control in the CSGIB

Although the proposed controller in the previous section can suppress the chaos in the CSGIB, it is possible to stabilize the system in a shorter time frame by using the finite time control theory. This theory is described by the lemma below.[19]

Lemma 1 Assume that a continuous, positive definite function V satisfies the condition

$$\dot{V}(t) \le -\varepsilon V^{\rho}(t), \quad \forall t \ge t_0, \quad V(t_0) \ge 0, \tag{10}$$

where $\varepsilon > 0$ and $0 < \rho < 1$. Then for any initial time t_0, $V(t)$ satisfies

$$V(t) \equiv 0, \quad \forall t \ge t_1. \tag{11}$$

Moreover, t_1 is given by

$$t_1 = t_0 + \frac{V^{1-\rho}(t_0)}{\varepsilon(1-\rho)}. \tag{12}$$

Now assume two control signals are applied to the system

$$\begin{cases} \dot{\omega} = \dfrac{1}{J}\left[T\cos(\varphi) - \dfrac{EV_{\mathrm{B}}}{X}\sin\delta - D\omega\right] + u_1, \\ \dot{\delta} = \omega + u_2. \end{cases} \tag{13}$$

The candidate Lyapunov function is considered as

$$V = \frac{1}{2}\left(\omega^2 + \delta^2\right). \tag{14}$$

The time-derivation of V is

$$\begin{aligned} \dot{V} &= \omega\dot{\omega} + \delta\dot{\delta} \\ &= \frac{\omega}{J}\left[T\cos(\varphi) - \frac{EV_{\mathrm{B}}}{X}\sin\delta - D\omega\right] \\ &\quad + \omega u_1 + \delta\omega + \delta u_2. \end{aligned} \tag{15}$$

The control signals are chosen as

$$u_1 = -\frac{T}{J}\cos(\varphi) + \frac{EV_{\mathrm{B}}}{JX}\sin\delta - \omega^k, \tag{16}$$

$$u_2 = -\omega - \delta^k, \tag{17}$$

where k is the control parameter. Replacing (16) in Eq. (15) leads to the equation below:

$$\begin{aligned} \dot{V} &= -\frac{D}{J}\omega^2 - \omega^{k+1} - \delta^{k+1} \le -\omega^{k+1} - \delta^{k+1} \\ &= -\left(\frac{1}{2}\right)^{-\frac{k+1}{2}}\left[\left(\frac{1}{2}\omega^2\right)^{\frac{k+1}{2}} + \left(\frac{1}{2}\delta^2\right)^{\frac{k+1}{2}}\right] \\ &\le -\left(\frac{1}{2}\right)^{-\frac{k+1}{2}}\left(\frac{1}{2}\omega^2 + \frac{1}{2}\delta^2\right)^{\frac{k+1}{2}} \\ &= -\left(\frac{1}{2}\right)^{-\frac{k+1}{2}}(V)^{\frac{k+1}{2}}. \end{aligned} \tag{18}$$

According the aforementioned lemma, if $-1 < k < +1$, then the states δ and ω converge to zero at finite time t_0. Control parameter k can vary in a limited domain to adjust the settling time and the control signal's amplitudes. The $k = 0.6$ is determined after trial and error. Then the settling time is calculated as

$$\varepsilon = \left(\frac{1}{2}\right)^{-\frac{k+1}{2}} = 1.7411, \quad \rho = \frac{k+1}{2} = 0.8,$$

$$t_{\mathrm{s}} = \frac{V^{1-\rho}(30)}{\varepsilon(1-\rho)} = 5.41 \text{ s}.$$

It is claimed that the proposed controllers in Eq. (16) can suppress chaos at the calculated settling time above. The simulation will show the validity of this statement.

5. Simulation

The proposed controllers can direct the state of the system to a desired point within a finite time. This statement is represented via simulation. The chaotic behavior appears when the dynamic equations are solved from a random point without a controller. Every controller is applied to the system at a certain time. Figures 5 and 6 show the speed and the rotor angle changes while the u controller is applied with $k = 10$ after 20 seconds. These figures show that the oscillations of both speed and rotor angle decrease to zero by applying the controller.

Although the controller can suppress the chaotic behavior well, the control time is fairly long. On the other hand, the control signal amplitude, an important control parameter, remains in a limited range. Figure 7 shows the control signal, which is very large.

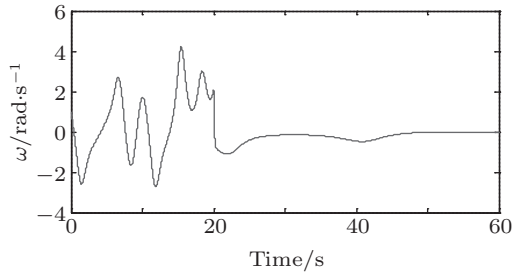


Fig. 5. Speed oscillation and control with u.

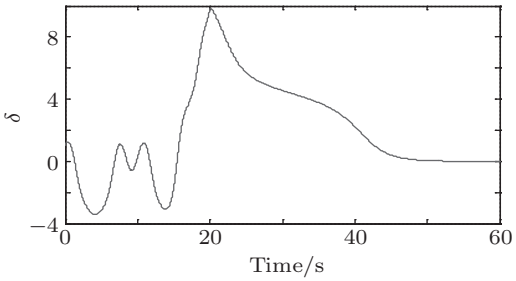


Fig. 6. Rotor angle oscillation and control with u.

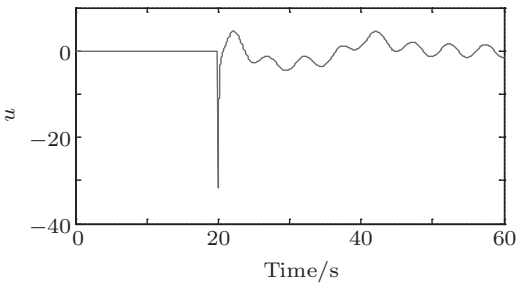


Fig. 7. Control signal of u.

The control signal amplitude is an important problem, but it receives less attention in many papers. Very large values of this factor can lead to actuator saturation and improper operation of a control system. Sometimes limiting the control signal amplitude is more important than optimizing the settling time or chaos control time.

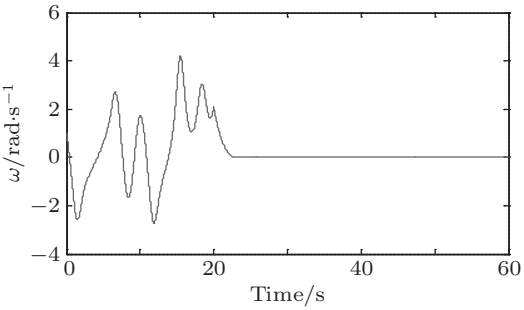


Fig. 8. Speed oscillation and control with u_1 & u_2.

Using two controllers, u_1 and u_2, the time to stabilization decreases greatly. Figures 8 and 9 show the changes of speed and of rotor angle, respectively, while u_1 and u_2 are applied after 20 seconds.

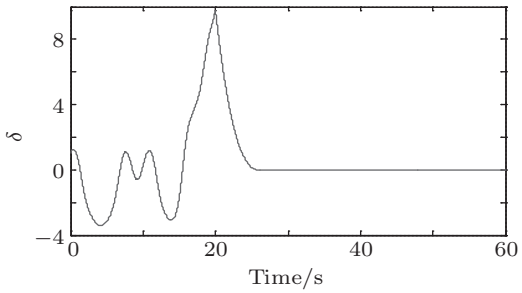


Fig. 9. Rotor angle oscillation and control with u_1 & u_2.

Furthermore, the validity of the settling time calculated in the previous section is evident in Figs. 8 and 9. Decreasing the chaos control time in the CSGIB can play an important role in keeping a power system stable when a chaotic state occurs. Moreover, Figs. 10 and 11 show that the control signal's amplitude has been decreased greatly by new controllers. Less total effort is required of these new controllers compared to the previous single controller. Therefore, the new controllers are easier to realize and may be cheaper.

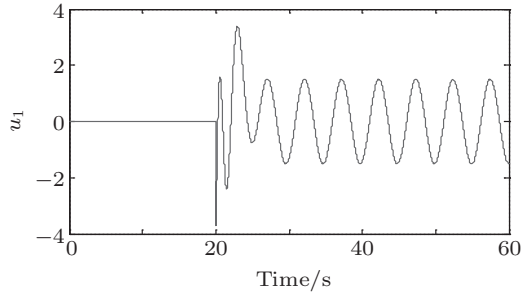


Fig. 10. Control signal of u_1.

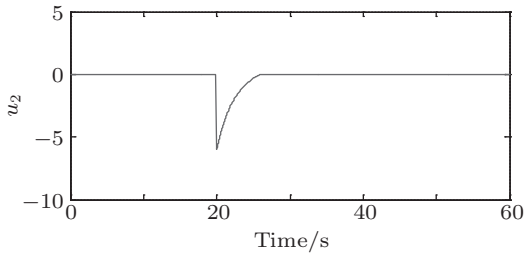


Fig. 11. Control signal of u_2.

6. Conclusion

Some engineered systems may experience chaotic states in special conditions. This situation often leads to undesirable operation of the system. Therefore, chaos detection, investigation, and control are important areas of research in all engineered systems. In this paper, a new chaotic state was detected by investigating the dynamic behavior of a typical power system. The oscillations of both speed and rotor angle are an unwanted result of the chaotic state. A simple controller was proposed to suppress the chaos based on the Lyapunov stability index. Then, an optimized controller was proposed based

on the finite time control theory to decrease the chaos control time and control effort. All of the obtained results were verified via simulation.

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