# Exciton transfer dynamics in a trimer system*

Tan Qing-Shou(谭庆收)[a], Xu Lan(许 兰)[b], and Kuang Le-Man(匡乐满)[a]†

[a] Key Laboratory of Low-Dimensional Quantum Structures and Quantum Control of Ministry of Education, and Department of Physics, Hunan Normal University, Changsha 410081, China

[b] Department of Mathematics and Science, Hunan First Normal University, Changsha 410205, China

(Received 29 May 2012; revised manuscript received 30 June 2012)

We study exciton transfer dynamics in a trimer system by investigating excitation transfer probability (ETP). We calculate ETP in the zero-temperature limit and theoretically predict the environment-assisted quantum critical effect, in which ETP exhibits a sudden change at the critical point of quantum phase transition for the trimer. In particular, we find that the steady-state ETP can be observed in the presence of the environment interaction.

**Keywords:** exciton transfer, quantum phase transition, trimer system

**PACS:** 03.65.Yz, 05.60.Gg, 71.35.–y **DOI:** 10.1088/1674-1056/22/1/010304

## 1. Introduction

Excitation energy transfer (EET) in natural and artificial photosynthetic systems has attracted much attention in the communities of not only physics and chemistry, but also energy resources.[1–4] Light-harvesting complexes in the photosynthetic process can capture the available light energy and transmit the resultant excitation energy to a reaction center at nearly unit efficiency. The highly efficient EET has inspired much work to uncover the basic mechanisms behind the perfect energy transfer.[5–10] Understanding the working principle of such a process and finding artificial analogs of natural light-harvesting complexes are immensely valuable for both fundamental research and practical applications.

Usually, the excitation transfer in photosynthetic systems is a very complicated physical process. However, all essential features can be illustrated by the use of dimer and trimer models which consist of monomeric chlorophylls (Chls).[11] The dimer model has been widely employed to study the coherent energy transfer in the photosynthetic systems.[12–19] A trimer system containing three monomeric Chls can form a simple light-harvesting complex under certain conditions. The extension of the dimer model to the trimer one allows one to study some relatively complicated photosynthetic complexes.

Quantum phase transition (QPT)[20] is a quantum critical phenomenon which occurs at zero temperature and is driven only by quantum fluctuation. It is shown that near the quantum critical point, a QPT system possesses ultrasensitivity.[21] An issue of considerable interest is whether the EET develops special features in the vicinity of the quantum critical point for the light-harvesting complexes. For instance, the basic unit cell in the LH2 light-harvesting complex[22] is a chromophore trimer.

In this paper, we are going to study the exciton transfer dynamics in a trimer system, and investigate the influence of QPT on the exciton transfer and the role of dephasing. We will show that the QPT can sensitively affect the exciton transfer probability (ETP) in the vicinity of the critical point of the QPT, leading to the environment-assisted quantum critical effect (QCE), in which exciton energy transfer exhibits a sudden change at the critical point of the QPT. It will be shown that due to the dephasing interaction between the system and its environment, a steady-state probability of exciton transfer can be observed.

## 2. Model and quantum master equation

The physical system under consideration is a trimer system shown in Fig. 1. It consists of two dimerized acceptors ($A_1$ and $A_2$) and one donor (D) which are modeled by three two-level systems. A single exciton transfer is from the donor to the acceptors. We shall deal with just one exciton in the trimer system interacting with three independent phonon baths. The Hamiltonian of the total system then consists of three parts (with $\hbar = 1$)

$$H = H_{\rm S} + H_{\rm SB} + H_{\rm B}. \tag{1}$$

The system part is

$$H_{\rm S} = \sum_{m=1}^{3} \varepsilon_m |m\rangle\langle m| + \sum_{m>n}^{3} J_{mn}(|m\rangle\langle n| + |n\rangle\langle m|), \tag{2}$$

where the site basis state $|m\rangle$ denotes the state in which only the $m$-th site is excited and all other sites are in their ground states (i.e., $|1\rangle = |100\rangle, |2\rangle = |010\rangle, |3\rangle = |001\rangle$), and $\varepsilon_m$ is the

*Project supported by the National Natural Science Foundation of China (Grant No. 11075050), the National Basic Research Program of China (Grant No. 2013CB921804), the Program for Changjiang Scholars and Innovative Research Team in University, China (Grant No. IRT0964), and the Hunan Provincial National Science Foundation, China (Grant No. 11JJ7001).

†Corresponding author. E-mail: lmkuang@hunnu.edu.cn

© 2013 Chinese Physical Society and IOP Publishing Ltd 

relative site energy. In the presence of interaction $J_{mn}$, a stationary single-excitation state should be delocalized and composed of a combination of the site basis states.

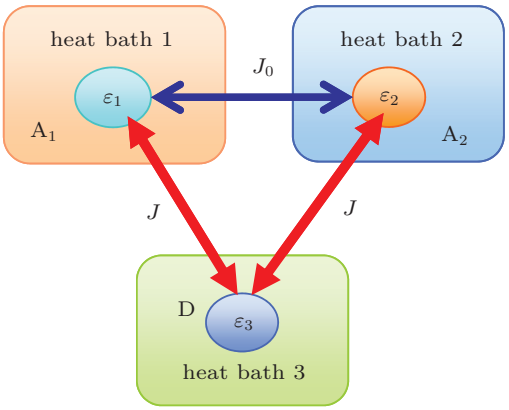



**Fig. 1.** (color online) Schematic diagram of the trimer model under consideration, which consists of a pair of acceptors and a donor immersed in three independent heat baths. A single exciton is transferred from the donor to the acceptors.

Here, we assume $\varepsilon_1 = \varepsilon_2 = \varepsilon$, $\varepsilon_3 = \varepsilon$, $J_{12} = J_0 > 0$, $J_{13} = J_{23} = J$, and an identical environment at every site. Under these assumptions, we can obtain the following three eigenstates of $H_S$:

$$|\lambda_1\rangle = \frac{|2\rangle - |1\rangle}{\sqrt{2}}, \quad |\lambda_{2,3}\rangle = \alpha_\pm |1\rangle + \alpha_\pm |2\rangle \pm \sqrt{2}\alpha_\mp |3\rangle, \quad (3)$$

where the two superposition parameters $\alpha_\pm$ are given by

$$\alpha_\pm = \frac{1}{2}\left[1 \pm \frac{\Delta\varepsilon + J_0}{\sqrt{(\Delta\varepsilon + J_0)^2 + 8J^2}}\right]^{1/2}, \quad (4)$$

and the site-energy detuning between a donor and an acceptor $\Delta\varepsilon$ is defined as $\Delta\varepsilon = \varepsilon - \varepsilon$. The corresponding eigenergies are given by

$$\lambda_1 = \varepsilon - J_0, \quad \lambda_{2,3} = \frac{1}{2}\left[\varepsilon + \varepsilon + J_0 \pm \sqrt{(\Delta\varepsilon + J_0)^2 + 8J^2}\right]. \quad (5)$$

We now consider the ground-state structure of the trimer system, which depends on the energy-level structure of the system. In order to do so, we introduce a critical parameter $\Delta\varepsilon_c = (J_0^2 - J^2)/J_0$. From Eqs. (3) and (5), we can see that $|\lambda_2\rangle$ is the eigenstate with the highest energy level. When $\Delta\varepsilon > \Delta\varepsilon_c$, we have $\lambda_1 > \lambda_3$, which implies that the eigenstate $|\lambda_3\rangle$ is the ground state of the trimer system described by $H_S$. When $\Delta\varepsilon < \Delta\varepsilon_c$, we have $\lambda_1 < \lambda_3$, which implies that the eigenstate $|\lambda_1\rangle$ is the ground state of the trimer system. When $\Delta\varepsilon = \Delta\varepsilon_c$, we have $\lambda_1 = \lambda_3$, which implies that $|\lambda_1\rangle$ and $|\lambda_3\rangle$ are two degenerate ground states of the trimer system. Hence, there exists a QPT in the trimer system under consideration, and the site-energy detuning $\Delta\varepsilon$ is a parameter controlling the QPT. The $\Delta\varepsilon = \Delta\varepsilon_c$ is the critical point of the QPT.

In Eq. (1), $H_B$ is the Hamiltonian of the heat baths of the system, which can be modeled as a harmonic-oscillator heat bath with the following Hamiltonian:

$$H_B = \sum_{m,j} \omega_{m,j} b^\dagger_{m,j} b_{m,j}, \quad (6)$$

where the sum is over all phonon modes at each site, and $b_{m,j}$ ($b^\dagger_{m,j}$) is the annihilation (creation) operator of the $j$-th harmonic oscillator with frequency $\omega_j$ of the environment for the $m$-th chromophore.

In Eq. (1), $H_{SB}$ describes the dephasing interaction between the trimer system and the heat baths, it reads

$$H_{SB} = \sum_{m,j} g_{m,j}(b^\dagger_{m,j} + b_{m,j}) |m\rangle \langle m|, \quad (7)$$

where each site is separately interacting with a set of phonon modes indicated by sub-index $m$, and $g_{m,j}$ describes the coupling strength of the $m$-th site to its environment. In this form, there is no energy exchange between the system and the heat baths.

In the zero-temperature limit, the state evolution of the trimer in the Born–Markov approximation can be described by the quantum master equation[13,23]

$$\begin{aligned}\dot{\rho}_S &= -i[H_S, \rho_S] \\ &\quad + \frac{1}{2}\sum_{(i,j)} [\Gamma_{ij}(2\sigma_{ji}\rho_S\sigma_{ij} - \sigma_{ii}\rho_S - \rho_S\sigma_{ii}) \\ &\quad + \Gamma_{ji}(2\sigma_{ij}\rho_S\sigma_{ji} - \sigma_{jj}\rho_S - \rho_S\sigma_{jj})], \end{aligned} \quad (8)$$

where $\sigma_{nm}$ $(m,n = 1,2,3)$ are the project operators in the eigenstate-representation defined by $\sigma_{nm} \equiv |\lambda_n\rangle \langle \lambda_m|$, and the summation parameters $(i,j)$ in Eq. (8) can take $(i,j) = (1,2),(1,3),(2,3)$. The effective decoherence rates $\Gamma_{ij}$ can be expressed in terms of the decoherence rate as

$$\begin{aligned} \Gamma_{12} &= |\alpha_+|^2\gamma(\omega_{12}), & \Gamma_{21} &= |\alpha_+|^2\gamma(\omega_{21}), \\ \Gamma_{13} &= |\alpha_-|^2\gamma(\omega_{13}), & \Gamma_{31} &= |\alpha_-|^2\gamma(\omega_{31}), \\ \Gamma_{23} &= 6|\alpha_+\alpha_-|^2\gamma(\omega_{23}), & \Gamma_{32} &= 6|\alpha_+\alpha_-|^2\gamma(\omega_{32}), \end{aligned} \quad (9)$$

where $\omega_{ij} = \lambda_i - \lambda_j$, and the decoherence rate $\gamma(\omega) = 2\pi J(\omega)$. Here, we assume an Ohmic spectral density with $J(\omega) = 0$ for $\omega < 0$ and $J(\omega) = (\lambda_R/\omega_c)\omega\exp(-\omega/\omega_c)$ elsewhere, where $\omega_c$ is the cutoff frequency, and $\lambda_R$ is the strength of the system–bath coupling.

Furthermore, from Eq. (9) we have $\Gamma_{ij} = 0$ if $\omega_{ij} < 0$. Thus the solutions of the Eq. (8) may be different according to two different regimes of the site-energy detuning: $\Delta\varepsilon > \Delta\varepsilon_c$ and $\Delta\varepsilon \le \Delta\varepsilon_c$, which correspond to the site-energy detuning $\Delta\varepsilon$ being in the right-hand-side and the left-hand-side regimes of the critical point of the QPT, respectively.

## 3. Exciton transfer dynamics

In this section, we will investigate exciton transfer dynamics by calculating the ETP in the trimer system for two different regimes of site-energy detuning ($\Delta\varepsilon > \Delta\varepsilon_c$ and $\Delta\varepsilon < \Delta\varepsilon_c$) respectively.

We assume that at time $t = 0$ the excitation is initially in the donor site D. Then we have $\rho_{33}(0) = 1$. In the eigenstate representation of $H_S$, we have $\langle\sigma_{11}(0)\rangle = 0, \langle\sigma_{22}(0)\rangle = 2|\beta|^2, \langle\sigma_{33}(0)\rangle = 2|\alpha|^2$, and $\langle\sigma_{23}(0)\rangle = -2|\alpha\beta|$, where $\langle\sigma_{mn}(t)\rangle = \mathrm{Tr}_S[\rho_S(t)\sigma_{mn}]$. The ETP from the donor site D to acceptor sites $A_1$ and site $A_2$ is given by

$$P_{3\to(1,2)}(t) = 1 - \rho_{33}(t), \tag{10}$$

where $\rho_{33}(t)$ is the population of the exciton at the donor site D, and can be expressed in the eigenstate representation as

$$\begin{aligned}\rho_{33}(t) &= 2|\beta|^2\langle\sigma_{22}(t)\rangle + 2|\alpha|^2\langle\sigma_{33}(t)\rangle \\ &\quad -4|\alpha\beta|\mathrm{Re}[\langle\sigma_{23}(t)\rangle].\end{aligned} \tag{11}$$

### 3.1. ETP in the regime of $\Delta\varepsilon > \Delta\varepsilon_c$

In this case, the site-energy detuning takes its values in the right-hand-side regime of the QPT point $\Delta\varepsilon = \Delta\varepsilon_c$, and $|\lambda_3\rangle$ is the ground state with $\lambda_2 > \lambda_1 > \lambda_3$. The solutions of Eq. (8) for elements $\langle\sigma_{mn}(t)\rangle$ can be obtained as

$$\begin{aligned}\langle\sigma_{11}(t)\rangle &= \mathrm{e}^{-\Gamma_{13}t}\langle\sigma_{11}(0)\rangle + \mathscr{A}(t)\langle\sigma_{22}(0)\rangle, \\ \langle\sigma_{22}(t)\rangle &= \mathrm{e}^{-(\Gamma_{21}+\Gamma_{23})t}\langle\sigma_{22}(0)\rangle, \\ \langle\sigma_{33}(t)\rangle &= \mathscr{B}(t)\langle\sigma_{11}(0)\rangle + \mathscr{C}(t)\langle\sigma_{22}(0)\rangle + \langle\sigma_{33}(0)\rangle, \\ \langle\sigma_{32}(t)\rangle &= \mathrm{e}^{-\mathrm{i}\omega_{32}t}\mathrm{e}^{-\frac{1}{2}[\Gamma_{21}+\Gamma_{23}]t}\langle\sigma_{32}(0)\rangle,\end{aligned} \tag{12}$$

where the three time-dependent coefficients are given by

$$\begin{aligned}\mathscr{A}(t) &= \frac{\Gamma_{21}}{\Gamma_{21}+\Gamma_{23}-\Gamma_{13}}\left[\mathrm{e}^{-\Gamma_{13}t} - \mathrm{e}^{-(\Gamma_{21}+\Gamma_{23})t}\right], \\ \mathscr{B}(t) &= 1 - \mathrm{e}^{-\Gamma_{13}t}, \\ \mathscr{C}(t) &= 1 - \frac{\Gamma_{21}\mathrm{e}^{-\Gamma_{13}t} + (\Gamma_{23}-\Gamma_{13})\mathrm{e}^{-(\Gamma_{21}+\Gamma_{23})t}}{\Gamma_{21}+\Gamma_{23}-\Gamma_{13}}.\end{aligned} \tag{13}$$

Substituting the above solutions into Eq. (10), from Eq. (11) we can obtain the transition probability

$$\begin{aligned}P_{3\to(1,2)}(t) &= 1 - 2|\alpha_+|^2 + \frac{4|\alpha_+\alpha_-|^2\Gamma_{21}}{\Gamma_{21}+\Gamma_{23}-\Gamma_{13}}\mathrm{e}^{-\Gamma_{13}t} \\ &\quad -4|\alpha_-|^2\left[|\alpha_-|^2 + \frac{|\alpha_+|^2(\Gamma_{13}-\Gamma_{23})}{\Gamma_{21}+\Gamma_{23}-\Gamma_{13}}\right]\mathrm{e}^{-(\Gamma_{21}+\Gamma_{23})t} \\ &\quad -8|\alpha_+\alpha_-|^2\cos(\omega_{23}t)\,\mathrm{e}^{-\frac{1}{2}(\Gamma_{21}+\Gamma_{23})t}.\end{aligned} \tag{14}$$

We now consider the long-time behaviors of transition probability for the trimer system in the pure dephasing case. When the evolution time approaches infinity, from Eqs. (4) and (14), we have

$$P_{3\to(1,2)}(\infty) = \frac{1}{2} - \frac{\Delta\varepsilon + J_0}{\sqrt{(\Delta\varepsilon+J_0)^2 + 8J^2}}, \tag{15}$$

which implies that there may exist a nonzero stable ETP that depends on the detuning $\Delta\varepsilon$ and the site-site couplings $J_0$ and $J$.

### 3.2. ETP in the regime of $\Delta\varepsilon < \Delta\varepsilon_c$

In this case, the ground state of the trimer system is $|\lambda_1\rangle$ due to $\lambda_2 > \lambda_3 > \lambda_1$, and the solutions of Eq. (8) for elements $\langle\sigma_{mn}(t)\rangle$ are

$$\begin{aligned}\langle\sigma_{11}(t)\rangle &= \langle\sigma_{11}(0)\rangle + \mathscr{D}(t)\langle\sigma_{22}(0)\rangle + \mathscr{E}(t)\langle\sigma_{33}(0)\rangle, \\ \langle\sigma_{22}(t)\rangle &= \mathrm{e}^{-(\Gamma_{21}+\Gamma_{23})t}\langle\sigma_{22}(0)\rangle, \\ \langle\sigma_{33}(t)\rangle &= \mathrm{e}^{-\Gamma_{31}t}\langle\sigma_{33}(0)\rangle + \mathscr{F}(t)\langle\sigma_{22}(0)\rangle, \\ \langle\sigma_{32}(t)\rangle &= \mathrm{e}^{-i\omega_{32}t}\mathrm{e}^{-\frac{1}{2}[\Gamma_{21}+\Gamma_{23}+\Gamma_{31}]t}\langle\sigma_{32}(0)\rangle,\end{aligned} \tag{16}$$

where the three time-dependent coefficients are given by

$$\begin{aligned}\mathscr{D}(t) &= 1 - \frac{\Gamma_{23}\mathrm{e}^{-\Gamma_{31}t} + (\Gamma_{21}-\Gamma_{31})\mathrm{e}^{-(\Gamma_{21}+\Gamma_{23})t}}{\Gamma_{21}+\Gamma_{23}-\Gamma_{31}}, \\ \mathscr{E}(t) &= 1 - \mathrm{e}^{-\Gamma_{31}t}, \\ \mathscr{F}(t) &= \frac{\Gamma_{23}}{\Gamma_{21}+\Gamma_{23}-\Gamma_{31}}\left[\mathrm{e}^{-\Gamma_{31}t} - \mathrm{e}^{-(\Gamma_{21}+\Gamma_{23})t}\right].\end{aligned} \tag{17}$$

Through some simple calculation, it is easy to see that the probability of transition is given by

$$\begin{aligned}P_{3\to(1,2)}(t) &= 1 - 4|\alpha_+|^2\left[|\alpha_+|^2 + \frac{|\alpha_-|^2\Gamma_{23}}{\Gamma_{21}+\Gamma_{23}-\Gamma_{31}}\right]\mathrm{e}^{-\Gamma_{31}t} \\ &\quad -4|\alpha_-|^2\left[|\alpha_-|^2 + \frac{|\alpha_+|^2\Gamma_{23}}{\Gamma_{21}+\Gamma_{23}-\Gamma_{31}}\right]\mathrm{e}^{-(\Gamma_{21}+\Gamma_{23})t} \\ &\quad -8|\alpha_+\alpha_-|^2\cos(\omega_{23}t)\,\mathrm{e}^{-\frac{1}{2}(\Gamma_{21}+\Gamma_{23}+\Gamma_{31})t}.\end{aligned} \tag{18}$$

The long time steady-state transition probability is obtained as

$$P_{3\to(1,2)}(\infty) = 1, \tag{19}$$

which implies that in this regime the environment can assist the excitation energy transport, and induces the probability of transition to a maximum steady-state value.

We can show that the steady-state excitation transfer does not occur if there does not exist the environment dephasing interaction. In fact, without the environment dephasing interaction ($\Gamma_{ij} = 0$), the ETP can be obtained as

$$\begin{aligned}P_{3\to(1,2)}(t) &= 1 - 4[|\alpha_+|^4 + |\alpha_-|^4 \\ &\quad +2|\alpha_+\alpha_-|^2\cos(\omega_{23}t)].\end{aligned} \tag{20}$$

This shows a periodic oscillation with respect to time, and is a continuous function for the full regime of site-energy detuning $\Delta\varepsilon$, hence a steady-state excitation transfer cannot be reached. The transient maximal ETP is given by

$$\mathrm{Max}[P_{3\to(1,2)}(t)] = \frac{8J^2}{(\Delta\varepsilon+J_0)^2 + 8J^2}, \tag{21}$$

which means that only in the case $\Delta\varepsilon = -J_0$, we can say that the excitation is transferred with certainty $[P_{3\to(1,2)}(t) = 1]$ at the time moment $t = (2k+1)\pi/\omega_{23}$, $(k = 0, 1, \ldots, n)$.

In order to further understand the phenomenon of dephasing-assisted excitation transfer in the trimer system, in Fig. 2 we plot the ETP as a function of time and the QPT controlling parameter, i.e., the site-energy detuning $\Delta\varepsilon$. Figures 2(a) and 2(b) show the results with and without the environment dephasing interaction, respectively. In both cases, we have chosen parameters $J_0 = 100\ \mathrm{cm}^{-1}$, $J = 50\ \mathrm{cm}^{-1}$, and $\omega_c = 50\ \mathrm{cm}^{-1}$. The critical value of the site-energy detuning at the QPT point is $\Delta\varepsilon_c = 75\ \mathrm{cm}^{-1}$. The dephasing parameter is $\lambda_R = 20\ \mathrm{cm}^{-1}$ in Fig. 2(a), while $\Gamma_{ij} = 0$ in Fig. 2(b).

Figure 2(a) indicates that in the dephasing case a sudden change of ETP occurs at the quantum critical point ($\Delta\varepsilon_c = 75\ \mathrm{cm}^{-1}$) in the trimer system in the whole time evolution process. Additionally, in the regime of $\Delta\varepsilon < \Delta\varepsilon_c$, the environment-assisted exciton transport enhancement phenomena can be clearly observed, while in the regime of $\Delta\varepsilon > \Delta\varepsilon_c$, the probability of transition is suppressed. In particular, it can be seen that a steady-state ETP is induced by the environment dephasing interaction.

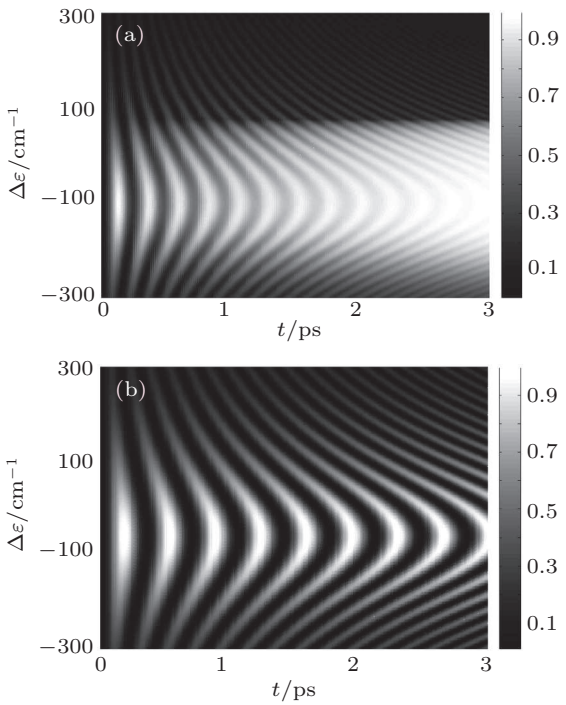



**Fig. 2.** Probability of transition $P_{3\to(1,2)}(t)$ in the trimer system as a function of time $t$ and site-energy detuning $\Delta\varepsilon$ in the low temperature limit case. Panel (a) shows the result in the presence of only the environment dephasing with $\lambda_R = 20\ \mathrm{cm}^{-1}$. Panel (b) shows the result in the absence of environment dephasing. The other parameters are chosen as $J_0 = 100\ \mathrm{cm}^{-1}$, $J = 50\ \mathrm{cm}^{-1}$, $\omega_c = 50\ \mathrm{cm}^{-1}$. Here the critical value of the site-energy detuning at the QPT point is $\Delta\varepsilon_c = 75\ \mathrm{cm}^{-1}$, and we have used the rate $1\ \mathrm{ps}^{-1} \equiv 5.3\ \mathrm{cm}^{-1}$ in units of $\hbar = 1$ and $2\pi c = 1$ with $c$ being the light speed in vacuum.

## 4. Conclusion

In conclusion, we have investigated exciton transfer dynamics in a trimer system and the influence of QPT on the excitation transfer probability. It is shown that the ETP exhibits a sudden change at the quantum critical point in the presence of the environment dephasing interaction. This is an environment-assisted quantum critical effect induced by the dephasing environment since the quantum critical effect does not appear without the dephasing environment interaction. We have also shown that a steady-state ETP can be generated in the presence of environment dephasing interaction. Finally, it should be pointed out that the trimer model is a simple toy model, but the revealed environment-assisted QCE and physical mechanisms of the ETP enhancement obtained from this model may provide insights into the working principle of natural and artificial light-harvesting systems. These promising results would motivate further study of energy transfer for more complex light-harvesting models and environments.

## References

[1] Scholes G D and Fleming G R 2006 *Adven. Chem. Phys.* **132** 57
[2] Benniston A C and Harriman A 2008 *Mater. Today* **11** 26
[3] Fleming G R and Ratner M A 2008 *Phys. Today* **61** 28
[4] Ghosh P K, Smirnov A Y and Nori F 2011 *J. Chem. Phys.* **134** 244103
[5] Scholes G D 2003 *Annu. Rev. Phys. Chem.* **54** 57
[6] Jang S, Newton M D and Silbey R J 2007 *J. Phys. Chem.* B **111** 6807
[7] Cheng Y C and Fleming G R 2009 *Annu. Rev. Phys. Chem.* **60** 2417
[8] van Grondelle R and Novoderezhkin V I 2006 *Phys. Chem. Chem. Phys.* **8** 793
[9] Wu J, Liu F, Shen Y, Cao J and Silbey R J 2010 *New J. Phys.* **12** 105012
[10] Sarovar M, Cheng Y C and Whaley K B 2011 *Phys. Rev.* E **83** 011906
[11] Novoderezhkin V I and Grondelle R V 2010 *Phys. Chem. Chem. Phys.* **12** 7352
[12] Sinayskiy I, Marais A, Petruccione F and Ekert A 2012 *Phys. Rev. Lett.* **108** 020602
[13] Rebentrost P, Chakraborty R and Aspuru-Guzik A 2009 *J. Chem. Phys.* **131** 184102
[14] Liao J Q, Huang J F, Kuang L M and Sun C P 2010 *Phys. Rev.* A **82** 052109
[15] McCutcheon D P S and Nazir A 2011 *Phys. Rev.* B **83** 165101
[16] Singh N and Brumer P 2011 arXiv: 1106.5911v1
[17] Pachón L A and Brumer P 2011 arXiv: 1107.0322v1
[18] Nazir A 2009 *Phys. Rev. Lett.* **103** 146404
[19] Mülken O and Schmid T 2010 *Phys. Rev.* E **82** 042104
[20] Sachdev S 1999 *Quantum Phase Transitions* (Cambridge: Cambridge University Press)
[21] Quan H T, Song Z, Liu X F, Zanardi P and Sun C P 2006 *Phys. Rev. Lett.* **96** 140604
[22] Yang S, Xu D Z and Sun C P 2010 *J. Chem. Phys.* **132** 234501
[23] Breuer H P and Petruccione F 2002 *The Theory of Open Quantum Systems* (Oxford: Oxford University Press)