Continuous variable entanglement generation in coupled cavities*

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We investigate continuous variable entanglement produced in two distant coupled cavities, in which two four-level atoms are driven by classical fields respectively. Under the large detuning condition, an effective Hamiltonian containing the square of the creation (annihilation) operator of the cavity field is derived. Due to the nonlinearity, entanglement formally created by the beam splitter type interaction is transformed into the nondegenerate parametric down conversion type. Employing the operator algebraic method, we study the time evolution of the entanglement condition, and show that the system provides us an advantage in achieving a larger photon number with better entanglement. We also discuss the dissipation of the cavities affecting the entanglement.

Keywords: continuous variable entanglement, coupled cavities, nondegenerate parametric down conversion, cavity dissipation

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1. Introduction

Entanglement, one of the important characteristics of quantum systems, is a marvelous phenomenon in quantum mechanics and plays a crucial role in the quantum information field. As an essential resource, entanglement has been widely applied in quantum computation and quantum communication such as quantum teleportation[1] and dense coding.[2] It is therefore not surprising that the generation of an entangled state has become a favorite topic in recent years. As an available candidate system, cavity quantum electrodynamics (QED) with atoms interacting with quantized cavity modes has been proved to be an excellent quantum device to produce entanglement.[3–6] Hence, the creation of entanglement in cavity QED has been extensively studied.[7–11]

Continuous variable (CV) entanglement, as a kind of elementary entanglement resource, has shown tremendous superiority in the treatment of quantum transport problems.[12,13] Therefore, numerous authors have been involved in the research of generating CV entanglement and put forward many theoretical and experimental schemes.[14–25] Generally, the creation of a CV entangled source can be acquired in a nondegenerate four-wave mixing[26,27] and nondegenerate parametric down conversion (NPDC) process.[28–30] The NPDC in a crystal has been proved to be a perfect method for producing CV entanglement,[28] and proposals have been presented to generate CV entanglement by engineering an NPDC Hamiltonian. Based on the study concerning NPDC, Adamyan *et al.*[18] and Zhou *et al.*[7] have presented schemes to produce CV entanglement of two-mode field with the NPDC form.

In recent years, there has been growing interest in the generation of entanglement,[31–33] realizing conditional quantum phase gates,[34] and simulation of anisotropic XYZ-spin chain[35] in two distant coupled cavities. The coupled cavities can be achieved in some physical systems, such as photonic crystals and cavity–fiber–cavity systems. More recently, Tan *et al.*[36] presented a proposal for producing CV entanglement between two nanocavities, but an initial squeezed state was required. Since the single atom laser has been realized in experiment,[37] entangling two distant cavities might be feasible experimentally by coupling two cavities of the same structure. Hence, we consider a scheme in which two trapped atoms with a similar atomic configuration as that in Ref. [37], and show that CV entanglement with the NPDC form can be created in two distant coupled cavities. Compared with the proposal of Tan *et al.*, we only need an initial vacuum state and two trapped atoms in two distant coupled cavities. Moreover, because our scheme joins the degenerate parametric down conversion and the beam splitter (BS) interaction, for a high-quality cavity, our system provides the advantage in achieving a larger photon number with better entanglement, while the system with BS[41] or NPDC[7] interaction cannot do that. In addition, by adjusting the product of classical Rabi frequencies, we can have a long time interval of entanglement and lose the restriction on the time interval.

2. Effective Hamiltonian and evolution of the state

We consider two identical single-mode cavities coupled directly with a hopping strength J or via a fiber. For the second case, if the fiber mode is far-detuned with the cavity mode,

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and with a short fiber, the intercavity interaction is the same as that in the first case.[38] In our scheme, each cavity contains a four-level atom. The system setup as well as the atomic configuration is plotted in Fig. 1. The atomic transitions $|d\rangle \leftrightarrow |e\rangle$ and $|c\rangle \leftrightarrow |g\rangle$ are coupled to the j-th cavity ($j = 1,2$) with coupling constants g_1 and g_2 and detunings δ_1 and δ_2, respectively. At the same time, the atomic levels between $|d\rangle \leftrightarrow |g\rangle$ and $|c\rangle \leftrightarrow |e\rangle$ are driven by two classical fields with Rabi frequencies Ω_3 and Ω_4 and detunings δ_3 and δ_4, respectively. Without loss of generality, the cavity field coupling strengths g_1, g_2 and the Rabi frequencies Ω_3, Ω_4 are assumed to be real numbers and the same for the two cavities. In the interaction picture, the Hamiltonian ($\hbar = 1$) can be written as

$$H = H_1 + H_2, \tag{1}$$

where

$$\begin{aligned} H_1 &= \sum_{j=1,2} (g_1 a_j \mathrm{e}^{-\mathrm{i}\delta_1 t}|d_j\rangle\langle e_j| + g_2 a_j \mathrm{e}^{-\mathrm{i}\delta_2 t}|c_j\rangle\langle g_j| \\ &\quad + \Omega_3 |d_j\rangle\langle g_j|\mathrm{e}^{-\mathrm{i}\delta_3 t} + \Omega_4 |c_j\rangle\langle e_j|\mathrm{e}^{-\mathrm{i}\delta_4 t} \\ &\quad + \mathrm{H.c.}), \\ H_2 &= J(a_1^\dagger a_2 + a_1 a_2^\dagger). \end{aligned} \tag{2}$$

The Hamiltonian H_1 describes the interaction within the cavities, and H_2 is the inter-cavity hopping with a hopping rate J. One can see that the creation of entanglement should result from the BS type H_2. To obtain the total effective Hamiltonian, we treat the Hamiltonian H_1 first. Under large detuning condition $g_i/\delta_i \ll 1$, $\Omega_l/\delta_l \ll 1$, $(i = 1,2, l = 3,4)$, the levels $|d_j\rangle$ and $|c_j\rangle$ can be adiabatically eliminated, and the Hamiltonian can be obtained as

$$\begin{aligned} H_3 &= \sum_{j=1,2} \left\{ \left(\frac{g_1^2}{\delta_1} a_j^\dagger a_j + \frac{\Omega_4^2}{\delta_4} \right) |e_j\rangle\langle e_j| \right. \\ &\quad + \left(\frac{g_2^2}{\delta_2} a_j^\dagger a_j + \frac{\Omega_3^2}{\delta_3} \right) |g_j\rangle\langle g_j| \\ &\quad + \left[\left(\frac{\Omega_3 g_1}{\delta_{13}} a_j \mathrm{e}^{\mathrm{i}\Delta_1 t} + \frac{\Omega_4 g_2}{\delta_{24}} a_j^\dagger \mathrm{e}^{-\mathrm{i}\Delta_2 t} \right) |g_j\rangle\langle e_j| \right. \\ &\quad \left. \left. + \mathrm{H.c.} \right] \right\}, \end{aligned} \tag{3}$$

where $\Delta_1 = \delta_3 - \delta_1$, $\Delta_2 = \delta_4 - \delta_2$, $1/\delta_{13} = (1/\delta_1 + 1/\delta_3)/2$, and $1/\delta_{24} = (1/\delta_2 + 1/\delta_4)/2$. In order to obtain the time-independent Hamiltonian, we perform the further unitary transformation $H_4 = \mathrm{e}^{\mathrm{i}H_0 t} H_3 \mathrm{e}^{-\mathrm{i}H_0 t}$ with

$$H_0 = \sum_{j=1,2} \left(\frac{\Omega_4^2}{\delta_4} |e_j\rangle\langle e_j| + \frac{\Omega_3^2}{\delta_3} |g_j\rangle\langle g_j| + \frac{\Delta_1 + \Delta_2}{2} a_j^\dagger a_j \right),$$

then if $\Delta = \Omega_3^2/\delta_3 - \Omega_4^2/\delta_4 + (\Delta_1 - \Delta_2)/2$ satisfies the condition $|\Delta| \gg \{|\Omega_3 g_1/\delta_{13}|, |\Omega_4 g_2/\delta_{24}|\}$, the adiabatic elimination is performed again, and the corresponding Hamiltonian can be given by

$$\begin{aligned} H_5 &= \sum_{j=1,2} \left\{ -\frac{\Delta_1 + \Delta_2}{2} a_j^\dagger a_j \right. \\ &\quad + \frac{g_1^2}{\delta_1} a_j^\dagger a_j |e_j\rangle\langle e_j| + \frac{g_2^2}{\delta_2} a_j^\dagger a_j |g_j\rangle\langle g_j| \\ &\quad + \frac{1}{\Delta} \left[\left(\frac{g_1 \Omega_3 a_j}{\delta_{13}} + \frac{g_2 \Omega_4 a_j^\dagger}{\delta_{24}} \right) \left(\frac{g_1 \Omega_3 a_j^\dagger}{\delta_{13}} + \frac{g_2 \Omega_4 a_j}{\delta_{24}} \right) \right. \\ &\quad \times |g_j\rangle\langle g_j| - \left(\frac{g_1 \Omega_3 a_j^\dagger}{\delta_{13}} + \frac{g_2 \Omega_4 a_j}{\delta_{24}} \right) \\ &\quad \left. \left. \times \left(\frac{g_1 \Omega_3 a_j}{\delta_{13}} + \frac{g_2 \Omega_4 a_j^\dagger}{\delta_{24}} \right) |e_j\rangle\langle e_j| \right] \right\}. \end{aligned} \tag{4}$$

Assuming that the atoms are initially in the ground state $|g_j\rangle$, we have the effective H_1 Hamiltonian

$$H_{1\mathrm{eff}} = \sum_{j=1,2} \left[\zeta a_j^\dagger a_j + \gamma \left(a_j^2 + a_j^{\dagger 2} \right) \right], \tag{5}$$

where $\zeta = \Omega_3^2 g_1^2/\Delta\delta_{13}^2 + \Omega_4^2 g_2^2/\Delta\delta_{24}^2 + g_2^2/\delta_2 - (\Delta_1 + \Delta_2)/2$, and $\gamma = \Omega_3 \Omega_4 g_1 g_2/(\Delta\delta_{13}\delta_{24})$. Equation (5) is a nonlinear equation, and quadratic terms $a_j^2 + a_j^{\dagger 2}$ are the generator of the squeezed states. We can clearly understand from Fig. 1(b) that after one cycle of transitions the atomic jumpings $|d\rangle \leftrightarrow |e\rangle$ and $|c\rangle \leftrightarrow |g\rangle$ will produce (annihilate) two photons when the atoms interact with the cavity modes. We will show the characteristic of the field entanglement under the action of the nonlinearity.

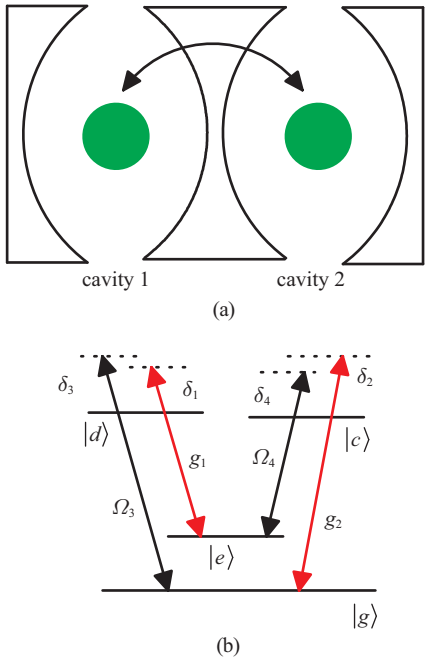


Fig. 1. (color online) (a) Two four-level atoms are trapped in two coupled cavities. (b) The atomic transition $|d\rangle \leftrightarrow |e\rangle$ ($|c\rangle \leftrightarrow |g\rangle$) is coupled with the cavity mode with coupling strength g_1 (g_2) and detuning δ_1 (δ_2). The two laser fields with Rabi frequencies Ω_3 and Ω_4 drive the atomic transitions $|d\rangle \leftrightarrow |g\rangle$ and $|c\rangle \leftrightarrow |e\rangle$ with detunings δ_3 and δ_4, respectively.

For convenience, we choose $\zeta = 0$. Including the hopping between the two cavities, the total effective Hamiltonian takes the form

$$H_{\rm eff} = \gamma\left(a_1^2 + a_1^{\dagger 2} + a_2^2 + a_2^{\dagger 2}\right) + J\left(a_1^\dagger a_2 + a_1 a_2^\dagger\right). \quad (6)$$

We assume that the initial state of the two cavities is in the vacuum state $|0,0\rangle$. The time evolution of the field state can be obtained as $|\psi(t)\rangle = {\rm e}^{-{\rm i}H_{\rm eff}t}|0,0\rangle$. By using the spilt-operator method, the unitary evolution operator can be decomposed as

$$\begin{aligned} {\rm e}^{-{\rm i}H_{\rm eff}t} &= {\rm e}^{x_1\left(a_1 a_2 - a_1^\dagger a_2^\dagger\right)}\, {\rm e}^{x_2\left(a_1^2 + a_1^{\dagger 2} + a_2^2 + a_2^{\dagger 2}\right)} \\ &\quad \times {\rm e}^{x_3\left(a_1^\dagger a_2 + a_1 a_2^\dagger\right)}, \end{aligned} \quad (7)$$

where coefficients x_1, x_2, and x_3 satisfy the differential equations given in Appendix A. It can be seen clearly from Eqs. (6) and (7) that the nonlinear two-photon process plus the BS interaction will induce the NPDC type. In Eq. (7), the last exponent consisting of the BS term acts upon the initial state $|0,0\rangle$, and the fields are still in the vacuum state $|0,0\rangle$, i.e., ${\rm e}^{x_3(a_1^\dagger a_2 + a_1 a_2^\dagger)}|0,0\rangle = |0,0\rangle$. The exponent term ${\rm e}^{x_2(a_1^2 + a_1^{\dagger 2} + a_2^2 + a_2^{\dagger 2})}$ denotes two-single-mode squeezing. Employing the SU(1, 1) Lie algebra,[39] we can obtain

$$\begin{aligned} &{\rm e}^{x_2\left(a_1^2 + a_1^{\dagger 2} + a_2^2 + a_2^{\dagger 2}\right)}|0,0\rangle \\ &= {\rm e}^{\left(\xi^* a_1^2 - \xi a_1^{\dagger 2}\right)/2}\, {\rm e}^{\left(\xi^* a_2^2 - \xi a_2^{\dagger 2}\right)/2}|0,0\rangle = |0,0\rangle_{\rm g}, \end{aligned} \quad (8)$$

where $|0\rangle_{\rm g}$ is a single-mode squeezed vacuum state, and is obtained by the single-mode squeeze operator ${\rm e}^{\left(\xi^* a_j^2 - \xi a_j^{\dagger 2}\right)/2}$ acting upon the vacuum field. Let $\xi = r{\rm e}^{{\rm i}\theta}$, r and θ are the single-mode squeezed parameters and satisfy the relation $A = -{\rm e}^{{\rm i}\theta}\tanh r = -{\rm i}\tanh\varphi$, $r = \tanh^{-1}|A|$, and $\varphi = 2{\rm i}x_2$. It is worthwhile to point out that the property of the squeezed parameter r depends not only on the coefficient γ of the nonlinear terms but also on the hopping rate J of the BS term. Finally, the state of the fields can be given by

$$|\psi(t)\rangle = {\rm e}^{x_1\left(a_1 a_2 - a_1^\dagger a_2^\dagger\right)}|0,0\rangle_{\rm g}. \quad (9)$$

It can be noticed that equation (9) means a two-mode squeeze operator taking action on the squeezed vacuum state. It is this term that produces entanglement, that is to say, the CV entanglement of the cavity fields is created in an NPDC process. However, the NPDC term depends on BS and nonlinear terms. The entanglement of the cavity field state (9) cannot be created without the BS term. Certainly, without the nonlinearity, we only have the BS type entanglement.

By applying Eq. (9), the total mean photon number of the two cavities is expressed as

$$\begin{aligned} N &= \langle a_1^\dagger a_1\rangle + \langle a_2^\dagger a_2\rangle \\ &= \cosh 2r\cosh 2x_1 - 1. \end{aligned} \quad (10)$$

For determining the entanglement of the cavity field state (9), we need an entanglement criterion for the CV system. Here, we choose the criterion presented in Ref. [40] to judge the entanglement. According to this criterion, the system is entangled if the total variance of two EPR-like operators $\hat{u}$ and $\hat{v}$ satisfy the following inequality:

$$\langle(\Delta\hat{u})^2 + (\Delta\hat{v})^2\rangle < 2, \quad (11)$$

where $\hat{u} = \hat{X}_1 + \hat{X}_2$, $\hat{v} = \hat{P}_1 - \hat{P}_2$, $\hat{X}_j = (a_j{\rm e}^{-{\rm i}\phi} + a_j^\dagger{\rm e}^{{\rm i}\phi})/\sqrt{2}$, and $\hat{P}_j = (a_j{\rm e}^{-{\rm i}\phi} - a_j^\dagger{\rm e}^{{\rm i}\phi})/({\rm i}\sqrt{2})$ $(j = 1, 2)$. The quadrature operators $\hat{X}_j$ and $\hat{P}_j$ describe the cavity fields and are used to calculate the entanglement condition. For Eq. (11), by taking $\phi = \pi$, we can obtain

$$\langle(\Delta\hat{u})^2 + (\Delta\hat{v})^2\rangle = 2\cosh 2r(\cosh 2x_1 - \sinh 2x_1). \quad (12)$$

From Eqs. (10) and (12), it is noticed that the average photon number and the entanglement condition depend on x_1 and r (i.e. ultimately depend on the product of Ω_3 and Ω_4). Therefore, we can change the average photon number of the fields and observe the entanglement characteristic of the cavity modes for different values of $\Omega_3\Omega_4$ by manipulating Ω_3 and Ω_4.

In Fig. 2, we plot the total average photon number and the summation of fluctuations of the fields from Eqs. (10) and (12) for two values of $\Omega_3\Omega_4$. Comparing the two cases, we observe that the amplitude of the average photon number and the oscillation periods of both N and $\langle(\Delta\hat{u})^2 + (\Delta\hat{v})^2\rangle$ increase with the increase of the product $\Omega_3\Omega_4$. The longer time interval helps us to acquire entanglement, because it loses the restriction of time interval. Most interestingly, when the total photon number achieves its maximum values, the summation of fluctuation acquires its minimum values. For example, when $gt = 475$ (see dashed line in Fig. 2), the total photon number is about 140, while the summation of fluctuation is nearly zero, that is to say, the larger the photon number, the smaller the summation of fluctuation, which means that we can have a larger photon number with better entanglement than that at other time instants. For a pure BS system, an entangled coherent state can be obtained, but the total photon number is conservative;[41] while for a two-mode coherent-squeezed state generated from NPDC, the entanglement disappears when the total photon number achieves its maximum value[7] (see Fig. 2). Compared with these two cases, the present system, joining nonlinear and BS interaction, provides us with the advantages of achieving a larger photon number and a better entanglement.

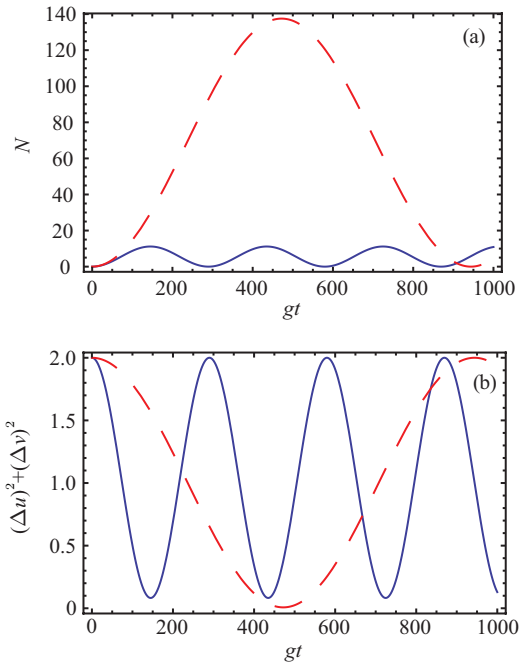


Fig. 2. (color online) (a) The time evolution of the total photon number. (b) The summation of quantum fluctuation $\langle(\Delta\hat{u})^2+(\Delta\hat{v})^2\rangle$ changes with time. The red-dashed and blue-solid lines correspond to parameters $\{\Omega_3=15g,\ \Omega_4=15g\}$ and $\{\Omega_3=13.67g,\ \Omega_4=14.89g\}$, respectively. The other parameters are $g_1=g$, $g_2=10.1g$, $\delta_1=100g$, $\delta_2=118g$, $\delta_3=115g$, $\delta_4=105g$, and $J=0.0278g$.

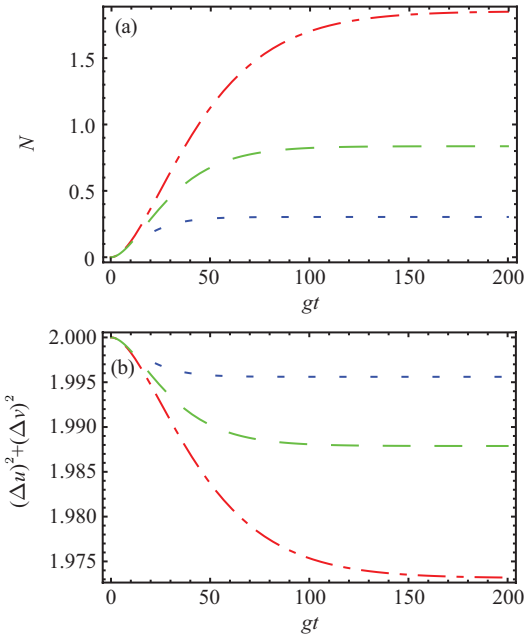


Fig. 3. (color online) The time evolutions of (a) the total photon number and (b) quantity $\langle(\Delta\hat{u})^2+(\Delta\hat{v})^2\rangle$ under different cavity decay rates κ. The blue-dotted, green-dashed, and red-dashed-dotted lines correspond to $\kappa=0.05g$, $0.03g$, and $0.02g$, respectively. For all plots, $g_1=g$, $g_2=10.1g$, $\Omega_3=15g$, $\Omega_4=15g$, $\delta_1=100g$, $\delta_2=118g$, $\delta_3=115g$, $\delta_4=105g$, and $J=0.0278g$.

3. Numerical solution including cavity decay

Due to the large detuning condition, the atomic spontaneous emission damping can be ignored. We now discuss the effect of cavity dissipation on the entanglement of the two cavities as well as on the summation of the mean photon number. For simplicity, the losses of the two cavities are assumed to be equal. By introducing the cavity damping terms, the master equation is

$$\begin{aligned}\dot{\rho} &= -\mathrm{i}\Big[\gamma\Big(a_1^2+a_1^{\dagger 2}+a_2^2+a_2^{\dagger 2}\Big)\\ &\quad +J\Big(a_1^{\dagger}a_2+a_1a_2^{\dagger}\Big),\rho\Big]\\ &\quad +\kappa\sum_{j=1,2}\Big(2a_j\rho a_j^{\dagger}-a_j^{\dagger}a_j\rho-\rho a_j^{\dagger}a_j\Big),\end{aligned}\tag{13}$$

where κ is the damping rate of the two cavities. From Eq. (13), the differential equation for mean value $\langle a_1^{\dagger}a_1\rangle$ can be given by

$$\begin{aligned}\frac{\mathrm{d}\langle a_1^{\dagger}a_1\rangle}{\mathrm{d}t} &= -\mathrm{i}\Big[-2\gamma\langle a_1^2\rangle+2\gamma\langle a_1^{\dagger 2}\rangle\\ &\quad +J\langle a_1^{\dagger}a_2\rangle-J\langle a_1a_2^{\dagger}\rangle\Big]-2\kappa\langle a_1^{\dagger}a_1\rangle.\end{aligned}\tag{14}$$

The remaining eleven equations of $\mathrm{d}\langle a_2^{\dagger}a_2\rangle/\mathrm{d}t$, $\mathrm{d}\langle a_1a_2\rangle/\mathrm{d}t$, $\mathrm{d}\langle a_1^2\rangle/\mathrm{d}t$, etc. are given in Appendix A.

We present the numerical solutions for the summation of fluctuations and the mean of the total photon number. Figure 3 displays the time evolution of the total photon number and the entanglement condition for several cavity decay rates κ. It exhibits the usual behavior that the smaller the loss of the cavity, the larger the mean photon number, and the better the entanglement. We would like to show further the role of product $\Omega_3\Omega_4$ under the dissipation of the cavity. In Fig. 4, we plot the time evolution of the average photon number and the entanglement condition for two $\Omega_3\Omega_4$ at the same decay

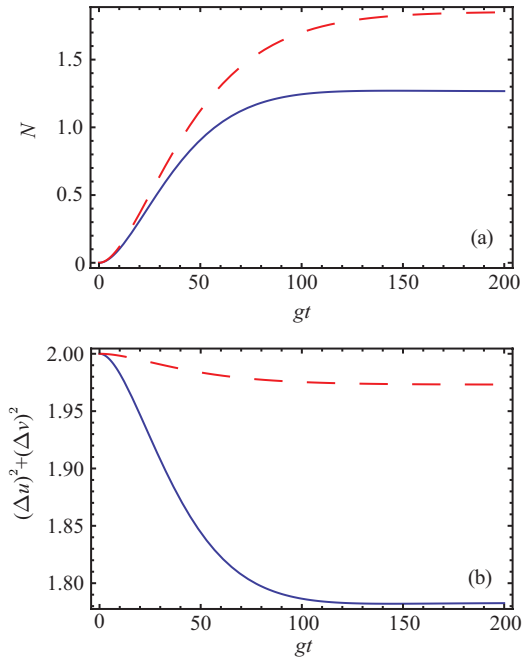


Fig. 4. (color online) (a) The mean value of the total photon number N, (b) the time evolution of the entanglement condition. The decay rate is $\kappa=0.02g$, the red-dashed and blue-solid lines correspond to $\{\Omega_3=15g,\ \Omega_4=15g\}$ and $\{\Omega_3=13.67g,\ \Omega_4=14.89g\}$, respectively. The other parameters are the same as those in Fig. 3.

rate $\kappa = 0.02g$. We can see that the average photon number increases with the increase of product $\Omega_3\Omega_4$. However, the summation of the quantum fluctuations could not decrease quickly with the increase of the product. This is because for a larger value of the product, the system requires a longer time to decrease to its lowest values even for the case without cavity loss (it is the result of the longer oscillation period). But the dissipation of the cavity loses the coherence quickly, so it leaves no time for the system to achieve its lowest value. Therefore, we need a moderate value of $\Omega_3\Omega_4$ to achieve satisfactory entanglement.

4. Discussion and conclusion

We present a brief discussion on the experimental feasibility of our program. It can be realized by employing a single caesium atom trapped in a high-finesse optical cavity, and a strong coupling is demanded for the experimental realization of a one-atom laser in Ref. [37]. With the groups of the cavity parameters $g_1 = g = g_0/2\pi = 16$ MHZ, $g_2 = 10.1g = 10.1g_0/2\pi = 161.6$ MHZ, the cavity decay rates $\kappa = 0.05g$, $0.03g$, and $0.02g$ correspond to $\kappa/2\pi \sim 0.127$ MHZ, 0.076 MHZ, and 0.051 MHZ. It satisfies the strong coupling condition $g_1, g_2 \gg \kappa$. Applying the above parameters, the time period of generating CV entanglement is about $t = 950/g \sim 10^{-5}$ s, which is much shorter than the experimental trap lifetime $\Delta t = 0.05$ s. Hence, there is enough time for the generation of CV entanglement, and the proposal might be feasible experimentally. Moreover, our study is performed under the large detuning condition, so we can ignore the decoherence induced by the atomic spontaneous emission. Using the selected set of parameters in Figs. 2–4, the first adiabatic elimination condition $g_i/\delta_i \ll 1$, $\Omega_l/\delta_l \ll 1$, $(i = 1,2, l = 3,4)$ is satisfied. At the same time, $|\Omega_3 g_1/\delta_{13}| = 2.243$ (2.045) MHZ, $|\Omega_4 g_2/\delta_{24}| = 21.814$ (21.654) MHZ, and $|\Delta| = 221.019$ (216.214) MHZ correspond to the parameters $\Omega_3 = \Omega_4 = 15g$ and $\Omega_3 = 13.67g$, $\Omega_4 = 14.89g$, so the second adiabatic elimination condition $|\Delta| \gg \{|\Omega_3 g_1/\delta_{13}|, |\Omega_4 g_2/\delta_{24}|\}$ is satisfied. Therefore, the large detuning condition can also be fulfilled simultaneously for the chosen parameters. Our scheme might be feasible in experiments along with the process of technology.

In conclusion, we present a scheme to generate the entanglement in two coupled cavities with two four-level atoms. In our scheme, a BS type hopping is transformed into an NPDC type with the help of the nonlinear two-photon terms, and the CV entanglement can be created. For the total effective Hamiltonian, we investigate the time evolution of the summation of quantum fluctuations and the average photon number and find that by increasing the product $\Omega_3\Omega_4$, we can obtain better entanglement with a larger photon number. Moreover, taking into account the cavity losses, we show that the entanglement is still present.

Appendix A: Coefficients and equations of the cavity fields

The differential equations for parameters x_1, x_2, and x_3 in Eq. (7) are

$$\begin{aligned} &x_1' + x_3' \sin 4x_2 = 0, \qquad \text{(A1)}\\ &2x_2' \sinh 2x_1 + x_3' \cos 4x_2 \cosh 2x_1 = -\mathrm{i}J,\\ &2x_2' \cosh 2x_1 + x_3' \cos 4x_2 \sinh 2x_1 = -2\mathrm{i}\gamma. \end{aligned}$$

In order to numerically calculate $\langle(\Delta\hat{u})^2 + (\Delta\hat{v})^2\rangle$ and the average photon number, the differential equations for various moments from Eq. (13) are listed below:

$$\begin{aligned} \frac{\mathrm{d}\langle a_2^\dagger a_2\rangle}{\mathrm{d}t} &= -\mathrm{i}[-2\gamma\langle a_2^2\rangle + 2\gamma\langle a_2^{\dagger 2}\rangle + J\langle a_1 a_2^\dagger\rangle - J\langle a_1^\dagger a_2\rangle]\\ &\quad - 2\kappa\langle a_2^\dagger a_2\rangle, \qquad \text{(A2)}\\ \frac{\mathrm{d}\langle a_1 a_2\rangle}{\mathrm{d}t} &= -\mathrm{i}[2\gamma\langle a_1^\dagger a_2\rangle + 2\gamma\langle a_1 a_2^\dagger\rangle + J\langle a_2^2\rangle + J\langle a_1^2\rangle]\\ &\quad - 2\kappa\langle a_1 a_2\rangle, \qquad \text{(A3)}\\ \frac{\mathrm{d}\langle a_1^2\rangle}{\mathrm{d}t} &= -\mathrm{i}[4\gamma\langle a_1^\dagger a_1\rangle + 2J\langle a_1 a_2\rangle + 2\gamma] - 2\kappa\langle a_1^2\rangle, \qquad \text{(A4)}\\ \frac{\mathrm{d}\langle a_1 a_2^\dagger\rangle}{\mathrm{d}t} &= -\mathrm{i}[2\gamma\langle a_1^\dagger a_2^\dagger\rangle - 2\gamma\langle a_1 a_2\rangle + J\langle a_2^\dagger a_2\rangle - J\langle a_1^\dagger a_1\rangle]\\ &\quad - 2\kappa\langle a_1 a_2^\dagger\rangle, \qquad \text{(A5)}\\ \frac{\mathrm{d}\langle a_2^2\rangle}{\mathrm{d}t} &= -\mathrm{i}[4\gamma\langle a_2^\dagger a_2\rangle + 2J\langle a_1 a_2\rangle + 2\gamma] - 2\kappa\langle a_2^2\rangle, \qquad \text{(A6)}\\ \frac{\mathrm{d}\langle a_1\rangle}{\mathrm{d}t} &= -\mathrm{i}[2\gamma\langle a_1^\dagger\rangle + J\langle a_2\rangle] - \kappa\langle a_1\rangle,\\ \frac{\mathrm{d}\langle a_2\rangle}{\mathrm{d}t} &= -\mathrm{i}[2\gamma\langle a_2^\dagger\rangle + J\langle a_1\rangle] - \kappa\langle a_2\rangle. \end{aligned}$$

The remaining equations are their Hermitian conjugate.

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