Effects of non-Gaussian noise on a calcium oscillation system*

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We investigate the effects of the non-Gaussian colored noise on a calcium oscillation system using stochastic simulation methods. It is found that the reciprocal coefficient of variance R has a maximum ($R_{\max}$) with increasing noise intensity Q. The non-Gaussian noise parameter q has an important effect on the system. For some values of q (e.g., $q = 0.9$, $q = 1.0$), R has a maximum with increasing correlation time τ. Non-Gaussian noise induced spikes are more regular than Gaussian noise induced spikes when q is small and Q has large values. The R has a maximum with increasing q. Therefore, non-Gaussian noise could play more effective roles in the calcium oscillation system.

Keywords: non-Gaussian noise, coherence resonance, calcium oscillation

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1. Introduction

In recent years, the constructive roles of noise in nonlinear systems have been widely investigated in many fields.[1–6] One of the important effects is the stochastic resonance (SR) phenomenon, which shows that noise can help us detect and amplify weak external signals.[7,8] The concept of SR was first proposed to explain the periodic oscillation of the earth's ice age.[9] The SR has been investigated in physics,[10–12] chemical reactions,[13,14] biological systems,[15–17] neuron systems,[18–21] and so on. In the early studies, it was believed that the occurrence of SR needed three necessary conditions: a nonlinear system, an external periodic signal, and environment noise. However, with the progress of the studies, it has been shown that the output of a nonlinear system becomes more coherent at an intermediate noise intensity even in the absence of the periodic signal input. This phenomenon is termed the coherence resonance (CR).[22–24]

It is known that noise is unavoidable in a biochemical reaction system, and some previous studies have demonstrated that the CR phenomenon can be induced by noise. In 1997, Pikovsky *et al.*[22] studied the CR phenomenon in a noise driven excitable FitzHugh–Nagumo system. Hu *et al.*[25] found the CR phenomenon in a system when the asymptotic state of the deterministic system is stationary. We know that Ca^{2+} is involved in many cellular functions, such as muscle contraction, sensory perception, bursting oscillations, and the injury of brain tissue. A lot of work has shown that intracellular Ca^{2+} is one of the most important second messengers in the cytosol of living cells. Ca^{2+} oscillations play an important role in the control of many cellular processes[26,27] and regulate a series of biological events including gene transcription, protein synthesis, and protein degradation. So the studies of the effects of noise on Ca^{2+} oscillations are important to reveal the essence of cellular living processes. In 2005, Li *et al.*[28] studied the internal noise SR phenomenon for intracellular calcium oscillations in a cell system and found that stochastic calcium oscillations appear when the internal noise is considered, while the deterministic model only yields steady states. Li *et al.*[29] studied noise induced SR in a Ca^{2+} oscillation system with both implicit and explicit internal signals based on the model proposed by Li and Rinzel,[30] and they found that noise can induce both implicit and explicit internal signal stochastic resonances and observed the internal signal stochastic bi-resonance. In 2009, Yu *et al.*[18] studied a calcium oscillation model simultaneously driven by external noise and internal noise using the chemical Langevin equation method and found that the internal noise CR can be suppressed by the external noise when the extracellular stimulation is subthreshold.

In most of the previous analysis about CR (or SR) phenomenon in Ca^{2+} oscillation systems, the fluctuations were assumed to be the Gaussian noise. Some experimental and theoretical results for one kind of crayfish and for rat skin offered strong indications that there could be non-Gaussian noise sources in those sensory systems.[31–34] Thus, stochastic systems with non-Gaussian noise need to be investigated. In the present article, we investigate the influence of non-Gaussian noise on a calcium oscillation system. In Section 2, the deterministic model is introduced, and then the stochastic model driven by non-Gaussian noise is derived. In Section 3, we study the effects of non-Gaussian noise on the system and find

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the CR phenomenon. In Section 4, we give the conclusion.

2. Models and equations

There are a number of models for Ca^{2+} oscillations, all of which show a great deal of fundamental similarity. The model used in the present article describes the dynamics of Ca^{2+} in a single cell of rat hepatocytes, which was proposed by Höfer[35,36] and Cracheva.[37] If the noise is ignored, the time evolution of the species is governed by the following macroscopic kinetics:

$$\frac{dx}{dt} = \rho\left[\nu_0 + \nu_c\frac{p}{k_0+p} - \nu_4\frac{x^2}{k_4^2+x^2} + \frac{\alpha k_r(x,p)}{\beta}(z-(1+\beta)x) - \alpha\nu_3\frac{x^2}{k_3^2+x^2}\right], \quad (1)$$

$$\frac{dz}{dt} = \rho\left(\nu_0 + \nu_c\frac{p}{k_0+p} - \nu_4\frac{x^2}{k_4^2+x^2}\right), \quad (2)$$

$$k_r(x,p) = k_1\left\{\frac{d_2(d_1+p)px}{(d_p+p)(d_a+x)[d_2(d_1+p)+x(d_3+p)]}\right\}^3 + k_2, \quad (3)$$

where x and z represent the concentrations of cytosolic Ca^{2+} and the total Ca^{2+} in the cell; the structural parameters are $\alpha = 2.0$ L·μmol^{-1}, $\beta = 0.1$ L·μmol^{-1}, $\rho = 0.2$ L·μmol^{-1}; $\nu_0 = 0.2$ μmol^{-1}·L^{-1}·s^{-1} describes a calcium leakage from the background; $\nu_c = 4.0$ μmol·L^{-1}·s^{-1} is the maximum rate of IP_3 (inositol 1, 4, 5-trisposphate) induced calcium formation (influx); $\nu_3 = 9.0$ μmol·L^{-1}·s^{-1} is the maximum rate of endoplasmic reticulum (ER) uptake of calcium from the cytosol; and $\nu_4 = 3.6$ μmol·L^{-1}·s^{-1} is the maximum rate of calcium extrusion through the plasma membrane. The other parameters are $k_0 = 4.0$ μmol·L^{-1}, $k_2 = 0.02$ s^{-1}, $k_3 = 0.12$ μmol·L^{-1}, $k_4 = 0.12$ μmol·L^{-1}, $d_1 = 0.3$ μmol·L^{-1}, $d_2 = 0.4$ μmol·L^{-1}, $d_3 = 0.2$ μmol·L^{-1}, $d_p = 0.2$ μmol·L^{-1}, $d_a = 0.4$ μmol·L^{-1}.

The p is the concentration of IP_3. When p is viewed as the control parameter, the bifurcation diagram is shown in Fig. 1, and the two bifurcation points appear at $p = 1.450$ μmol·L^{-1} and $p = 8.892$ μmol·L^{-1}. The $k_r(x,p)$ is the IP_3R (IP_3 receptor) release function and describes the gating kinetics of the IP_3 receptor. In the absence of noise, the system is in the stable steady state at $p_0 = 1.40$ μmol·L^{-1}. To study the effects of noise on this system, we apply the non-Gaussian noise to parameter p, so $p = p_0 + \xi(t)$, where $\xi(t)$ is the non-Gaussian noise. For $\xi(t)$, we have

$$\frac{d\xi(t)}{dt} = -\frac{1}{\tau}\frac{d}{d\xi}V_q(\xi) + \frac{\sqrt{2Q}}{\tau}\Gamma(t), \quad (4)$$

where the generalized potential is

$$V_q(\xi) = \frac{Q}{\tau(q-1)}\ln\left[1+\frac{\tau}{Q}(q-1)\frac{\xi^2}{2}\right], \quad (5)$$

$\Gamma(t)$ is the Gaussian white noise with $\langle\Gamma(t)\rangle = 0$ and $\langle\Gamma(t)\Gamma(t')\rangle = \delta(t-t')$, Q and τ are the noise intensity and the correlation time, respectively, and q stands for the departure from the Gaussian behavior. In the limit of $q \to 1$, the process $\xi(t)$ coincides with the Gaussian colored noise with self-correlation time τ. If $q \neq 1$, it is a non-Gaussian noise term. The theoretical analysis shows that the stationary probability distribution has the form

$$P_q^{st}(\xi) = \frac{1}{Z_q}\left[1+\frac{\tau}{Q}(Q-1)\frac{\xi^2}{2}\right]^{\frac{1}{1-q}}, \quad (6)$$

where Z_q is the normalization factor. This distribution can be normalized solely for $q < 3$. The first moment is always equal to zero, $\langle\xi\rangle = 0$, and the second moment $\langle\xi^2\rangle = 2Q/[\tau(5-3q]$ is finite solely for $q < 5/3$. So the value of q will be restrained as $q < 5/3$ in our simulations.

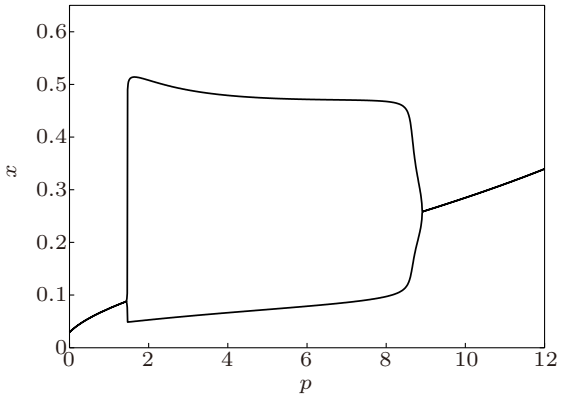


Fig. 1. Schematic bifurcation diagram for the deterministic dynamics. The Hopf bifurcation points are $p = 1.450$ μmol·L^{-1} and $p = 8.892$ μmol·L^{-1}.

3. Results and discussion

In order to give a simple and clear analysis about the noise near the bifurcation point, we have investigated the CR phenomenon induced by the non-Gaussian noise. Equations (1) and (7) are integrated by using a simple forward Eular[1] algorithm with a fixed time step of $\Delta t = 0.05$ s. In each calculation, the time evolution of the system lasts 1000 min after the transient behavior is discarded. It is found that the non-Gaussian noise can induce calcium spikes, and the regularity of the spikes is different at different noise intensity and correlation time. To measure the regularity of the spikes, we introduce the reciprocal coefficient of variance

$$R = \frac{\langle T\rangle}{\sqrt{\langle T^2\rangle - \langle T\rangle^2}}, \quad (7)$$

where T denotes the inter-spike intervals.[38] The R is a measure of the inter-spike interval regularity, represents the degree of CR, and is similar to the signal-to-noise (SNR) (see Refs. [25] and [39] for detailed description of the SNR).

The dependence of R on the noise intensity Q for different deviation parameters q is plotted in Fig. 2. It is found that there exists a peak in the R–Q curve, which means that the CR phenomenon occurs. The maximum ($R_{\max}$) position occurs at about $Q = 0.1$ for different values of q; and the smaller q is, the larger $R_{\max}$ is. We also find that R decreases with increasing q at large Q (e.g., $Q = 0.1$ or $Q = 0.15$). When q is small and Q is large, non-Gaussian noise induced spikes are more regular than Gaussian noise (non-Gaussian noise coincides with Gaussian colored noise at $q = 1.0$) induced spikes.

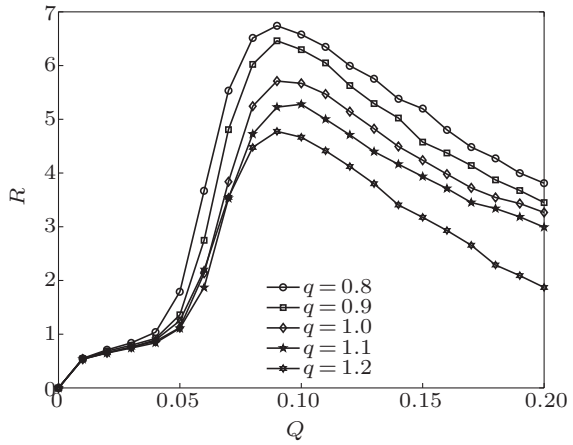


Fig. 2. The R as a function of noise intensity Q for different values of q at $\tau = 2.8$.

In Fig. 3, R as a function of the correlation time τ for different values of q is plotted. It shows that for some values of q (e.g., $q = 0.9$, $q = 1.0$), R has a maximum. In Fig. 3, we also find that R increases with the increasing τ at $2 \leq \tau \leq 4$. But as τ becomes larger ($\tau \geq 4$), there is almost no change for R when τ changes. In Fig. 3, at $1 \leq \tau \leq 5$, R value for $q = 0.8$ is larger than that for $q = 1.1$ or $q = 0.7$, which implies that the R–q curve should have a peak.

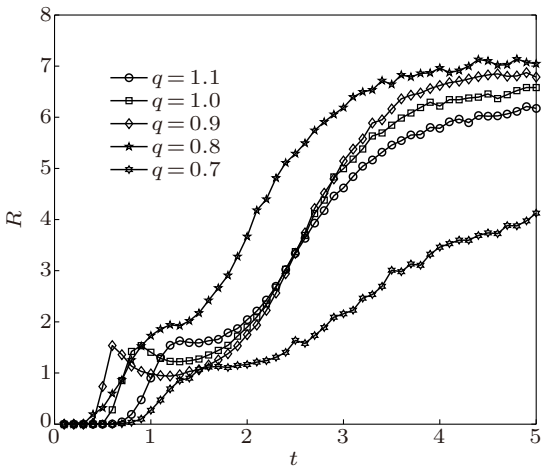


Fig. 3. The R as a function of correlation time τ for different values of q at $Q = 0.1$.

In Fig. 4, R as a function of the departure q for fixed $\tau = 2.8$ and different Q is plotted. It is clear that R has a maximum with increasing values of q. This phenomenon is in agreement with the result in Fig. 3. We also find that for some values of Q (e.g., $Q = 0.075$, $Q = 0.1$, and $Q = 0.125$), the position of $R_{\max}$ is at about $q = 0.85$ but not at $q = 1$, which means that an appropriate departure q of non-Gaussian noise can enhance the optimal CR phenomenon.

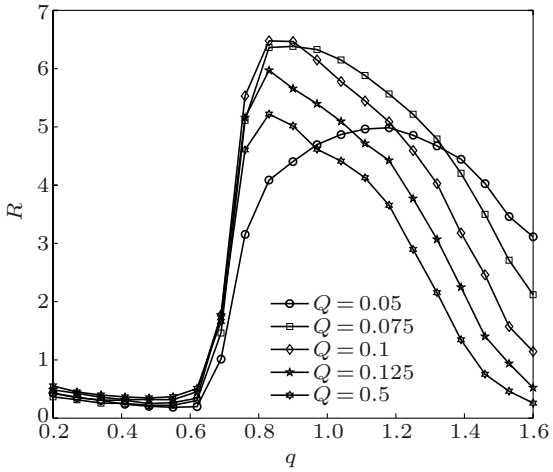


Fig. 4. The R as a function of departure q for different values of Q at $\tau = 2.8$.

4. Conclusion

In this paper, we have studied the effects of the non-Gaussian noise on a calcium oscillation system. It is found that the reciprocal coefficient of variance R has a maximum with increasing noise intensity Q. For some values of q (e.g., $q = 0.9$, $q = 1.0$), there exists a peak in the R–τ curve. The R increases with increasing τ at $2 \leq \tau \leq 4$. But when τ has a large value ($\tau \geq 4$), there is almost no change in R when τ changes. We find that an appropriate departure q of non-Gaussian noise can enhance the optimal CR phenomenon.

References

[1] Richardson K A, Imhoff T T, Grigg P and Collins J J 1998 *J. Chaos* **8** 599
[2] Duan D H, Xu W, Su J and Zhou B C 2011 *Chin. Phys.* B **20** 030501
[3] Mei D, Xie C and Zhang L 2003 *Phys. Rev.* E **68** 051102
[4] Yi M, Jia Y, Ma J, Tang J, Yu G and Li J 2008 *Phys. Rev.* E **77** 022902
[5] Ai B Q, Xie H Z and Liu L G 2007 *Phys. Rev.* E **75** 061126
[6] Guo Y, Xu W, Liu H, Li D and Wang L 2011 *Commun. Nonlinear Sci. Numer. Simulat.* **16** 522
[7] Gammaitoni L, Hanggi P, Jung P and Marchesoni F 1998 *Rev. Mod. Phys.* **70** 223
[8] Zhu R, Li Q S and Liu Z C 2002 *Chem. Phys. Lett.* **351** 410
[9] Benzi R, Sutera A and Vulpiani A 1981 *J. Phys.* A **14** L453
[10] Gammaitoni L 1995 *Phys. Rev.* E **52** 4691
[11] Cao L and Wu D J 2007 *Physica* A **376** 191
[12] Li J H and Han Y X 2006 *Phys. Rev.* E **74** 051115
[13] Guderian A, Dechert G, Zeyer K P and Schneider F W 1996 *J. Phys. Chem.* **100** 4437
[14] Ross J and Vlad M O 1999 *Ann. Rev. Phys. Chem.* **50** 51
[15] Douglass J K, Wilkens L, Pantazelou E and Moss F 1993 *Nature* **365** 337
[16] Petracchi D, Gebeshuber I C, DeFelice L J and Holden A V 2000 *Chaos Soliton. Fract.* **11** 1819
[17] Tanabe S and Pakdaman K 2001 *Phys. Rev.* E **63** 031911

[18] Yu Y , Wang W, Wang J and Liu F 2001 *Phys. Rev.* E **63** 021907
[19] Baltanás J P and Casado J M 2002 *Phys. Rev.* E **65** 041915
[20] Wu D and Zhu S 2008 *Phys. Lett.* A **372** 5299
[21] Tang J, Jia Y, Yi M, Ma J and Li J 2008 *Phys. Rev.* E **77** 061905
[22] Pikovsky A S and Kurths J 1997 *Phys. Rev. Lett.* **78** 775
[23] Neiman A, Saparin P I and Stone L 1997 *Phys. Rev.* E **56** 270
[24] Linder B and Schimansky-Geier L 2000 *Phys. Rev.* E **61** 6103
[25] Hu G, Ditzinger T, Ning C Z and Haken H 1993 *Phys. Rev. Lett.* **71** 807
[26] Li W H, Llopis J, Whitney M, Zlokarnik G and Tsien R Y 1998 *Nature* **392** 936
[27] Berridge M J 1997 *J. Exp. Biol.* **200** 315
[28] Li H, Hou Z and Xin H 2005 *Phys. Rev.* E **71** 061916
[29] Li Y P and Li Q S 2006 *Chem. Phys. Lett.* **417** 498
[30] Li Y X and Rinzel J 1994 *J. Theor. Biol.* **166** 461
[31] Borland L 1998 *Phys. Lett.* A **245** 67
[32] Tsallis C 1988 *J. Stat. Phys.* **52** 479
[33] Fuentes M A, Wio H S and Toral R 2002 *Physica* A **303** 91
[34] Bouzat S and Wio H S 2005 *Physica* A **351** 69
[35] Höfer T 1999 *Biophysical J.* **77** 1244
[36] Höfer T, Politi A and Heinrich R 2001 *Biophysical J.* **80** 75
[37] Gracheva M E, Toral R and Gunton J D 2001 *J. Theor. Biol.* **212** 111
[38] Miyakawa K, Tanaka T and Iskawa H 2003 *Phys. Rev.* E **67** 066206
[39] Hou Z and Xin H 2003 *J. Chem. Phys.* **119** 11508