Response of plasma turbulence against externally-controlled perturbations*

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A theory of the dynamic response of the turbulent plasma against the externally-controlled perturbations is reported. Based on the Mori's method [Prog. Theor. Phys. **33** 423 (1965)], the nonlinear force is assumed to be separated into the memory function and the nonlinear fluctuating force. The former corresponds to the damping term, and the latter is categorized into the noise term. The response of the turbulent plasma against the externally-controlled source is formulated. The response kernel, which connects the externally-controlled source and the response of the turbulent field, is shown to have both the nonlocal property (in space) and the non-Markovian response (in time). A discussion is made on the nonlocal and non-Markovian response, including the case of disparate-scale interactions. A new method is proposed to observe experimentally the nonlocal interaction in the drift wave turbulence via the zonal flows.

Keywords: turbulent transport, nonlocal and non-Markovian response

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1. Introduction

Physics of turbulence in the magnetized plasmas has shown remarkable progresses recently. The new physics pictures of turbulence, such as the excitation of fluctuations via nonlinear mechanisms (subcritical instabilities, zonal flows, etc.), the co-existence of fluctuations of multiple scales, and statistical and stochastic properties (stochastic transition, finite life time, probability density functions of physical quantities, etc.), are widely recognized to be important.[1−7] So far, emphases have been placed upon understanding the structure formation in real space.

Those progresses have been led by theories, and the experimental progress is also noticeable. As a result of theoretical prediction and simulational guidance, the direct experimental measurement of the nonlinear processes in the plasma turbulence has been in progress and flourished. Examples include the excitation of zonal flows and the nonlinear interaction of drift wave fluctuations.[8−19] The nonlocality in space in the transport process has just been pointed out.[2,4,20−24] However, the understanding of the stationary structure is not enough, and the quantitative observation and analyses, which may lead to the induction of the law of dynamic evolution, are important.

In the study of turbulent transport, the importance of dynamic behavior is widely recognized, and the important role of memory (the non-Markovian property) has been pointed out.[3,4] Experiments on the dynamic response of plasma transport have illuminated many challenging problems.[25−29] In the study of relaxation processes, the observation of the dynamic response against the externally controlled perturbation has been one of the standard methods of analyses.[30] In plasma turbulence, many basic quantities must be studied. For example, the time scales include the decorrelation rate (determined by the turbulence), the memory function of the test field, the source of incoherent excitation for the test field, and so on. There are other essential time scales, which govern the physics, and they also must be measured. It is expected that studying the dynamic response of turbulent plasma against the externally-controlled perturbations will further advance the study of plasma

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turbulence. Devoted experiments along this line of thought have been developed with the use of basic plasma experimental devices. The chaos control and synchronization of drift waves (i.e., the physics of small degrees of freedom) have been reported on various devices,[31−33] and the experimental study of the response in turbulence has emerged.[34,35] An explicit formula, which describes the response of plasma turbulence against the external field, is desirable. In addition, the dynamical response in coupled macro-micro systems has attracted much attention, and the propagating wave eigenmode in the diffusion reaction system has been pointed out.[36]

In this article, we report a theoretical approach to study the response of turbulent plasma against the prescribed externally-controlled perturbations. The statistical approach is based on the Mori's method[37] of nonlinear theory to formulate the stochastic equations of variables. According to the Mori's method, the course-grained dynamical equation for the mode of interest can be derived, in which the nonlinear force is separated into the memory function and the nonlinear fluctuating force. The former corresponds to the damping term, and the latter is categorized into the noise. In the preceding literature,[38] theories have been proposed to derive an approximate inter relationship of formalisms of memory functions and the fluctuating force for the plasma turbulence. In this article, the response of turbulent plasma against the externally controlled source is formulated using the model formula for the memory function and the random fluctuating force. It is shown that the response kernel, which connects the source and the response of turbulent field, has both the nonlocal property in space and the Non-Markovian response in time. A discussion is made about the nonlocal and non-Markovian response. A case of disparate-scale interaction is also discussed for the drift wave turbulence–zonal flow system. The dynamic response against an external oscillating source for the zonal flows is studied. The oscillation of the envelope of the drift wave turbulence has a spatially-nonuniform part, which oscillates at different phases (seesaw oscillation) compared to the spatially-uniform part. This provides an experimental method to observe the nonlocal interaction in the drift wave turbulence via the zonal flows, which has been predicted theoretically.[39−41] This report describes the theoretical framework, and provides a basis for the forthcoming applications.

2. Model equation for plasma turbulence

We consider the magnetized slab plasma, which is inhomogeneous in the x direction. Cartesian coordinates (x, y, z) are used. The magnetic field is given as $\boldsymbol{B} = B_0\,(0, sx, 1)$ with $B_0\,(x) = \left(1 + L_B^{-1}x + \cdots\right) B_0$, where L_B denotes the inhomogeneity of the magnetic field strength. In this system, a microscopic turbulence can be sustained. To describe the system, the reduced set of equations for electrostatic potential ϕ, current J, and pressure p is employed. This choice is for illustration, and does not influence the arguments in this article. The global plasma inhomogeneity and the turbulent fluctuations are separated as $\phi = \phi_0 + \tilde{\phi}$, $J = J_0 + \tilde{J}$, and $p = p_0 + \tilde{p}$, where the mean values with suffix 0 (ϕ_0, J_0, and p_0) are defined by the long time averages (i.e., the averaging time is much longer than the characteristic decorrelation time of fluctuations of interest). Parameters $\boldsymbol{\nabla} p_0$ and $\boldsymbol{\nabla}_\perp^2 \phi_0$ represent the inhomogeneity of the system and the potential free energy source to be released. The dynamical nonlinear equation of fluctuations is given by

$$\frac{\partial}{\partial t}\boldsymbol{f} + \mathcal{L}^{(0)}\boldsymbol{f} = \mathscr{N}\left(\boldsymbol{f}\right) + \boldsymbol{S}, \tag{1}$$

where $\mathscr{L}^{(0)}$ denotes the linear operator, $\boldsymbol{f}$ denotes the fluctuating fields with symbol $\tilde{}$ being suppressed

$$\boldsymbol{f}^{\mathrm{T}} = (\phi, J, p)\,, \tag{2}$$

and $\mathscr{N}\left(\boldsymbol{f}\right)$ stands for the nonlinear terms, $\mathscr{N}\left(\boldsymbol{f}\right)^{\mathrm{T}} = -\left(\boldsymbol{\nabla}_\perp^{-2}\left[\phi, \boldsymbol{\nabla}_\perp^2\phi\right], [\phi, J], [\phi, p]\right)$, and $\boldsymbol{S}$ indicates the external source, whose phase and amplitude are controlled externally. In the nonlinear operator, the bracket $[f, g]$ denotes the Poisson bracket, $[f, g] = (\boldsymbol{\nabla} f \times \boldsymbol{\nabla} g)\cdot\boldsymbol{b}, (\boldsymbol{b} = \boldsymbol{B}_0/B_0)$. Details of the basic dynamical equations have been explained in Refs. [38] and [42], and are not repeated here.

We introduce the Fourier representation in space

$$\boldsymbol{f} = \sum_k \boldsymbol{f}_k\,(\boldsymbol{x}, t)\,, \quad \boldsymbol{S} = \sum_k \boldsymbol{S}_k\,(\boldsymbol{x}, t)\,,$$

where the Fourier components $\boldsymbol{f}_k\,(\boldsymbol{x}, t)$ and $\boldsymbol{S}_k\,(\boldsymbol{x}, t)$ have the dependence $\exp\left(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}\right)$. In terms of the Fourier components, equation (1) is rewritten as

$$\begin{aligned} &\partial\boldsymbol{f}_k/\partial t + \mathscr{L}_k^{(0)}\boldsymbol{f}_k = \mathscr{N}_k + \boldsymbol{S}_k, \\ &\mathscr{N}_k \equiv \sum_{k=p+q} \boldsymbol{M}_{kpq}\boldsymbol{f}_p\boldsymbol{f}_q, \end{aligned} \tag{3}$$

where the explicit form of the nonlinear interaction matrix is given by

$$\boldsymbol{M}_{1,kpq} = (\boldsymbol{p}\times\boldsymbol{q})\cdot\boldsymbol{b}\frac{\left(p_\perp^2 - q_\perp^2\right)}{k_\perp^2},$$
$$\boldsymbol{M}_{(2,3),kpq} = (\boldsymbol{p}\times\boldsymbol{q})\cdot\boldsymbol{b}. \tag{4}$$

Depending on the plasma profiles, the global parameters such as $\mathrm{d}p_0/\mathrm{d}x$ in operator $\mathscr{L}^{(0)}$ may not be constants, but depend on x. In order to analyze the case with general profiles, the two-scale approach must be employed.[43] In this approach of a hierarchical system of equations, the expansion parameter is $1/kL$, where L is the scale length of global inhomogeneity. Equation (3) corresponds to the lowest order equation in this formalism. We choose this simplification in order to examine the dressed-test-mode method from the viewpoint of the Mori's method.

In the study of nonlinear equation, Mori has shown a methodology[37] to deduce the nonlinear interaction term for the memory function $\Gamma_k(s)$ under the condition that the long-term average exists, and the projection operator $\mathscr{P}$ can be defined as

$$\mathscr{P}\boldsymbol{Y} = \left\langle \boldsymbol{Y}(t)\boldsymbol{f}^\dagger(0)\right\rangle \left\langle \boldsymbol{f}(0)\,\boldsymbol{f}^\dagger(0)\right\rangle^{-1}\boldsymbol{f}(0), \tag{5a}$$

where the long time average is defined as

$$\left\langle A_l(t)A_m^\dagger(0)\right\rangle \equiv \lim_{T\to\infty}\frac{1}{T}\int_0^T \mathrm{d}s\, A_l(t+s)\,A_m^\dagger(s). \tag{5b}$$

If the above condition is satisfied, the nonlinear term can be decomposed into a memory function, a frequency shift, and a random force, and equation (3) can be rewritten as

$$\mathscr{N}_k = \mathrm{i}\Omega_k\boldsymbol{f}_k + \mathscr{R}_k(t) - \int_0^t \mathrm{d}s\,\Gamma_k(s)\boldsymbol{f}_k(t-s), \tag{6}$$

where Ω_k is the nonlinear frequency shift, and $\mathscr{R}_k(t)$ is a rapidly-changing fluctuating force term

$$\partial\boldsymbol{f}_k/\partial t + \mathscr{L}_k^{(0)}\boldsymbol{f}_k - \mathrm{i}\Omega_k\boldsymbol{f}_k + \int_0^t \mathrm{d}s\,\Gamma_k(s)\boldsymbol{f}_k(t-s) = \mathscr{R}_k(t) + \boldsymbol{S}_k. \tag{7}$$

Equation (7) can be rewritten as

$$\partial\boldsymbol{f}_k/\partial t + \boldsymbol{H}_k\boldsymbol{f}_k = \mathscr{R}_k(t) + \boldsymbol{S}_k, \tag{8a}$$

with

$$\boldsymbol{H}_k\boldsymbol{f}_k \equiv \mathscr{L}_k^{(0)}\boldsymbol{f}_k - \mathrm{i}\Omega_k\boldsymbol{f}_k + \int_0^t \mathrm{d}s\,\Gamma_k(s)\boldsymbol{f}_k(t-s). \tag{8b}$$

In Eq. (8a), the term $\boldsymbol{H}_k\boldsymbol{f}_k$ is apparently linear with respect to the field variable $\boldsymbol{f}_k$, and equation (8a) describes the response of $\boldsymbol{f}_k$ against the source $\boldsymbol{S}_k$ and the fluctuating force term $\mathscr{R}_k(t)$, which includes the effects from the ensemble of fluctuating components.

It might be useful to revisit the fundamental condition under which the Mori's method (separation of memory function from random force) can be applied to this problem. The principal condition is that the long time average (Eq. (5b)) exists. In the present analysis, we assume that the dynamical but stationary state of turbulence exists, and we study its response. Under this condition, the long-time average exists, and the Mori's method works. The magnitude of the memory function is related to the rate of decorrelation of the test mode, and the width of function $\Gamma(s)$ is related with the length of memory in the non-Markovian response. These relations are explained in Ref. [44].

It should also be noted that the terms $\boldsymbol{H}$ in the left hand side (LHS) of Eq. (8a) and $\mathscr{R}$ in the right hand side (RHS) of Eq. (8a) are of internal origin, while the term $\boldsymbol{S}$ in the RHS of Eq. (8a) is the contribution of the external source. The discrimination of the nonlinear term and the other source terms has been discussed in Ref. [45]. The essential point is that the phase of the term $\boldsymbol{H}\boldsymbol{f}$ (in the LHS) is controlled by the phase of the test mode $\boldsymbol{f}$, while that of $\boldsymbol{S}$ is prescribed by the external source.

3. Response to external source

3.1. Green's function

We next introduce a Green's function

$$\frac{\partial}{\partial t}g_k(t,t') + \boldsymbol{H}_k(t)g_k(t,t') = \delta(t-t'), \tag{9}$$

with $g_k(t,t) = 1$, where $\delta(t-t')$ is the Dirac's delta function. By using $g_k(t,t')$, the solution of Eq. (8a) is formally given as

$$\boldsymbol{f}_k(t) = \boldsymbol{\zeta}_k(t) + \int_0^t \mathrm{d}t'\,g_k(t,t')\,\boldsymbol{S}_k(t'), \tag{10a}$$

with

$$\boldsymbol{\zeta}_k(t) = g_k(t,0)\,\boldsymbol{f}_k(0) + \int_0^t \mathrm{d}t'\,g_k(t,t')\,\mathscr{R}_k(t'), \tag{10b}$$

where $\boldsymbol{k}_2 = \boldsymbol{k} - \boldsymbol{k}_1$, and $\boldsymbol{\zeta}_k(t)$ represents the response against the fluctuating force terms, which is originated from $\mathscr{N}_k$.

The Green's function has been explained in terms of the fluctuating fields. The result is briefly reproduced here. Details can be found in Ref. [38]. We

introduce the correlation function of the fluctuation field

$$\langle \boldsymbol{f}_k(t)\boldsymbol{f}_{-k}(t')\rangle = C_k(t-t'). \tag{11}$$

The Laplace transformation of the Green's function

$$G_k(p) = \int_0^\infty \mathrm{d}\tau \, \exp(-p\tau)\, g_k(\tau) \tag{12}$$

is given by the recurrence formula

$$G_k(p) = \frac{1}{p + \mathscr{L}_k^{(0)} - \sum{}' 2\boldsymbol{M}_{k,k_1,k_2}\boldsymbol{M}_{k_2,k,-k_1}U_{k_1,k_2}(p)}, \tag{13}$$

where $U_{k_1,k_2}(p)$ is the Laplace transform of function $g_{k_2}(s)C_{k_1}(s)$. This form is given for the simplified case with $\Omega_k = 0$. The RHS of Eq. (13) is a symbolic expression of the operator

$$\left[p + \mathscr{L}_k^{(0)} - \sum{}' 2\boldsymbol{M}_{k,k_1,k_2}\boldsymbol{M}_{k_2,k,-k_1}U_{k_1,k_2}(p)\right]^{-1}.$$

As has been discussed in Ref. [38], a simpler expression can be obtained if the strongly-damped eigenmodes are much weakly excited in comparison with the least stable eigenfunction

$$\left[p + \mathscr{L}_k^{(0)} - \sum{}' 2\boldsymbol{M}_{k,k_1,k_2}\boldsymbol{M}_{k_2,k,-k_1}U_{k_1,k_2}(p)\right]^{-1} = \frac{1}{p + \lambda_k - \sum{}' 2\boldsymbol{M}_{k,k_1,k_2}\boldsymbol{M}_{k_2,k,-k_1}U_{k_1,k_2}(p)}. \tag{14}$$

If we employ the assumption that the decay of the Green's function is faster than that of the correlation function, the term $U_{k_1,k_2}(p)$ in Eq. (13) is given by $U_{k_1,k_2}(p) = C_{k_1}G_{k_2}(p)$, and equation (13) then takes the form

$$G_k(p) = \frac{1}{p + \lambda_k - \sum{}' 2\boldsymbol{M}_{k,k_1,k_2}\boldsymbol{M}_{k_2,k,-k_1}C_{k_1}G_{k_2}(p)}. \tag{15}$$

Substituting the expression of G_{k_n} into Eq. (15), we obtain

$$G_k(p) = \cfrac{1}{p_k + \sum{}' V_{k,1,2}\left(\cfrac{1}{p_2 + \sum{}' V_{2,3,4}\left(\cfrac{1}{p_4 + \sum{}' V_{4,5,6}\left(\cfrac{1}{p_6 + \cdots}\right)}\right)}\right)}, \tag{16}$$

where $(k,1,2)$ is an abbreviation of (k,k_1,k_2), suffices $(2,3,4,\ldots)$ indicate $(k_2,k_3,k_4,\ldots)$, and

$$p_n = p + \lambda_{k_n}^{(0)},$$
$$V_{k,k_1,k_2} \equiv -2\boldsymbol{M}_{k,k_1,k_2}\boldsymbol{M}_{k_2,k,-k_1}C_{k_1}(s=0). \tag{17}$$

The Laplace transform of the memory function is given under the assumption that is used for deriving Eq. (15) as

$$\bar{\Gamma}_k(p) = \sum{}' V_{k,k_1,k_2}G_{k_2}(p). \tag{18}$$

Similarly, the continued fraction expression is obtained as

$$\bar{\Gamma}_k(p) = \sum{}' \cfrac{V_{k,1,2}}{p_2 + \sum{}' V_{2,3,4}\left(\cfrac{1}{p_4 + \sum{}' V_{4,5,6}\left(\cfrac{1}{p_6 + \cdots}\right)}\right)}. \tag{19}$$

These relations form the set of equations that describes the response of $\boldsymbol{f}_k$ against the external source $\boldsymbol{S}_k$. The Green's function and the memory function are given if the correlation functions of fluctuations are given by Eqs. (16) and (17). Once the Green's function and the memory function are given, the coherent part of $\boldsymbol{f}_k$ against the imposition of $\boldsymbol{S}_k$ is calculated by using Eq. (10). The response of turbulent plasma against the external source is analyzed.

3.2. Nonlocal and non-Markovian response

The response relation equation (10a) is formulated into the nonlocal and non-Markovian relation. The turbulence field and the source in the observable coordinates are written as

$$\boldsymbol{f}(x,t) = \int_{-\infty}^{\infty} \mathrm{d}k \, \exp(\mathrm{i}kx)\, \boldsymbol{f}_k(t), \tag{20a}$$
$$\boldsymbol{S}_k(t') = \int_{-\infty}^{\infty} \mathrm{d}x' \exp(-\mathrm{i}kx')\, \boldsymbol{S}(x',t'). \tag{20b}$$

Substituting Eq. (20b) into Eq. (10a), which is then substituted into Eq. (20a), we obtain

$$\begin{aligned}\boldsymbol{f}(x,t) = & \int_{-\infty}^{\infty} \mathrm{d}k \, \exp(\mathrm{i}kx)\, \boldsymbol{\zeta}_k(t) \\ & + \int_{-\infty}^{\infty} \mathrm{d}k \, \exp(\mathrm{i}kx) \int_0^t \mathrm{d}t' \, g_k(t,t') \\ & \times \int_{-\infty}^{\infty} \mathrm{d}x' \exp(-\mathrm{i}kx')\, \boldsymbol{S}(x',t'). \end{aligned} \quad (21)$$

The second term is the RHS of Eq. (21) has the form

$$\begin{aligned}&\int_{-\infty}^{\infty} \mathrm{d}x' \int_0^t \mathrm{d}t' \int_{-\infty}^{\infty} \mathrm{d}k \, \exp(\mathrm{i}kx - \mathrm{i}kx') \\ &\times g_k(t,t')\, \boldsymbol{S}(x',t').\end{aligned}$$

We introduce a nonlocal and non-Markovian kernel as the Fourier transformation of the Green's function

$$K(x,x':t,t') = \int_{-\infty}^{\infty} \mathrm{d}k \, \exp(\mathrm{i}kx - \mathrm{i}kx')\, g_k(t,t'). \quad (22)$$

By using this nonlocal and non-Markovian kernel, the turbulent field is rewritten as

$$\begin{aligned}\boldsymbol{f}(x,t) = & \int_{-\infty}^{\infty} \mathrm{d}k \, \exp(\mathrm{i}kx)\, \boldsymbol{\zeta}_k(t) \\ & + \int_{-\infty}^{\infty} \mathrm{d}x' \int_0^t \mathrm{d}t' K(x,x':t,t')\, \boldsymbol{S}(x',t'), \end{aligned} \quad (23)$$

where the second term in the RHS denotes the coherent response of the fluctuating field to the externally-controlled source $\boldsymbol{S}(x',t')$.

Equation (23) shows that the nonlocality in space and the memory effect in time (the non-Markovian property) coexist. It is emphasized that the nonlocality in space appears simultaneously with the memory effect in time. The range of the nonlocal interaction is determined by the dependence of the Green's function on the wave number. If the response function does not depend on the wave number of fluctuations (as the case of fluctuations in thermal equilibrium, where the Onsager's ansatz of scale-independence of the kinetic coefficient holds[30,43]), putting $g_k \to g$ in Eq. (22), we have

$$\begin{aligned}K(x,x':t,t') &= g(t,t') \int_{-\infty}^{\infty} \mathrm{d}k \, \exp(\mathrm{i}kx - \mathrm{i}kx') \\ &= g(t,t')\, \delta(x-x'). \end{aligned} \quad (24)$$

The local relation between the source and the response holds. On the other hand, if the response function has a peak width Δk in the wave number space, e.g., $\propto \exp\left(-(k-k_0)^2 (\Delta k)^{-2}\right)$, the nonlocal kernel has the dependence

$$K(x,x':t,t') \propto \exp\left(-(x-x')^2 \Delta k^2\right). \quad (25)$$

The interaction length in the x direction is in proportion to $1/\Delta k$.

The properties of the memory function and the Green's function have been discussed in the literature. Here we employ simplified approximations to study qualitative perspectives of the nonlocality and the memory. For like-scale interactions, where the relation $\bar{\Gamma}_k(p) \simeq \bar{\Gamma}_{k'}(p)$ holds between memory functions from k and k' for the like-scale wave numbers $|k'| \sim |k|$, equation (19) is approximated as

$$\bar{\Gamma}_k(p) = {\sum}' \frac{V_{k,1,2}}{p_2 + \bar{\Gamma}_{k_2}(p)} \sim \frac{{\sum}' V_{k,1,2}}{p_k + \bar{\Gamma}_k(p)}. \quad (26)$$

This crude estimate gives approximate relations for the memory function

$$\bar{\Gamma}_k(p) \sim \bar{\Gamma}_k^{(0)} = \sqrt{{\sum}' V_{k,1,2}} \quad \text{for} \quad p_k \ll \bar{\Gamma}_k^{(0)}, \quad (27a)$$

$$\bar{\Gamma}_k(p) \sim \frac{\bar{\Gamma}_k^{(0)^2}}{p_k} \quad \text{for} \quad p_k \gg \bar{\Gamma}_k^{(0)}. \quad (27b)$$

The relation (27a) corresponds to the Corrsin approximation,[3] which is often used in the strong turbulence case. The like-scale estimation also provides an approximate relation from Eq. (16)

$$\begin{aligned}\bar{\Gamma}_k(p) &= {\sum}' V_{k,k_1,k_2} G_{k_2}(p) \sim G_k(p) {\sum}' V_{k,k_1,k_2} \\ &= G_k(p)\, \bar{\Gamma}_k^{(0)^2}, \end{aligned} \quad (28)$$

so we have

$$\begin{aligned}G_k(p) &\sim \frac{1}{\bar{\Gamma}_k^{(0)}} \quad \text{for} \quad p_k \ll \bar{\Gamma}_k^{(0)}, \\ G_k(p) &\sim \frac{1}{p_k} \quad \text{for} \quad p_k \gg \bar{\Gamma}_k^{(0)}. \end{aligned} \quad (29)$$

This evaluation of the Green's function, Eq. (29), indicates that the memory in the Green's function remains substantial during the period of $\left(\mathrm{Re}\,\lambda_k + \bar{\Gamma}_k^{(0)}\right)^{-1}$, where $\mathrm{Re}\,\lambda_k$ is the linear damping rate. This may be more explicitly seen by employing an interpolation formula for Eq. (29)

$$G_k(p) \sim \frac{1}{p + \lambda_k + \bar{\Gamma}_k^{(0)}}, \quad (30)$$

which gives the Green's function

$$\begin{aligned}&g(t-t') \propto \exp\left(-\left(\lambda_k + \bar{\Gamma}_k^{(0)}\right)(t-t')\right) \\ &\quad \text{for} \quad t > t'. \end{aligned} \quad (31)$$

From these considerations, we observe that the spatial range of the nonlocal kernel Eq. (22) is determined by the width in the wave number space, and that the characteristic value of memory is given by the nonlinear decorrelation rate of fluctuations.

3.3. Deduction of nonlinear decorrelation rate from observations

Equations (10a) and (31) can be used in interpreting the experimental observation of the response of turbulent plasma against the externally-controlled signals. If one takes the correlation between the fluctuating field and the externally-controlled signals, one can observe the coherent part in the fluctuations. If one focuses on the temporal memory effect and assumes that each Fourier component is observed (for the simplicity of argument), one may have the relation

$$\langle \boldsymbol{f}_k(t)\boldsymbol{S}_k(t)\rangle = \left\langle \boldsymbol{S}_k(t)\int_0^t \mathrm{d}t'\, g_k\left(t,t'\right)\boldsymbol{S}_k\left(t'\right)\right\rangle. \quad (32)$$

The cross correlation function

$$C_{S,f}\left(\tau\right) = \frac{\langle \boldsymbol{f}_k\left(t\right)\boldsymbol{S}_k^*\left(t-\tau\right)\rangle}{\sqrt{\langle \boldsymbol{f}_k^2\rangle\langle \boldsymbol{S}_k^2\rangle}} \quad (33)$$

provides the information of the nonlinear decorrelation rate of the turbulent plasma. For a sinusoidal source, $\boldsymbol{S}_k(t) = \boldsymbol{S}_{k,\omega}\exp\left(-\mathrm{i}\omega t\right)$, the approximate relation (Eq. (31)) yields

$$C_{S,f}\left(\tau\right) = \frac{\mathrm{e}^{-\mathrm{i}\omega\tau}}{\bar{\Gamma}_k^{(0)} + \mathrm{Re}\,\lambda_k + \mathrm{i}\left(\mathrm{Im}\,\lambda_k - \Omega_k - \omega\right)}. \quad (34)$$

This result indicates that depending on the choice of the external frequency, the ratio between the amplitude of sinusoidal element $\boldsymbol{f}_k$ and that of the external source varies. The ratio becomes maximal at the applied frequency with which the imaginary part of the denominator vanishes. Thus, by observing the response against the prescribed source in experiments, the nonlinear decorrelation rate can be measured. It should also be noted again that the nonlinear decorrelation rate is influenced by the external source. This relation can be applied in the bicoherence analysis of the turbulent plasmas. An application to actual experiments will be reported in a separate article.

3.4. Nonlocal case of disparate-scale interaction

The results in Sections 3.2 and 3.3 are derived under the assumption that $\bar{\Gamma}_k\left(p\right) \simeq \bar{\Gamma}_{k'}\left(p\right)$ holds for the like-scale wave numbers $|k'| \sim |k|$. The zonal flows (with longer spatial scales) are excited by the microscopic drift wave turbulence in the confined plasmas. In this disparate-scale interaction, the response of the zonal flows has a much longer timescale compared to the decorrelation time of the drift waves. By taking advantage of this separation of time scale, a predator–prey model has been proposed.[5] In that mode, the dynamical variables are reduced to only two, i.e., the energy density of the drift waves W and that of the zonal flow Z. In addition, the rapid changes in the time scales of drift wave frequency or of rapidly-varying random force are filtered out. The latter has been discussed in Refs. [5], [46], and [47]. In the reduction process of the variables to W and Z, the memory function and the noise source (both of which have the quadratic order, either W^2 or WZ) are absorbed into the nonlinear term W^2 or the quasi-linear term WZ. Even though highly simplified, the predator–prey model is a useful model that explains the dynamical evolution of the energies of zonal flow and turbulence in the time scale of meso-scale physics. The response of the drift waves–zonal flows system against the external forcing (which is a smooth function in time, say sinusoidal wave form) is investigated here. The objective in this subsection is to analyze the nonlocal and non-Markovian interaction in the meso-scale time relevant to the zonal flow and the drift wave envelope.

Coupled dynamical equations for the drift wave and zonal flow system are derived in the normalized form as a set of nonlocal interaction equations[39]

$$\frac{\partial}{\partial t}W = \gamma_{\mathrm{L}}W - \omega_2 W^2 - \alpha ZW, \quad (35a)$$

$$\frac{\partial}{\partial t}Z = -\gamma_{\mathrm{damp}}Z + \alpha Z\frac{1}{2l}\int_{x-l}^{x+l}\mathrm{d}x'\,W\left(x'\right) + S(t), \quad (35b)$$

where W and Z are the energy densities of the drift waves and the zonal flows (normalized to $m_{\mathrm{i}}n_{\mathrm{i}}V_{\mathrm{d}}^2/2$, with V_{d} being the electron diamagnetic drift velocity), respectively, and $S(t)$ denotes the external source of the zonal flow excitation. The length l indicates the autocorrelation length of the zonal flow, which is much longer than that of the drift waves. The γ_{L} and ω_2 are the linear growth rate and the nonlinear damping rate of the drift waves, respectively, and γ_{damp} is the damping rate of the zonal flow. The quasi-linear coupling rate between the zonal flows and the drift waves, α, as well as ω_2 and γ_{damp} is assumed to be constant and uniform here for simplicity. Explicit expressions of these parameters are given in Ref. [3]. Figure 1 illustrates the radial distributions of W and Z, and denotes the schematic wave forms for the microscopic

turbulence and the zonal flows. Thus, the drift wave fluctuations can interact with the drift waves across long distances via the coupling with the zonal flows. The spatial profile of the energy exchange between the drift waves and the zonal flows has been studied experimentally, and the initial observation has shown the nonlocal energy exchange between the drift waves through the channel of the zonal flows.[48] This experimental progress motivates the analysis of the nonlocal dynamics of Eq. (35).

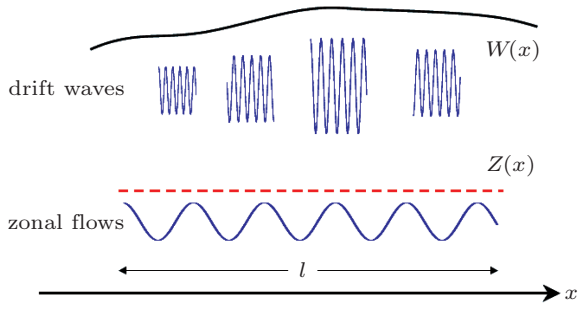


Fig. 1. (colour online) Conceptual description of microscopic turbulence and zonal flow. Correlation length of microscopic fluctuations is much shorter than that of zonal flow. Profiles of fluctuation intensity and that of zonal flow, W and Z, are also shown by solid and dashed lines, respectively.

Before proceeding to the detailed analysis of Eq. (35), it is useful to illustrate the limitation of the model studied in this article. First, this model takes into account only the meso-scale zonal flows, and does not include the effects of possible global characters. There are a couple of processes that can induce multiple-scale interactions. For instance, the dependence of diamagnetic drift velocity on radial location can introduce other types of nonlocal interactions (e.g., see Ref. [49]). These effects can introduce more complex dynamical responses. Second, even if the analysis is limited up to the meso-scale, GAMs can also play an important role in addition to the zonal flows. The coupled dynamics between drift waves, zonal flows, and GAMs has been analyzed in the absence of the source in Ref. [50]. The conclusion in Ref. [50] is that depending on the global plasma parameters, the plasma chooses a state with/without zonal flows and/or GAMs. The boundary between GAMs and zonal flows has been explicitly derived. Thus, for plasma in which GAMs have a larger damping rate than the zonal flows (in other words, the safety factor is low, and the plasma is in the collisionless regime), the zonal flows, instead of the GAMs, are preferentially induced. The analysis here is limited to the turbulence state without the GAMs, i.e., to the plasmas with relevant parameters. Noting these limitations, we emphasize that the analysis here is a prototypical example studying the multiple scale dynamics of turbulence, provides a basis for more complete theories that describe the nonlocal interactions of the drift wave turbulence.

Equation (35b) provides the nonlocal and non-Markovian response of the zonal flows to the external source $S(t)$

$$Z(x,t) = \int_{-\infty}^{t} \mathrm{d}t' \exp\left\{ -\gamma_{\mathrm{damp}}\left(t-t'\right) + \frac{\alpha}{2l} \times \int_{t'}^{t} \mathrm{d}t'' \int_{x-l}^{x+l} \mathrm{d}x' \, W\left(x',t''\right) \right\} S\left(t'\right). \quad (36)$$

The second term in the argument of the exponential function denotes the nonlocal coupling due to the long radial correlation length of the zonal flows.

If the time scale separation holds, the steady state of Eq. (35a) is reached during the evolution of Z,

$$W = \frac{1}{\omega_2}\left(\gamma_{\mathrm{L}} - \alpha Z\right). \quad (37)$$

Substituting Eq. (36) into Eq. (37) gives

$$W(x,t) = \frac{\gamma_{\mathrm{L}}}{\omega_2} - \frac{\alpha}{\omega_2} \int_{-\infty}^{t} \mathrm{d}t' \times \exp\left[-\gamma_{\mathrm{damp}}\left(t-t'\right) + \frac{\alpha}{2l} \times \int_{t'}^{t} \mathrm{d}t'' \int_{x-l}^{x+l} \mathrm{d}x' \, W\left(x',t''\right) \right] S\left(t'\right). \quad (38)$$

The drift wave fluctuations at far distance x' can interact with the fluctuations at location x within a correlation length of zonal flow, i.e., $|x'-x| < l$. Equation (38) represents the nonlocal and non-Markovian response of the microscopic turbulence in the presence of zonal flows.

In the presence of the temporally changing source, the simplified relation (37) does not always hold. The response near the stationary fixed point (without source) is studied here. The stationary solution $\left(\bar{W},\bar{Z}\right)$ of Eq. (35) in the absence of source ($S(t)=0$) is given by

$$\bar{W} = \frac{\gamma_{\mathrm{damp}}}{\alpha} + \frac{\gamma_{\mathrm{L}}}{\omega_2} - \frac{1}{2l}\int_{x-l}^{x+l} \mathrm{d}x' \frac{\gamma_{\mathrm{L}}\left(x'\right)}{\omega_2}, \quad (39a)$$

$$\bar{Z} = \frac{1}{2l}\int_{x-l}^{x+l} \mathrm{d}x' \frac{\gamma_{\mathrm{L}}\left(x'\right)}{\alpha} - \frac{\gamma_{\mathrm{damp}}\omega_2}{\alpha^2}. \quad (39b)$$

The second and the third terms in the RHS of Eq. (39a) indicate the influence of the seesaw mechanism, i.e., the pumping of fluctuations at distance

influences the local level of fluctuations. For the transparency of the argument, we keep the spatial dependence of $\gamma_{\rm L}$, and assume that the other parameters are constant 1 over the correlation length of the zonal flows. The perturbations driven by the source in the vicinity of nonlinear fixed point $\left(\bar{W}, \bar{Z}\right)$ is studied. We put

$$W(x,t) = \bar{W}(x) + w(x,t), \quad Z(t) = \bar{Z} + z(t).$$

The linearlized coupled equations are given as

$$\frac{\partial}{\partial t} w = -\Gamma_{11}(1-\varepsilon) w - \Gamma_{12}(1+\varepsilon) z, \quad (40a)$$

$$\frac{\partial}{\partial t} z = \Gamma_{21} \frac{1}{2l} \int_{x-l}^{x+l} {\rm d}x'\, w(x') + S(t), \quad (40b)$$

that is,

$$\begin{aligned} &\frac{\partial^2}{\partial t^2} w + (1-\varepsilon)\Gamma_{11} \frac{\partial}{\partial t} w \\ &\quad + (1+\varepsilon)\Gamma_{12}\Gamma_{21} \frac{1}{2l} \int_{x-l}^{x+l} {\rm d}x'\, w(x') \\ &= -(1+\varepsilon)\Gamma_{12} S(t), \end{aligned} \quad (40c)$$

where

$$\Gamma_{11} = \frac{\gamma_{\rm damp}\omega_2}{\alpha}, \quad (41a)$$

$$\Gamma_{12} = \gamma_{\rm damp}, \quad (41b)$$

$$\Gamma_{21} = \frac{1}{2l} \int_{x-l}^{x+l} {\rm d}x'\, \gamma_{\rm L}(x') - \frac{\gamma_{\rm damp}\omega_2}{\alpha}, \quad (41c)$$

$$\varepsilon(x) = \frac{\alpha}{\gamma_{\rm damp}\omega_2} \left(\gamma_{\rm L}(x) - \frac{1}{2l} \int_{x-l}^{x+l} {\rm d}x'\, \gamma_{\rm L}(x') \right). \quad (41d)$$

The term $\varepsilon(x)$ represents the influence from the nonlocal interaction. In the local limit, where $l \to 0$ holds, the coefficient $\varepsilon(x)$ vanishes. And, if $\gamma_{\rm L}$ is uniform across distance l, $\varepsilon(x) \to 0$.

We can observe the nonlocal effects by using the response against the external source. The response $w(x,t) \propto \exp(-{\rm i}\omega t)$ against the source $S(t) = S_0 \exp(-{\rm i}\omega t)$ is analyzed. Let us put

$$w(x,t) = w_0(t) + w_1(x,t), \quad (42a)$$

where $w_0(t)$ and $w_1(x,t)$ are spatially uniform and nonuniform parts of $w(x,t)$, respectively, i.e.,

$$\begin{aligned} &\frac{1}{2l} \int_{x-l}^{x+l} {\rm d}x'\, w(x',t) = w_0(t), \\ &\frac{1}{2l} \int_{x-l}^{x+l} {\rm d}x'\, w_1(x',t) = 0. \end{aligned} \quad (42b)$$

Putting Eq. (42a) into Eq. (40c), and separating the uniform and the nonuniform parts, up to the first order correction of ε, we have

$$w_0(t) = \frac{\Gamma_{12} S(t)}{\omega^2 + {\rm i}\omega\Gamma_{11} - \Gamma_{12}\Gamma_{21}}, \quad (43a)$$

$$w_1(x,t) = \frac{\omega + 2{\rm i}\Gamma_{11}}{\omega + {\rm i}\Gamma_{11}} \varepsilon(x)\, w_0(t), \quad (43b)$$

$$z(t) = \frac{{\rm i}\omega - \Gamma_{11}}{\omega^2 + {\rm i}\omega\Gamma_{11} - \Gamma_{12}\Gamma_{21}} S(t). \quad (43c)$$

Equation (43a) indicates the resonance (where the relative amplitude w_0/S takes the maximum) at $\omega = \omega_{\rm r}$

$$\omega_{\rm r} = \sqrt{\Gamma_{12}\Gamma_{21} - \Gamma_{11}^2/2}. \quad (44)$$

The correlation to the resonance frequency is of the order ε^2.

By using the sinusoidal oscillation source $S(t) \propto \exp({\rm i}\omega t)$, the resonance at $\omega = \omega_{\rm r}$ is searched. The resonance frequency is realized for the response, in which the perturbation w varies concomitantly in the domain of correlation length l. The result indicates that the influence of nonlocal effect appears in component $w_1(x,t)$, which has an unambiguous phase difference from $w_0(t)$ given by Eq. (43b). Although S is a constant in space, the spatially-inhomogeneous modulation of the envelope of the fluctuation intensity takes place. This spatially-inhomogeneous part oscillates at different phases (the seesaw oscillation). This provides an experimental method to observe the nonlocal interaction between the drift wave turbulence via the zonal flows. It is noted that in this section, the envelope of the drift wave turbulence (not the waveform of each wave component) is analyzed. At the resonant frequency $\omega = \omega_{\rm r}$, the zonal flow intensity changes in time sinusoidally, and the intensity of the drift wave turbulence also oscillates in time sinusoidally. Additional spatial modification of the envelope of turbulence $w_1(x,t)$ is induced. The relative phase delay in response is described by Eq. (43c).

We can directly measure and study the nonlocal response of microscopic fluctuations through the externally-modulated zonal flow amplitude. This damped oscillation has some analogy with the damped-propagation wave solution in the system of turbulence intensity and mean pressure gradient.[36] In Ref. 36], a coupling between the microscopic turbulence and the macroscopic profile is the origin of the oscillatory solution. In the case of the drift waves–zonal flows system, the oscillation takes place owing to

the coupling between microscopic and mesoscopic fluctuations. The novel feature in this article is that the effect of nonlocal interaction can be directly observed in experiments by measuring the phase difference between the spatially-homogeneous and inhomogeneous parts in response $w(x,t)$. The experimental observation of this oscillation by identifying the resonance in the response to the external oscillation will illuminate the nonlocal interaction between the drift wave turbulence and the zonal flows.

4. Summary

In this article, we report a theoretical framework that describes the formula of response of turbulent plasmas against the externally-controlled prescribed perturbations. We assume that the conditions required by the Mori's method are satisfied, and separate nonlinear interactions into the memory function and the fluctuating force. Making use of the memory function, we derive the formula of the response.

It is shown that the response kernel, which connects the source and the response of the turbulent field in real space and time, has the nonlocal property (in space) and the non-Markovian response (in time). The time scale of the memory and the spatial range of the nonlocal interaction are discussed. With the further simplifications that the memory function is weakly dependent on the wave number of interests, the approximate forms of the memory function and the Green's function are discussed. The response of the turbulent field is discussed. This provides a relation to deduce the memory function from the observed response. The case of disparate-scale interaction is also discussed. The response of the seesaw mechanism is analyzed, and an explicit relation for the nonlocal and non-Markovian transport is derived. The dynamic response against an external oscillating source for the zonal flows is studied. The oscillation of the envelope of the drift wave turbulence has a spatially-nonuniform part, which oscillates at different phases (the seesaw oscillation) compared to the spatially-uniform part. This provides a new experimental method to observe the nonlocal interaction in the drift wave turbulence via the zonal flows.

We note that the analysis is limited to the turbulence state without GAMs. This means that the analysis is valid for the plasma parameters in the domain of preferential excitation of the zonal flows. It is also noted that if we choose the plasma parameters with which the state with GAMs without zonal flows is preferred, we can directly extend to obtain a theory of similar structure. The basic dynamical equation is given by a similar formula (with different coefficients) as long as the time scale separation between the GAM frequency and the ambient turbulence holds. The analysis here will be straightforwardly extended to such circumstances. The analysis for the case, where GAMs and zonal flows coexist, could be much more complex, and needs furhter studies.

We now discuss possible methods to excite the zonal flows. Two cases have been considered. The first is an experimental study of the fundamental process of zonal flow–drift wave turbulence. This approach has been developed in the magnetized linear plasmas, and the essential physics has been discovered.[48] In this system, the end-plate biasing is a promising method to induce the zonal flow externally. Initial investigations have been presented, e.g., Ref. [51]. The other is the case of toroidal plasma. It is shown that the biased limiter is able to induce a solitary strong radial electric field at edge,[52] and explains the biased limiter experiments.[53,54] This process may be extended to drive zonal flows externally with temporal oscillation.

The basic framework in this article would be extended in the following directions. The expression here would be formulated in such a way that the wide range of spectrum is induced (e.g., avalanche). More explicit formulas are given for the interrelations among various physical quantities (e.g., density, momentum, heat).[55] The theory can be extended to the higher order moments, such as the bicoherence. An explicit description between the external circuit and the internal source (S in this article) is desirable for applying this result to the experimental work. These are subjects of near future work.

Dedication

This article is dedicated to the memory of Prof. H. Mori.

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