# Creation of quantum correlations via the initial classical-mixed states*

Han Wei(韩 伟), Zhang Ying-Jie(张英杰), and Xia Yun-Jie(夏云杰)†

*Key Laboratory of Laser Polarization and Information Technology of Shandong Province, Department of Physics, Qufu Normal University, Qufu 273165, China*

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Using the pseudomode method, we theoretically analyze the creation of quantum correlations between two two-level dipole–dipole interacting atoms coupled with a common structured reservoir with different coupling strengths. Considering certain classes of initial separable-mixed states, we demonstrate that the sudden birth of atomic entanglement as well as the generation of stationary quantum correlations occur. Our results also suggest a possible way to control the occurrence time of entanglement sudden birth and the stationary value of quantum correlations by modifying the initial conditions of states, the dipole–dipole interaction, and the relative coupling strength. These results are helpful for the experimental engineering of entanglement and quantum correlations.

**Keywords:** quantum correlation, quantum entanglement, quantum discord, pseudomode method

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## 1. Introduction

Quantum correlations are crucial to many quantum information tasks, as shown by the role of entanglement in speeding up quantum algorithms, making quantum teleportation[1] and cryptography,[2] and the enabling role of quantum discord (QD)[3] in the deterministic quantum computation with one pure qubit[4] and Grover?s search algorithm.[5] However, all realistic quantum systems are inevitably connected to the decohering environments which decay the quantumness of the systems. The dynamics of QD and entanglement of two qubits in contact with an environment have been studied in the last decade theoretically as well as experimentally.[6–23] It was shown that entanglement is very fragile and entanglement sudden death can occur,[6] while QD is more robust under decoherence and only presents an instantaneous disappearance at some time points in a non-Markovian regime and asymptotic decay in a Markovian regime. In particular, QD can be completely unaffected by decoherence channels for certain initial states, and this phenomenon has been observed experimentally.[8]

Now, besides the above studies for entanglement and QD, another major problem is to assess how quantum correlations can be created from the initial separable states. Recently, preexisting quantum correlations can exhibit an increasing behavior at some stages of their time evolution in the presence of system-reservoir initial correlations, as demonstrated for both entanglement and QD.[14,15] Then quantum noise arising from a common bath reservoir can create quantum correlations initially fully absent,[24] a phenomenon also well known to occur for entanglement.[25] In such instances, the increasing and creating of quantum correlations stems from the ability of a common reservoir to mediate an effective interaction, and in these theoretical models the qubits? dipole–dipole interaction and different coupling constants are neglected. In general, when the qubits exit in a common reservoir, they may be in the different locations and be quite near among them, at a distance no bigger than the wavelengths of the center mode of reservoir, the dipole–dipole interaction between the qubits and different coupling constants have to be considered.

In this paper, we study two two-level dipole–dipole interacting atoms in a common structured reservoir, and focus on the generation of the atomic entanglement and QD by considering certain classes of initial separable-mixed states. By analytical and numerical analysis we find that, for the case of the initial mixed states which contain the basis $|eg\rangle\langle eg|$ or $|ge\rangle\langle ge|$, the atomic entanglement and QD would be produced and then reach their stationary values. However, for those initial mixed states which are only composed by the basis $|ee\rangle\langle ee|$ and $|gg\rangle\langle gg|$, the atomic entanglement is never generated and the QD could appear and always eventually goes to zero, while by choosing the initial mixed state which contains the basis $|ee\rangle\langle ee|$ and $|eg\rangle\langle eg|$, the atomic entanglement sudden birth can appear. In addition, we also investigate the influences of the initial conditions of states, the dipole–dipole interaction and the relative coupling strength on the occurrence time of atomic entanglement sudden birth and the stationary value of quantum correlations.

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†Corresponding author. E-mail: yjxia@mail.qfnu.edu.cn

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## 2. Theoretical framework

We consider two two-level atoms A and B interacting with a common structured reservoir at zero temperature, and take their dipole–dipole interaction into account from the very beginning. The Hamiltonian of such a composite two-atom system plus the reservoir under the rotating-wave approximation is given by $H = H_0 + H_{\rm int} + H_{\rm dd}$, which, in the basis $\{|gg\rangle, |eg\rangle, |ge\rangle, |ee\rangle\}$ reads

$$H_0 = \omega_0(\sigma_+^{\rm A}\sigma_-^{\rm A} + \sigma_+^{\rm B}\sigma_-^{\rm B}) + \sum_k \omega_k a_k^\dagger a_k, \tag{1}$$

$$H_{\rm int} = (\mu_{\rm A}\sigma_+^{\rm A} + \mu_{\rm B}\sigma_+^{\rm B})\sum_k g_k a_k + {\rm h.c.}, \tag{2}$$

$$H_{\rm dd} = D(\sigma_+^{\rm A}\sigma_-^{\rm B} + \sigma_-^{\rm A}\sigma_+^{\rm B}), \tag{3}$$

where $\sigma_\pm^{\rm A}$ and $\sigma_\pm^{\rm B}$ are, respectively, the Pauli raising and lowering operators for atoms A and B, $\omega_0$ is the Bohr frequency of two atoms, $a_k$ and $a_k^\dagger$ are the annihilation and creation operators for the field mode $k$, which is characterized by the frequency $\omega_k$ and the coupling constant $g_k$. To measure the coupling strength of the atoms to the field mode determined by the atoms relative position in the reservoir, we introduce the dimensionless constant $\mu_{\rm A,B}$.[26] The dipole–dipole interaction $D$ is proportional to the atomic relative position and the electric dipole moment of the atom. For the sake of simplicity, in the following we assume that two atoms interact resonantly with a Lorentzian structured reservoir.

In order to find the dynamics of two atoms, we solve the master equation by using the pseudomode approach. This exact master equation describes the coherent interaction between the atoms and the pseudomodes in the presence of decay of the pseudomodes due to the interaction with a Markovian reservoir.[27] The number of the pseudomodes relies on the shape of the reservoir spectral distribution. Since a Lorentzian function $D(\omega) = \Gamma/[(\omega-\omega_{\rm c})^2 + (\Gamma/2)^2]$ has only one pole in the lower half complex plane, two atoms interact with one pseudomode which leaks into a Markovian reservoir. Here, the $\omega_{\rm c}$ and $\Gamma$ are, respectively, the oscillation frequency and the decay rate of the pseudomode and they determine the position of the pole $z_1 \equiv \omega_{\rm c} - {\rm i}\Gamma/2$. In our theoretical model, two atoms are unequally coupled to the reservoir and have dipole–dipole interaction between them; the dynamics of the two atoms can be effectively described by a four-state system. In the basis $\{|0\rangle = |gg\rangle, |+\rangle = (|eg\rangle + |ge\rangle)/\sqrt{2}, |-\rangle = (|eg\rangle - |ge\rangle)/\sqrt{2}, |2\rangle = |ee\rangle\}$, the Hamiltonians (1)–(3) can be rewritten

$$H_0 = 2\omega_0|2\rangle\langle 2| + \omega_0(|+\rangle\langle +| + |-\rangle\langle -|) + \sum_k \omega_k a_k^\dagger a_k, \tag{4}$$

$$H_{\rm int} = \Big[\frac{\mu_{\rm A}+\mu_{\rm B}}{2}(|+\rangle\langle 0| + |2\rangle\langle +|) + \frac{\mu_{\rm A}-\mu_{\rm B}}{2}(|-\rangle\langle 0| - |2\rangle\langle -|)\Big]\sum_k g_k a_k + {\rm h.c.}, \tag{5}$$

$$H_{\rm dd} = D(|+\rangle\langle +| - |-\rangle\langle -|). \tag{6}$$

The $|+\rangle$ and $|-\rangle$ states are, respectively, the super-radiant and subradiant states. Since the different atomic relative position $\mu_{\rm A/B}$, the super-radiant state $|+\rangle$ and subradiant state $|-\rangle$ are both coupled to states $|0\rangle$ and $|2\rangle$ via the field mode. The transitions $|0\rangle \leftrightarrow |+\rangle$, $|0\rangle \leftrightarrow |-\rangle$, $|+\rangle \leftrightarrow |2\rangle$, and $|-\rangle \leftrightarrow |2\rangle$ have the same frequencies and identically coupled with the common bosonic reservoir. Using the pseudomode approach, the dynamics of two atoms interacting with a Lorentzian structured reservoir can be treated using the following pseudomode master equation

$$\frac{{\rm d}\rho}{{\rm d}t} = -{\rm i}[H,\rho] - \frac{\Gamma}{2}\left[a^\dagger a\rho - 2a\rho a^\dagger + \rho a^\dagger a\right], \tag{7}$$

where $\rho$ is the density operator for two atoms and the pseudomode of the structured reservoir, and

$$\begin{aligned} H \;=\; & 2\omega_0|2\rangle\langle 2| + \omega_0(|+\rangle\langle +| + |-\rangle\langle -|) + \omega_{\rm c}a^\dagger a \\ & + \frac{\mu_{\rm A}+\mu_{\rm B}}{2}\Omega[(|+\rangle\langle 0| + |2\rangle\langle +|)a + (|0\rangle\langle +| + |+\rangle\langle 0|)a^\dagger] \\ & + \frac{\mu_{\rm A}-\mu_{\rm B}}{2}\Omega[(|-\rangle\langle 0| - |2\rangle\langle -|)a + (|0\rangle\langle -| - |-\rangle\langle 0|)a^\dagger] \\ & + D(|+\rangle\langle +| - |-\rangle\langle -|). \end{aligned} \tag{8}$$

Here $a$ and $a^\dagger$ are the annihilation and creation operators of the pseudomode. The two atoms interact coherently with the pseudomode (the strength of the coupling $\Omega$). In order to find the dynamics of the two atoms, we need to solve a set of differential equations obtained from the pseudomode master Eqs. (7) and (8). Finally, we obtain the reduced density matrix of the atomic system by tracing out the pseudomode degree of freedom.

In order to analyze the roles of different initial separable-mixed states on the generation of quantum correlations between two atoms, we consider the initial classical-mixed states between two atoms in the form $\rho_{\rm cc} = \sum_{m,n} p_{mn}|m^{\rm A}\rangle\langle m^{\rm A}| \otimes |n^{\rm B}\rangle\langle n^{\rm B}|$, where $\{|m^{\rm A}\rangle\} = \{|e^{\rm A}\rangle, |g^{\rm A}\rangle\}$ and $\{|n^{\rm B}\rangle\} = \{|e^{\rm B}\rangle, |g^{\rm B}\rangle\}$ are sets of orthogonal states of atoms A and B, with non-negative probabilities $p_{mn}$ that add up to 1. For initial states of the above form, in this paper the reduced density matrix for the atomic system $\rho_{\rm AB}(t)$ in the basis $\{|gg\rangle, |ge\rangle, |eg\rangle, |ee\rangle\}$ has an $X$ structure defined by nonzero elements only along the main diagonal and antidiagonal, with non-real coherences $\rho_{23} = \rho_{32}^* \neq 0$ and $\rho_{41} = \rho_{14}^* = 0$.

The measure of total quantum correlations used here is the quantum discord (QD).[3] For this $X$ class of density matrix, QD can be calculated analytically:[28] ${\rm QD}(\rho_{\rm AB}) = {\rm MI}(\rho_{\rm AB}) - {\rm CC}(\rho_{\rm AB})$, where the quantum mutual information (MI) is ${\rm MI}(\rho_{\rm AB}) = S(\rho_{\rm A}) + S(\rho_{\rm B}) + \sum_{j=0}^{3}\lambda_j \log_2 \lambda_j$, and the classical correlation (CC) is in the form ${\rm CC}(\rho_{\rm AB}) = S(\rho_{\rm A}) - \min_{\{B_i\}}[S(\rho|\{B_i\})]$. Here $\lambda_j$ is the $j$-th eigenvalue of the density matrix $\rho_{\rm AB}(t)$, $S(\rho_{\rm B})$ denotes the von Neumann entropy of $\rho_{\rm B} = {\rm Tr}_{\rm A}\rho_{\rm AB}$ and $S(\rho|\{B_i\})$ is the quantum conditional entropy with respect to a von Neumann measurement $\{B_i\}$ for subsystem B. The entanglement, a kind of quantum correlation, is computed through concurrence.[29] For an $X$-form density matrix $\rho_{\rm AB}(t)$ the concurrence is $2\max\{0, |\rho_{23}| - \sqrt{\rho_{11}\rho_{44}}\}$. For the pure states, the QD is equal to concurrence,

but there are some mixed correlated states with null entanglement and nonzero quantum discord.

## 3. Numerical results and discussion

In the following, we mainly investigate the creation of atomic entanglement and QD which is related to the different initial states, the dipole–dipole interaction and the relative coupling strength. First, we consider the weak coupling limit, such that $\Gamma = 5\Omega$, and no dipole–dipole interaction between two atoms $D = 0$. For the different initial classical-mixed states, we show the dynamics behavior of the atomic concurrence, CC and QD with the same relative positions $\mu_A/\mu_B = 1$ in Fig. 1. For two atoms initially prepared in the state $|ee\rangle$,

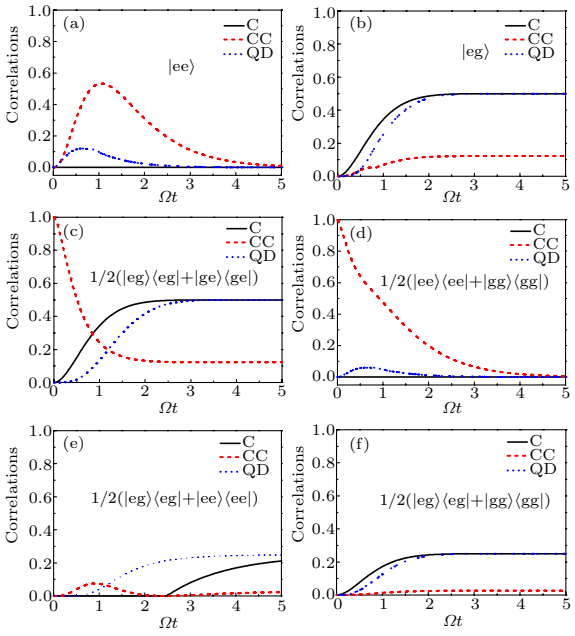



**Fig. 1.** (color online) Time evolution of the atomic concurrence (solid dark line), CC (dashed red line), and QD (dotted blue line) as a function of the dimensionless quantity $\Omega t$ with the different initial classical-mixed states, under $\Gamma = 5\Omega$, $\mu_A/\mu_B = 1$, and $D = 0$.

we see that the concurrence is always zero, and the QD would reach the maximum at first, and then gradually decrease to zero, while for the initial factorized state $|eg\rangle$ or $|ge\rangle$, the atomic concurrence and QD both generate continuously until they reach their stationary value. However, for the initial atomic separable-mixed states, as shown in Figs. 1(c)–1(f), one can find that concurrence and QD of two atoms are always produced and then reach their stationary values for the initial mixed states which contain the basis $|eg\rangle\langle eg|$ or $|ge\rangle\langle ge|$. But for those initial mixed states which only are composed by the basis $|ee\rangle\langle ee|$ and $|gg\rangle\langle gg|$, the atomic entanglement does not generate, but the QD could appear and always goes to zero. In Fig. 1, we also show the time evolution of the atomic CC for different initial states. It is clearly found that for the initial pure state $|ee\rangle$ or those initial mixed states only composed by the basis $|ee\rangle\langle ee|$ and $|gg\rangle\langle gg|$, the CC is always much larger than the concurrence and QD in the earlier time regime and then also goes to zero. On the contrary, for the initial pure state $|eg\rangle$ ($|ge\rangle$) or the initial mixed states which contain $|eg\rangle\langle eg|$ and $|ge\rangle\langle ge|$, the stationary value of CC is smaller than that of concurrence and QD. Moreover, it is interesting to find that when the initial mixed state contains the basis $|ee\rangle\langle ee|$ and $|eg\rangle\langle eg|$, the entanglement sudden birth of these two atoms can appear (see Fig. 1(e)).

Thus, to gain the entanglement sudden birth occurrence time and the stationary values of entanglement and QD, we analyze the atomic concurrence and QD as a function of $\Omega t$ with the different initial classical-mixed states which contain the basis $|ee\rangle\langle ee|$ and $|eg\rangle\langle eg|$ in Fig. 2. This appreciable result for the atomic entanglement shows that the occurrence time of entanglement sudden birth and the stationary value of entanglement, are both determined by the weight of $|ee\rangle\langle ee|$ or $|eg\rangle\langle eg|$ in the initial mixed states. The speed of the occurrence of entanglement sudden birth can increase as the weight of $|eg\rangle\langle eg|$ increases in the initial mixed state. Figures 2(a) and 2(b) show that the asymptotic stationary values of concurrence and QD also depend on the weight of $|ee\rangle\langle ee|$ or $|eg\rangle\langle eg|$ in the initial mixed states. In order to generate the much larger atomic entanglement and QD via the reservoir-mediated interaction between two atoms, one can choose the initial classical-mixed state in which the weight of $|eg\rangle\langle eg|$ is much bigger. What is the physics behind the phenomena? We then try to give an enlightening discussion for this result based on the evolution of the basis $|ee\rangle\langle ee|$ and $|eg\rangle\langle eg|$. In the evolution processes,

$$\begin{aligned} |ee0\rangle &\rightarrow c_1|ee0\rangle + c_2|+1\rangle + c_3|gg2\rangle, \\ |eg0\rangle &\rightarrow d_1|+0\rangle + d_2|-0\rangle + d_3|gg1\rangle. \end{aligned} \tag{9}$$

From Eq. (9), we can obtain that the basis $|ee0\rangle$ cannot produce any entanglement in the time evolution process, while the basis $|eg0\rangle$ can lead to the occurrence of the atomic entanglement and QD. For the initial classical-mixed states in the form $p_{eg}|eg\rangle\langle eg| + p_{ee}|ee\rangle\langle ee|$, with $p_{eg} + p_{ee} = 1$, the atomic entanglement can be calculated

$$C = 2\max\{0, |p_{eg}d_1d_2^*| - \sqrt{p_{ee}|c_1|^2(p_{eg}|d_3|^2 + p_{ee}|c_3|^2)}\}.$$

In the condition

$$|p_{eg}d_1d_2^*| \leq \sqrt{p_{ee}|c_1|^2(p_{eg}|d_3|^2 + p_{ee}|c_3|^2)},$$

the atomic $C$ is equal to 0, and in the condition

$$|p_{eg}d_1d_2^*| > \sqrt{p_{ee}|c_1|^2(p_{eg}|d_3|^2 + p_{ee}|c_3|^2)},$$

the entanglement of two atoms would suddenly birth. Thus, when $p_{eg}$ increases from 0 to 1, the occurrence of entanglement sudden birth will accelerate. In what follows, we will give a simple interpretation for the relationship between $p_{eg}$ and the stationary values of concurrence and QD. As we know, the asymptotic stationary state of the system has the form

$$\rho_{AB}(t \rightarrow \infty) = \frac{p_{eg}}{2}|-\rangle\langle -| + \left(1 - \frac{p_{eg}}{2}\right)|gg\rangle\langle gg|,$$

so the asymptotic stationary values of concurrence and QD increase with increasing $p_{eg}$.

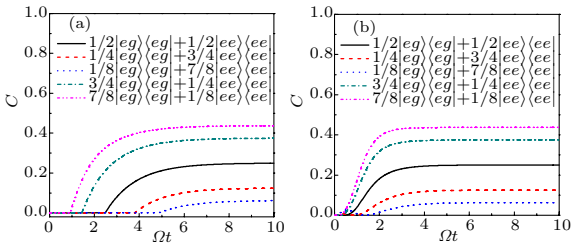



**Fig. 2.** (color online) Time evolution of the atomic concurrence and QD as a function of the dimensionless quantity $\Omega t$ with the different initial classical-mixed states which contain the basis $|ee\rangle\langle ee|$ and $|eg\rangle\langle eg|$, under $\Gamma = 5\Omega$, $\mu_A/\mu_B = 1$, and $D = 0$.

In this paper, we also understand the influences of the dipole–dipole interaction $D$ and the relative coupling strength $\mu_A/\mu_B$ on the generation of atomic entanglement and QD in the weak coupling regime which satisfies $\Gamma = 5\Omega$. From Ref. [30], we acquire that the condition for the existence of the asymptotic stationary state is $D = 0$ or $\mu_A = \mu_B$. Thus, we first fix $D = 0$ to study the relationship between the generation of atomic quantum correlation and $\mu_A/\mu_B$ in Fig. 3, with the initial state $(1/2)(|ee\rangle\langle ee| + |eg\rangle\langle eg|)$. It is clear to see that the fastest entanglement sudden birth happens when the relative coupling strength $\mu_A/\mu_B$ increases. Both the atomic entanglement and QD tend to their asymptotic stationary values inevitably as time increases, which means that the quantum system has a steady state

$$\rho_{AB}(t\to\infty) = \frac{\mu_B^2}{2(\mu_A^2+\mu_B^2)}|\psi_-\rangle\langle\psi_-| + \left(1-\frac{\mu_B^2}{2(\mu_A^2+\mu_B^2)}\right)|gg\rangle\langle gg|,$$

with the subradiant state

$$|\psi_-\rangle = \frac{\mu_B}{\sqrt{\mu_A^2+\mu_B^2}}|eg\rangle - \frac{\mu_A}{\sqrt{\mu_A^2+\mu_B^2}}|ge\rangle$$

in the case of $\mu_A \neq \mu_B$ and $D = 0$. Finally, by choosing $\mu_A = \mu_B$, an intuitive impression of the role of dipole–dipole interaction $D$ can be obtained from Fig. 4. It is clearly found that the speed of occurrence of entanglement sudden birth decreases with increasing $D$, and the atomic QD performs increased oscillations and tends to an asymptotic stationary value. In addition, both the atomic concurrence and QD will stay at their stationary values as time increases, irrespective of the dipole–dipole interaction $D$.

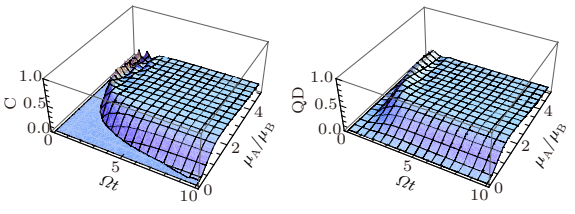



**Fig. 3.** (color online) Time evolution of the atomic concurrence ($C$) and QD as a function of the dimensionless quantity $\Omega t$ and $\mu_A/\mu_B$ with the initial classical-mixed state $(|ee\rangle\langle ee| + |eg\rangle\langle eg|)/2$, under $\Gamma = 5\Omega$ and $D = 0$.

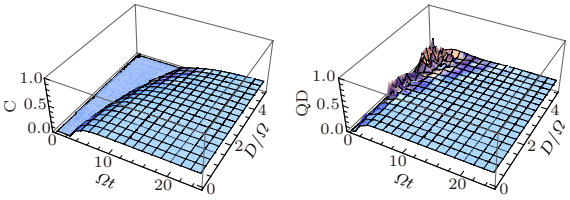



**Fig. 4.** (color online) Time evolution of the atomic concurrence ($C$) and QD as a function of the dimensionless quantity $\Omega t$ and $D$ with the initial classical-mixed state $(|ee\rangle\langle ee| + |eg\rangle\langle eg|)/2$, under $\Gamma = 5\Omega$ and $\mu_A = \mu_B = \Omega$.

## 4. Conclusion

In conclusion, we extend the study of the generation of entanglement and quantum correlation to a realistic situation where the dipole–dipole interaction is not neglected and the couplings of the atoms to the field are different. By considering certain classes of initial separable-mixed states, we find that the atomic entanglement sudden birth takes place and generation of the stationary quantum correlations occurs. In addition, the speed of occurrence of entanglement sudden birth and the stationary value of quantum correlation are related to the initial conditions of states, the dipole–dipole interaction and the relative coupling strength. These results are helpful for the practical engineering of entanglement in the future.

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