Influences of finite gain bandwidth on pulse propagating in parabolic fiber amplifiers with distributed gain profiles*

Zhao Jia-Sheng(赵佳生)[a)], Li Pan(李 磐)[a)], Chen Xiao-Dong(陈晓东)[a)], Feng Su-Juan(冯素娟)[a)], and Mao Qing-He(毛庆和)[a)b)†]

[a)] *Anhui Provincial Key Laboratory of Photonics Devices and Materials, Anhui Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Hefei* 230031, *China*

[b)] *University of Science and Technology of China, Hefei* 230026, *China*

(Received 13 January 2012; revised manuscript received 25 February 2012)

The evolutions of the pulses propagating in decreasing and increasing gain distributed fiber amplifiers with finite gain bandwidths are investigated by simulating with the nonlinear Schrödinger equation. The results show that the parabolic pulse propagations in both the decreasing and the increasing gain amplifiers are restricted by the finite gain bandwidth. For a given input pulse, by choosing small initial gain coefficient and gain variation rate, the whole gain for the pulse amplification limited by the gain bandwidth may be higher, which is helpful to the enhancement of the output linearly chirped pulse energy. Compared to the decreasing gain distributed fiber amplifier, the increasing gain distributed amplifier may be more conducive to suppress the pulse spectral broadening and increase the critical amplifier length for achieving a larger output linearly chirped pulse energy.

Keywords: finite gain bandwidth, parabolic pulse, linearly chirped pulse energy

PACS: 42.65.Re, 42.65.–k **DOI:** 10.1088/1674-1056/21/9/094217

1. Introduction

High-repetition-rate and large energy ultra-short laser pulses are very important for applications of nonlinear frequency conversion, ultrafast dynamics exploration, and optical precision processing.[1−6] Rare-earth doped silica-based fibers are very good gain media for building mode-locking lasers generating such ultra-short pulses.[7−9] However, the nonlinear Kerr effect of the single-mode transmission fibers may induce nonlinear chirps to cause a pulse distortion. This makes it difficult to obtain large energy pulses with fiber devices.[10] The previous research results have shown that the parabolic pulse, which carries linear chirps, is efficiently compressed and may be capable of resisting the wave breaking and preserving the parabolic shape during propagation through positive dispersion fibers.[11] Such excellent parabolic pulses have been attracting much attention in the past years.[12−14] In 1996, Tamura and Nakazawa showed that a nonlinear pulse propagating in a fiber amplifier with positive dispersion may generate parabolic pulses.[15] Then, Ferman *et al.* proved theoretically by using the self-similarity technique that the nonlinear Schrödinger equation (NLSE) for the constant gain and normal dispersion fiber has an exact asymptotic parabolic solution, and demonstrated experimentally that any pulse can evolve into the linearly chirped parabolic pulse in a sufficient long fiber amplifier with normal dispersion.[16,17] Since then, a lot of research work has been done for the parabolic fiber amplifiers, including the theoretical verification of the exact self-similar solution to the NLSE with distributed dispersion, nonlinearity, and gain.[18−20] Moreover, it has also been found that the finite gain bandwidth and the stimulated Raman scattering (SBS) are the two main factors in parabolic fiber amplifiers that may lead to the chirp nonlinearity and, consequently, to distortions of the recompressed pulses.[21] As far as the gain bandwidth is concerned, it has shown for the constant gain fiber amplifier that the finite gain bandwidth seriously limits the performance of parabolic amplification

*Project supported by the National High Technology Research and Development Program of China (Grant No. 2011AA030203) and the National Basic Research Program of China (Grant No. 2007CB936603).

†Corresponding author. E-mail: mqinghe@aiofm.ac.cn

© 2012 Chinese Physical Society and IOP Publishing Ltd

by distorting both the chirp and the frequency envelope, thus preventing efficient pulse compression required for obtaining high peak power pulses.[22] However, since the gain distribution of any practical fiber amplifier is not flat, and the influences of the finite gain bandwidth on the parabolic pulses propagating in different gain distributed fiber amplifiers may be different, it is significant to investigate the parabolic pulses amplified in different distributed gain fiber amplifiers under the restriction of the limited gain bandwidth.

In this paper, the evolution behaviors of pulses propagating in different gain distributed fiber amplifiers with finite gain bandwidths are investigated by simulating with the NLSE. Our results show that under the restriction of a given gain bandwidth, it is helpful to choose small initial gain coefficient and gain variation rate for obtaining the high whole gain to enhance the output linearly chirped pulse energy. Compared to the decreasing gain distributed amplifier, the increasing gain distributed fiber amplifier may be more conducive to suppress the pulse spectral broadening due to the self-phase modulation (SPM) effect and the increased critical amplifier length for achieving larger output linearly chirped pulse energy.

2. Theoretical model

Ultra-short pulses propagating and amplified in a fiber amplifier may be described by the generalized NLSE. Considering that the influences of the high-order dispersion, for instance, the third-order dispersion, and the high-order nonlinearity, in particular, the SBS, on the pulses propagating in fiber amplifiers have been investigated by many research groups,[22–24] for simplicity, we ignore these high-order dispersion and nonlinearity in our theoretical model. Thus, ultra-short pulses amplified in a fiber amplifier can be described by the following frequency domain NLSE:[25]

$$\frac{\partial \bar{\Psi}}{\partial z} = \frac{\mathrm{i}}{2}\beta_2\omega^2\bar{\Psi} - \frac{\alpha}{2}\bar{\Psi} + \mathrm{i}\gamma\left|\bar{\Psi}\right|^2\bar{\Psi} + \frac{g}{2}\bar{\Psi},$$

where $\bar{\Psi}$ is the pulse frequency domain amplitude, β_2 is the GVD parameter, γ is the nonlinearity parameter, ω is the angular frequency, α and g are the loss and the gain coefficients of the amplifier, respectively. In order to investigate the nonlinear pulse propagation in the fiber amplifier with a finite gain bandwidth, the gain coefficient is described by $g = g(z)f(\omega)$. The super Gaussian profile $f(\omega) = \exp\left(-(\omega/\Omega_g)^{10}\right)$ is used, corresponding to the flat gain properties of the fiber amplifier, with Ω_g of 10 THz (FWHM) being the gain bandwidth. For simplicity and without loss of generality, the gain coefficient along the fiber amplifier is expressed in the exponential function $g(z) = g_0\exp(\delta z)$, with g_0 being the gain coefficient at the input end of the amplifier. When δ is a negative (positive) real number, the gain coefficient along the fiber amplifier may have a decreasing (increasing) distribution, corresponding to the co-directional (counter-directional) pumped fiber amplifier. In accordance with Eq. (1), by using the standard split step Fourier method,[24] the pulse propagation in the fiber amplifier with the finite gain bandwidth may be simulated with the parameters displayed in Table 1.[23]

Table 1. Simulation parameters.

Parameter	Symbol	Value
GVD	β_2	20×10^{-27} s^2·m^{-1}
Nonlinearity parameter	γ	0.27 W^{-1}·km^{-1}
Loss coefficient	α	0.002 m^{-1}
Gain bandwidth	Ω_g	10 THz
Center wavelength of pulse	λ	1060 nm
Center wavelength of gain	λ'	1060 nm

3. Results and discussion

The evolution behaviors of the pulses propagating in the distributed gain fiber amplifiers with the finite gain bandwidth are first calculated. Figure 1 shows the simulation results for $g_0 = 3$, $\delta = -0.2$ and $g_0 = 1$, $\delta = 0.2$ respectively. Here all the input pulses are chirp-free Gaussian pulses with the duration of 200 fs and the energy of 0.35 nJ. For the sake of comparison, the evolutions of the pulses propagating in the same fiber amplifiers without the gain bandwidth limit are also displayed. Since the spectral width of the input pulse (5 nm) is much smaller than the gain bandwidth of the amplifier (35 nm), when the pulse is just launched into the amplifier, the gain bandwidth does not restrict the pulse propagation and evolution. As can be seen, near the input ends of the fiber amplifiers, all the pulses are temporally broadened in both time and frequency domains, and gradually evolve into the parabolic pulses with linearly chirped (see Fig. 2, which shows the chirp evolution behaviors of the pulses propagating along the fiber amplifiers) no matter whether the fiber amplifiers have increasing or

decreasing gain distributions.[19] As the pulses are further amplified along the amplifiers, the spectral widths of the pulses continue to be broadened until they reach the amplifier gain bandwidth when the pulses have propagated for particular lengths respectively in the amplifiers. Beyond those particular lengths, the spectral widths of the pulses exceed the gain bandwidth of the amplifiers, and the pulses no longer evolve self-similarly into parabolic pulses. The pulse spectral envelopes in the two gain-distributed amplifiers with the gain bandwidth limit exhibit two sidebands with multi-peak oscillation structures (see Fig. 1). The reason may be explained as follows. When the pulse spectral width exceeds the gain bandwidth, only the center part of the pulse spectral envelope within the amplifier gain bandwidth can be efficiently amplified. This corresponds to narrowing down the widths of the pulse spectral envelopes compared to the case without the gain bandwidth limit, resulting that the pulses cannot continue to be broadened required for the self-similarly evolution into the parabolic pulses, then the leading and the tailing edges of the pulses become steep. The steep leading and tailing edges in turn induce the increasing nonlinear chirps to the pulses due to the SPM effect as they continue to be amplified, as shown in Fig. 2. Such nonlinear chirps may make different parts of the pulse spectral envelopes have the same instantaneous frequency, thus interfering with each other, which leads to the two sidebands with the multi-peak oscillation structures.[25]

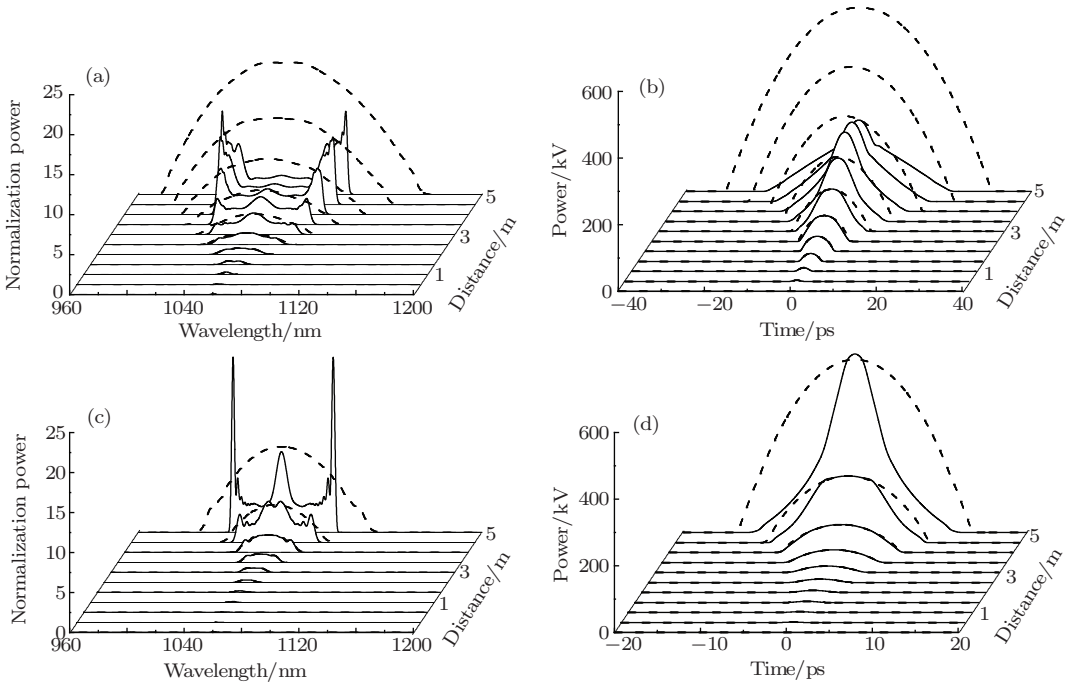


Fig. 1. Evolutions of the chirp-free Gaussian pulses propagating along distributed gain fiber amplifiers with (solid curves) and without (dash curves) the limit of the gain bandwidth. Panels (a) and (b) show normalization frequency profile and waveform for $g_0 = 3$ and $\delta = -0.2$, respectively, panels (c) and (d) show the results for $g_0 = 1$ and $\delta = 0.2$.

The above results show that the evolutions of the pulses propagating in the distributed gain fiber amplifiers are restricted by the gain bandwidth. However, since the pulse amplifications along the decreasing and the increasing gain distributed fiber amplifiers are different, the spectral envelope broadenings due to the SPM effect are also different, resulting in different critical amplifier lengths for the two kinds of amplifiers within which the gain bandwidth does not restrict the pulse's self-similar evolving into the parabolic pulse. The critical amplifier lengths for the decreasing and the increasing gain distributed fiber amplifiers shown in Fig. 1 are 2.1 m and 3.9 m, respectively. Since different critical amplifier lengths combined with the specific gain distributions give different whole gain values to the pulses, which makes the output linearly chirped pulses have different energies. In the cases shown in Fig. 1, the output linearly chirped pulse energies for the decreasing and the increasing gain distributed fiber amplifiers with the critical amplifier lengths are 65.7 nJ and 141 nJ, respectively. This implies that the output linearly chirped pulse energy may depend on the gain distribution of the fiber amplifier for a finite gain bandwidth.

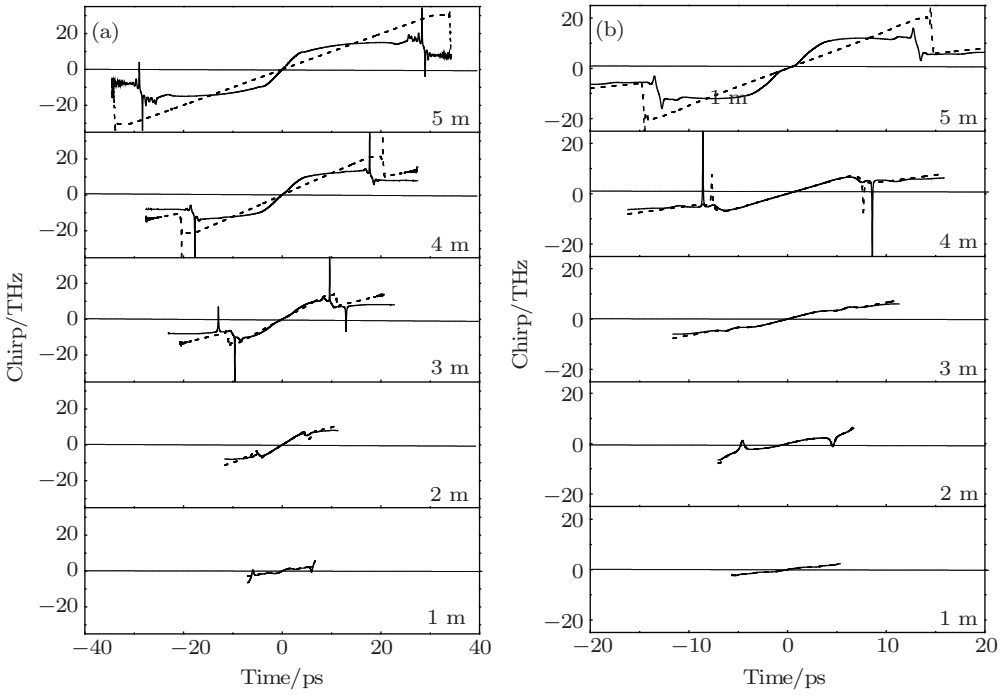


Fig. 2. Chirps of the pulses propagating in (a) decreasing (b) increasing gain fiber amplifiers with different lengths. Solid and dash curves correspond to the cases with and without the limit of gain bandwidth, respectively.

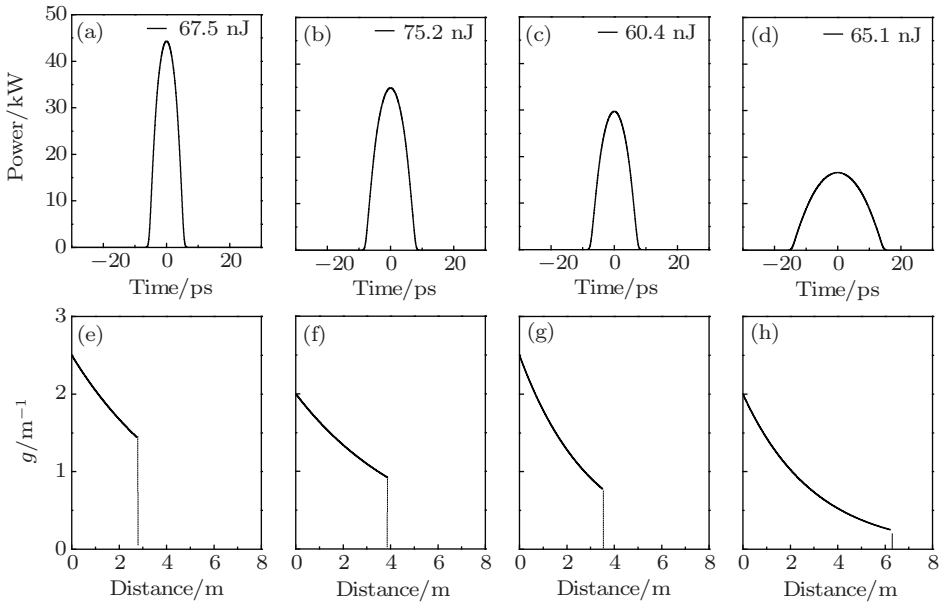


Fig. 3. Output linearly chirped parabolic pulse shapes and the optimized critical amplifier lengths for different decreasing gain amplifiers ((a) and (e): $\delta = -0.2$, $g_0 = 2.5$; (b) and (f): $\delta = -0.2$, $g_0 = 2.0$; (c) and (g): $\delta = -0.3$, $g_0 = 2.5$; (d) and (h): $\delta = -0.3$, $g_0 = 2.0$) with a gain bandwidth of 10 THz.

Figure 3 shows the output linearly chirped parabolic pulse shapes for different exponential decreasing gain distributed amplifiers with a gain bandwidth of 10 THz when the amplifier lengths have been optimized. As shown, for a given initial gain coefficient of 2.5, the critical amplifier lengths for $\delta = -0.2$ and $\delta = -0.3$ are 2.7 m and 3.5 m, respectively. This is because the energy compensation to the pulse per unit length near the input end of the amplifier for $\delta = -0.2$ is more than that for $\delta = -0.3$. Thus, the SPM effect and the pulse spectral broadening for $\delta = -0.2$ are stronger and faster, leading to a shorter critical amplifier length. Since a shorter critical amplifier length gives a lower dispersion accumulation, as shown in

Fig. 3, the output pulse duration for $\delta = -0.2$ is narrower than that for $\delta = -0.3$. However, the whole gain supplied by the amplifier for $\delta = -0.2$ is 22.8 dB, which is higher than that (22.3 dB) for $\delta = -0.3$, the output pulse energy for $\delta = -0.2$ (67.5 nJ) is higher than that (60.4 nJ) for $\delta = -0.3$. The critical amplifier lengths, the durations, and the pulse energies with $g_0 = 2.0$ for different δ exhibit the same properties as those with $g_0 = 2.5$. However, when g_0 is decreased to 2.0, since the pulse spectral broadening due to the SPM effect becomes weakened, the critical amplifier lengths are longer and the whole gains supplied by the amplifiers are slightly higher, giving larger output pulse energies. For instance, when δ is fixed at -0.3, the critical amplifier length and the output linearly chirped pulse energy for $g_0 = 2.0$ is 6.2 m and 65.1 nJ, respectively, while they are only 3.5 m and 60.4 nJ for $g_0 = 2.5$. This indicates that a smaller initial gain may be helpful to achieve a larger output linearly chirped pulse energy for the decreasing gain distributed amplifier with a finite gain bandwidth.

Figure 4 shows the output linearly chirped parabolic pulse shapes for different increasing gain distributed amplifiers with a gain bandwidth of 10 THz when the amplifier lengths have been optimized. For given initial gain coefficients of 0.8 and 0.6, the critical amplifier lengths are 4.7 m and 5.7 m, and 3.8 m and 4.5 m for $\delta = 0.2$ and $\delta = 0.3$, respectively. This also shows that a smaller initial gain and a smaller gain increasing rate allow the pulse propagating in a longer amplifier to exceed the finite gain bandwidth. Similarly, the output pulse energies for the amplifiers with $\delta = 0.2$ are higher than those with $\delta = 0.3$, since the whole gains obtained by the pulses from the amplifiers with $\delta = 0.2$ are larger than those with $\delta = 0.3$ in their critical amplifier lengths, while the output pulse durations with $\delta = 0.3$ are narrower than those with $\delta = 0.2$, as the latter have longer amplifier lengths, which gives larger dispersion accumulations. As shown in Fig. 4, the output pulse energy from the exponential increasing gain distributed amplifier with $g_0 = 0.6$ and $\delta = 0.2$ may be as high as 226 nJ. This indicates that by choosing smaller initial gain and gain increasing rate, the output linearly chirped pulse energy for the increasing gain distributed amplifier with a finite gain bandwidth may be increased.

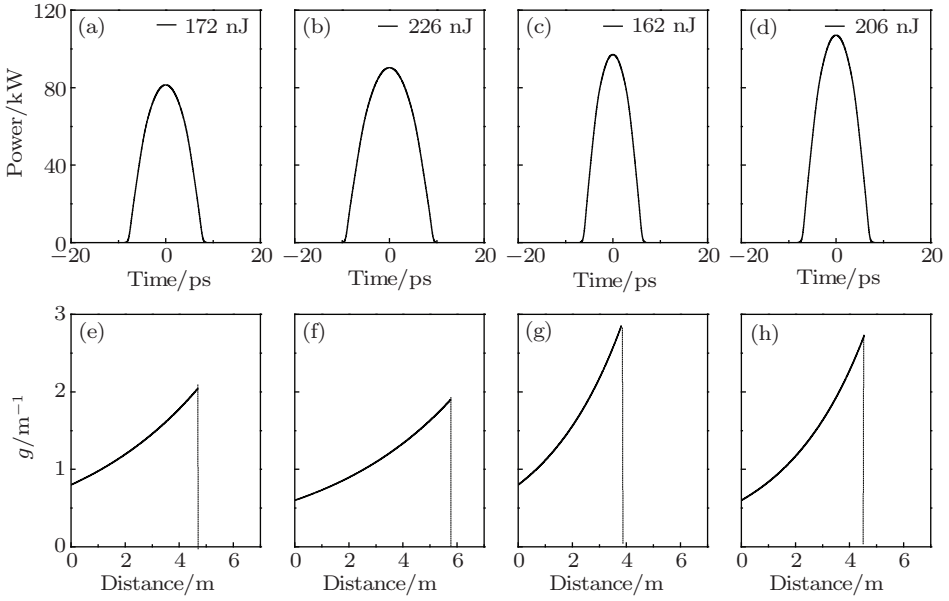


Fig. 4. Output linearly chirped parabolic pulse shapes and the optimized critical amplifier lengths for different increasing gain amplifiers ((a) and (e): $\delta = 0.2$, $g_0 = 0.8$; (b) and (f): $\delta = 0.2$, $g_0 = 0.6$; (c) and (g): $\delta = 0.3$, $g_0 = 0.8$; (d) and (h): $\delta = 0.3$, $g_0 = 0.6$;) with a gain bandwidth of 10 THz.

By comparing the results shown in Fig. 3 to those in Fig. 4, we can clearly see that the output linearly chirped pulse energy for the exponential increasing gain distributed amplifier seems to be larger than that for the exponential decreasing gain distributed amplifier when their gain bandwidths are the same. In order to further investigate the difference between the pulse propagations in the two kinds of amplifiers, we simulate the output linearly chirped pulse shapes of the two exponential gain distributed amplifiers at their critical

lengths for the same gain variation rate, while the initial gain coefficient of the increasing gain distributed amplifier is chosen to be equal to the minimum gain coefficient of the increasing gain distributed amplifier at its critical length. Figure 5 shows the simulation results. Here we can see that for the same minimum gain coefficient and gain variation rate, the critical amplifier length and the output linearly chirped pulse energy for the increasing gain distributed amplifier are respectively longer and larger than those for the decreasing gain distributed amplifier. This shows that for the same minimum gain coefficient and variation rate, compared to that with the decreasing gain distribution, the fiber amplifier with the increasing gain distribution may be more conducive to suppress the pulse spectral broadening due to the SPM effect and increase the critical amplifier length to achieve a larger output linearly chirped pulse energy. As shown in the figure, for the increasing gain distributed amplifier, when the initial gain coefficients are 0.92, 0.71, and 0.49, the whole gains obtained by the pulse from the amplifier with $\delta = 0.2$ is 26.3 dB, 27.4 dB, and 29.0 dB, and the output linearly chirped pulse energies are 151 nJ, 193 nJ, and 276 nJ, respectively. This shows that the increasing gain distributed amplifier with a small initial gain coefficient may be helpful to achieve a larger output linearly chirped pulse energy when the gain bandwidth is finite.

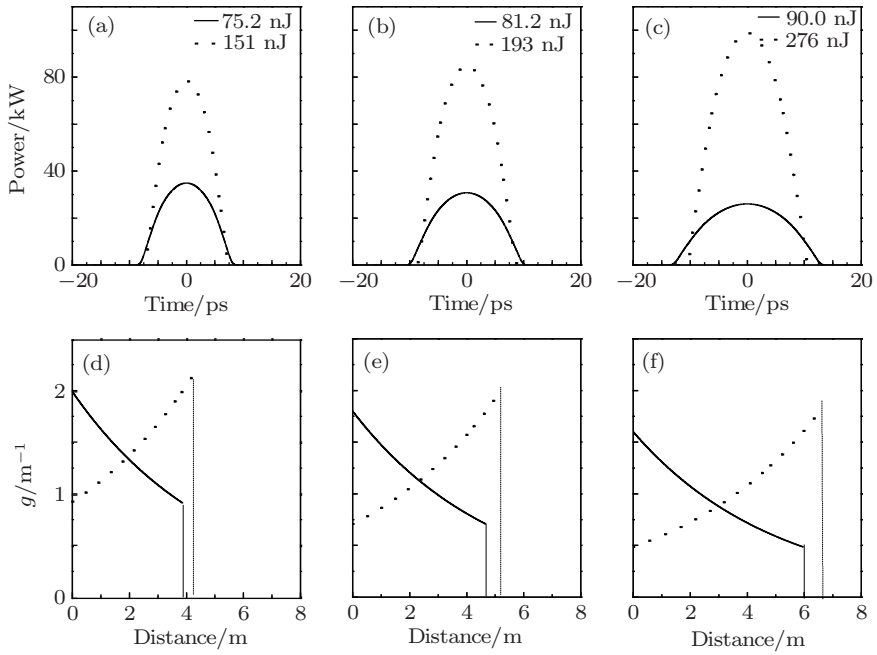


Fig. 5. Output linearly chirped parabolic pulse shapes and the optimized critical amplifier lengths for exponential increasing (dash curve) and decreasing (solid curve) gain fiber amplifiers with the same smallest gain coefficient and a gain bandwidth of 10 THz. In panels (a) and (d), $\delta = \pm 0.2$, $g_{\min} = 0.92$; in panels (b) and (e), $\delta = \pm 0.2$, $g_{\min} = 0.71$; in panels (c) and (f), $\delta = \pm 0.2$, $g_{\min} = 0.49$.

Figure 6 shows the output linearly chirped pulse shapes from an increasing gain distributed amplifier with a gain bandwidth of 10 THz when its length is optimized for different 200 fs chirp-free Gaussian input pulse energies. As can be seen, for the given initial gain coefficients of 0.6 and 0.8 and the fixed gain variation rate of 0.2, when the input pulse energies are 0.35 nJ, 0.7 nJ, and 1.4 nJ, the critical amplifier lengths are 5.7 m, 5.3 m, 4.8 m and 4.6 m, 4.2 m 3.7 m, the output parabolic pulse energies are 226 nJ, 202 nJ, 167 nJ and 172 nJ, 150 nJ, 119 nJ, respectively. This indicates that when the gain bandwidth is finite, the amplifier length may have to be designed according to the input pulse energy even for a given gain distribution. For a given increasing gain distributed amplifier, the output linearly chirped pulse energy may be increased by lengthening the critical amplifier length when the input pulse energy is decreased. Moreover, by choosing a small initial gain coefficient to suppress the SPM effect near the input end of the amplifier, the critical amplifier length may be efficiently increased to supply the higher whole gain, and then to enhance the output parabolic pulse energy.

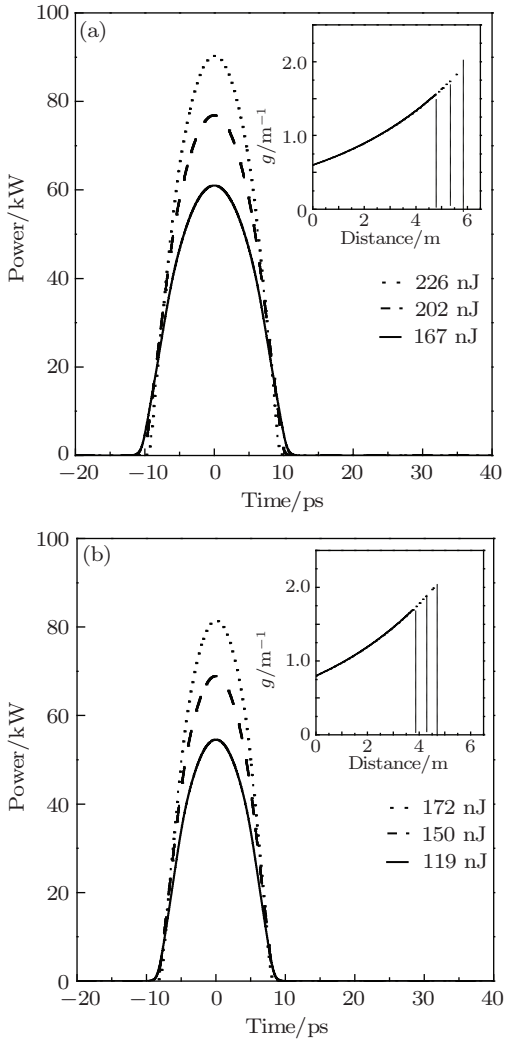


Fig. 6. Pulse outputs from increasing gain fiber amplifiers with different initial gains and gain variation rates ((a) $\delta = 0.2$, $g_0 = 0.6$; (b) $\delta = 0.2$, $g_0 = 0.8$) when the amplifier lengths are optimized to be the critical amplifier lengths. The insets are the corresponding distributed gains of the amplifiers, where dot, dash, and solid curves correspond to the energies of 0.35 nJ, 0.7 nJ, and 1.4 nJ, respectively.

In practical applications, the gain bandwidth of a given doped fiber amplifier is basically unchanged, the gain coefficient distribution along the fiber amplifier is mainly determined by the dopant concentration and the distributions of pump and signal powers along the doped fiber. When a low dopant concentration fiber is used as the gain fiber, the gain coefficient variation rate of the fiber amplifier could be small enough, because the low dopant concentration gives a small pump absorption. Therefore, according to our simulation results above, in order to obtain a large output linearly chirped pulse energy, we would better choose the low dopant concentration fiber for the fiber amplifier. Moreover, by using the counter-directional pumping scheme to form an increasing gain distributed amplifier for a small initial gain coefficient, the pulse spectral broadening near the input end of the amplifier due to the SPM effect may be weakened, which is helpful to increase the critical amplifier length and the whole gain obtained by the pulse, and thus, to enhance the output linearly chirped pulse energy under the restriction of a finite amplifier gain bandwidth. It would be worthwhile to point out that the recent research results have shown that the linear chirp large energy dissipative soliton pulses may be generated with fiber lasers directly, which may also supply an alternative scheme for improving the pulse energies of fiber lasers in the future.[26,27]

4. Conclusion

We have investigated the evolution behaviors of the pulses propagating in decreasing and increasing gain distributed fiber amplifiers with a finite gain bandwidth by simulating with the nonlinear Schrödinger equation. Our results have shown that the pulse amplifications in both exponential decreasing and increasing gain distributed amplifiers are restricted by the finite gain bandwidth, which limits the output linearly chirped pulse energies achieved. For a given input pulse, by choosing small initial gain coefficient and gain variation rate, the whole gain for the pulse amplification limited by the gain bandwidth may be higher, which is helpful to enhance the output linearly chirped pulse energy. Compared to the decreasing gain distributed amplifier, the increasing gain distributed fiber amplifier may be more conducive to suppress the pulse spectral broadening due to the SPM effect and increase the critical amplifier length for achieving a larger output linearly chirped pulse energy if they have the same minimum gain coefficient and variation rate. For a given increasing gain distribution, the amplifier length may have to be designed according to the input pulse to achieve a large output linearly chirped pulse energy. Therefore, in practical applications, the low dopant concentration fiber may be benefit to enhancing the output linearly chirped pulse energy for a fiber amplifier using the counter-directional pumping scheme if the finite gain bandwidth is taken into accounted.

References

[1] Schaffer C B, Brodeur A, García J F and Mazur E 2001 *Opt. Lett.* **262** 93

[2] Wang Y M, Shen H, Hua L Q, Hu C J and Zhang B 2009 *Opt. Express* **17** 10506

[3] Korte F, Adams S, Egbert A, Fallnich C, Ostendorf A, Nolte S, Will M, Ruske J, Chichkov B N and Tuennermann A 2000 *Opt. Express* **7** 41

[4] Wei Z Y, Wang Z H, Wang P, Ling W J, Zhu J F, Han H N and Zhang J 2008 *IFSA* 032003

[5] Ma J, Liu Y and Zhang L J 2011 *Acta Phys. Sin.* **60** 024211 (in Chinese)

[6] Meng F, Cao S Y, Cai Y, Wang G Z,Cao J P, Li T C and Fang Z J 2011 *Acta Phys. Sin.* **60** 110601 (in Chinese)

[7] Ortac B, Hideur A, Chartier T, Brunel M and Özkul C 2003 *Opt. Lett.* **28** 1305

[8] Liu X M 2010 *Phys. Rev.* A **81** 053819

[9] Liu X M 2009 *Opt. Express* **17** 9549

[10] Dudler J, Fino C, Richard D J and Guymillot 2007 *Nature Phys.* **3** 597

[11] Anderson D, Desaix M, Karlsson M, Lisak M and Quiroga-Teixeiro M L 1993 *J. Opt. Soc. Am.* B **10** 1185

[12] Haghgoo S and Ponomarenko S A 2011 *Opt. Express* **19** 9750

[13] Zhang J Y, Tian Q, Wang Y Y and Dai C Q 2010 *Phys. Rev.* A **81** 023832

[14] Renninger W H, Chong A and Wise F W 2010 *Phys. Rev.* A **82** 021805

[15] Tamura K and Nakazawa M 1996 *Opt. Lett.* **21** 68

[16] Fermann M E, Kruglov V I, Thomsen B C, Dudley J M and Harvey J D 2000 *Phys. Rev. Lett.* **84** 6010

[17] Kruglov V I, Peacock A C and Harvey J D 2002 *J. Opt. Soc. Am.* B **19** 461

[18] Kruglov V I, Peacock A C and Harvey J D 2003 *Phys. Rev. Lett.* **90** 113902

[19] Kruglov V I, Peacock A C and Harvey J D 2000 *Opt. Lett.* **25** 1753

[20] Zhang J F, Tian Q, Wang Y Y, Dai C Q and Wu L 2010 *Phys. Rev.* A **81** 023832

[21] Chang G, Galvanauskas A, Winful H G and Norris T B 2004 *Opt. Lett.* **29** 2647

[22] Boyer G 1999 *Opt. Lett.* **24** 945

[23] Soh B S D, Nilsson J and Grudinin A B 2006 *J. Opt. Soc. Am.* B **23** 10

[24] Zaouter Y, Papadopoulos D N, Hanna M, Druon F, Cormier E and Georges P 2002 *Opt. Express* **15** 9372

[25] Agrawal G P 2002 *Nonlinear Fiber Optics* (New York: Academic Press) p. 67

[26] Liu X M and Mao D 2010 *Opt. Express* **18** 8847

[27] Liu X M 2009 *Opt. Express* **17** 22401