Reflecting single attosecond pulse by periodic Mo/Si multilayer mirrors with different layers

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The reflecting of single attosecond pulse from a periodic Mo/Si multilayer was investigated. With changing the number of bi-layers, the periodic multilayer showed greatly different spectral and temporal responses of the attosecond pulse reflection, which has been discussed in detail in this paper. The capability of attosecond pulse reflection of the periodic multilayers with different bi-layer numbers has been evaluated using suitable temporal parameters. In addition, the condition for obtaining high-efficiency reflected pulses has been analyzed by comparing the pulse responses of the periodic multilayer with different layers. The transfer-matrix method together with the fast Fourier transform has been used in our simulation.

Keywords: attosecond pulse, periodic multilayer, pulse response

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1. Introduction

Generation of attosecond pulses, such as high order harmonics (HOH) generated by the interaction of femtosecond laser with rare gas, has attracted much attention over the last decade.[1] Serious progresses have been reported worldwide in the study of attosecond pulse trains[2,3] and single isolated pulses[4−7] based on the high harmonic generation (HHG), which is generally located at short wavelengths in the extreme ultraviolet (EUV) or even soft X-ray (SXR) spectral range. In addition, many efforts have been made in theory to decrease the duration of isolated pulses as much as possible,[8−11] showing their wide application prospects in ultrafast optics.

As the vital reflector and filter for attosecond pulses in EUV and XUV regions, both periodic[4,7] and aperiodic[5,6] multilayers have often been used in the attosecond experiments due to their efficient broadband reflections as well as their excellent reflection phase characteristics, which are both necessary for the attosecond pulse control. Furthermore, since the chirped (aperiodic) multilayer structure has been designed for attosecond pulse compression by compensating the intrinsic chirp of HHG,[12,13] a lot of researches about the attosecond pulse reflection and compression using the aperiodic multilayer have been done,[14,15] following the previous work done by Beigman *et al.*[16,17]

Because of the totally irregular thickness distribution of layers in the aperiodic multilayer, the aperiodic multilayer structure can achieve a more flexible spectrum and a better phase control than the periodic one[18] during the pulse reflection and compression. However, for a single attosecond pulse generated in the cut-off region of HHG, the intrinsic chirp can become very small when the optimized conditions for the production of short attosecond pulses have been fulfilled.[19] In this case, the additional dispersion, which can cause pulse broadening, should not be induced when using a multilayer to reflect such a single pulse. However, for the periodic multilayer, it is easy to achieve the nearly no-dispersion reflection in its Bragg reflection band.[20]

Furthermore, the periodic multilayer with a small bi-layer number can also afford enough broadband in EUV or SXR region, which makes it suitable for reflecting narrow single attosecond pulses. Besides, the simple structure characteristics of the periodic multilayer make it not only easy in design and optimization for attosecond pulse reflection, but also very convenient in practical fabrication.

In this paper, the spectral and the temporal pulse responses of periodic multilayers with different layers for a single attosecond pulse are discussed in de-

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tail. Several temporal parameters are used to evaluate the abilities of reflecting attosecond pulse by periodic multilayers with different layers. A common method, which has been widely used in the study of reflecting femtosecond pulse by multilayers[21,22] and crystals,[23] is used to obtain the temporal pulse responses of difference multilayers in the attosecond domain.

2. Numerical approach

The computing method in this paper is based on the research of Beigman *et al.*,[16,17] which combines the transfer-matrix method (TMM) with the fast Fourier transform (FFT) to obtain the spectral and the temporal responses of periodic multilayers for a single attosecond pulse. The computing procedures are described as follows.

Firstly, the complex amplitude reflection coefficient $r(\omega)$ of multilayer should be calculated by using the TMM. The index of refraction n of every material in the multilayer should be known. The index of refraction for a material can be easily obtained as

$$n = 1 - n_{\mathrm{a}} r_{\mathrm{e}} \lambda^2 (f_1 + \mathrm{i} f_2)/2\pi, \tag{1}$$

where n_{a}, r_{e}, and λ represent the density of the material, the classical electron radius, and the wavelength, respectively, and the atomic scattering factors $f_{1,2}$ for nearly all materials have been measured by Henke and Gullikson.[24] For example, the indexes of refraction for Mo and Si at 93 eV can be calculated based on Eq. (1) to be $n_{\mathrm{Mo}} = 0.927 + \mathrm{i}0.006$, $n_{\mathrm{Si}} = 1.0003 + \mathrm{i}0.0018$, respectively.

With $r(\omega)$, we can determine the spectral reflectivity $R(\omega)$ and the phase of multilayer $\phi(\omega)$ by

$$R(\omega) = |r(\omega)|^2, \tag{2}$$

$$\phi(\omega) = \arctan\left(\frac{\mathrm{Im}(r(\omega))}{\mathrm{Re}(r(\omega))}\right), \tag{3}$$

where $\mathrm{Re}(r(\omega))$ and $\mathrm{Im}(r(\omega))$ represent the real and the imaginary parts of complex amplitude reflection coefficient $r(\omega)$. Generally, for the periodic multilayer, the maximum reflectivity will show with the light whose wavelength fulfills the Bragg condition

$$2d\cos\theta = m\lambda, \tag{4}$$

where d, θ, m, and λ represent the period of the multilayer, the incident angle of the light, the order of reflection, and the wavelength of the incident light, respectively.

Secondly, we perform the following calculations:

$$E_0(\omega) = \int_{-\infty}^{\infty} E_0(t)\exp(-\mathrm{i}\omega t)\,\mathrm{d}t, \tag{5}$$

$$E(\omega) = E_0(\omega) r(\omega), \tag{6}$$

$$E(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} E(\omega)\exp(\mathrm{i}\omega t)\,\mathrm{d}t. \tag{7}$$

Because the complex amplitude reflection coefficient $r(\omega)$ is used in calculating the reflected pulse field $E(t)$, the phase of $r(\omega)$ as well as the modulus of $r(\omega)$ plays an important role in determining $E(t)$.

In order to simulate the light source, we assume the incident pulse to be a Gaussian-shaped pulse, which is given by

$$E_0(t) = \exp\left[-\frac{\pi}{2}(t/\tau_0)^2\right]\exp[\mathrm{i}\omega_0 t], \tag{8}$$

where parameters τ_0 and ω_0 represent the duration and the center frequency of the incident pulse, respectively.

Three parameters are employed to describe the features of the reflected pulse by the multilayer. The first one is the pulse duration τ, which describes the effective pulse width in the time domain

$$\tau = \int_{-\infty}^{\infty} I(t)\,\mathrm{d}t/I_{\max}, \tag{9}$$

where $I(t) = |E(t)|^2$ gives the envelope intensity of the pulse, and $I_{\max}$ is the maximum of $I(t)$. The second one is the pulse reflection coefficient R, which is defined as the reflected-to-incident pulse energy ratio

$$R = \int_{-\infty}^{\infty} I(t)\,\mathrm{d}t \Big/ \int_{-\infty}^{\infty} I_0(t)\,\mathrm{d}t. \tag{10}$$

The last one is the power reflection coefficient R_{p} defined by

$$R_{\mathrm{p}} = R(\tau/\tau_0) = I_{\max}/I_{0\max}, \tag{11}$$

which will be used to evaluate the ability of reflecting pulse by the multilayer.

Actually, because the power reflection coefficient R_{p} has the same value with the peak power $I_{\max}$ of the reflected pulse, and the peak power $I_{\max}$ comes from the coherent superposition of reflection power at each layer in the multilayer, R_{p} can be used to describe the coherent performance of the reflection system.

3. Reflectivity and group delay dispersion of periodical multilayer

As discussed previously, to obtain a nearly transform limited reflected pulse with high reflection efficiency from a no-chirp incident pulse by using the multilayer, not only should the high spectral reflectivity in the broadband region of the incident pulse be required, but also the dispersion of the reflecting mirror, which may cause serious pulse broadening to the incident pulse, needs to be suppressed at the whole reflection region. In this case, a periodic Mo/Si multilayer is a good choice for reflecting attosecond pulses in the EUV region, because the broadband high reflectivity together with the low dispersion in the reflection band can be achieved easily by selecting proper parameters for the periodic multilayer.

Figure 1 shows the reflectivity and the group delay dispersion (GDD) of a classic periodic Mo/Si multilayer with 20 bi-layers under the normal incidence ($\theta = 0°$). Period $d = d_{\rm Si} + d_{\rm Mo}$ and thickness ratio $\gamma = d_{\rm Mo}/(d_{\rm Si} + d_{\rm Mo})$ of the multilayer are 6.8 nm and 1/3, respectively. These parameters of the multilayer are designed to make sure that the Bragg reflection peak is located at 93 eV under the normal incidence ($\theta = 0°$) according to Eq. (4). As predicted, the reflection spectrum of the periodic multilayer shown in Fig. 1 centers at 93 eV with a bandwidth of 5.3 eV, and nearly 60% peak reflectivity $R_{\rm peak}$ is achieved.

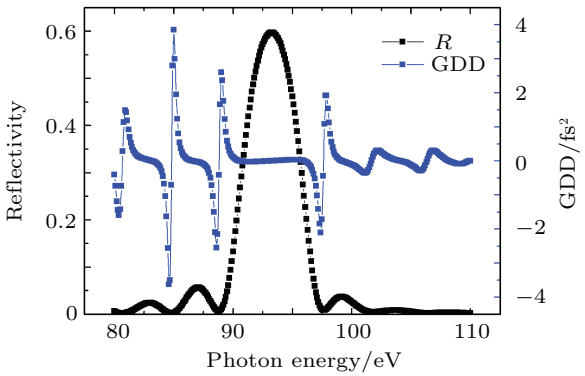


Fig. 1. (colour online) Reflectivity and group delay dispersion as functions of photon energy under the normal incidence ($\theta = 0°$) for periodic Mo/Si multilayer with 20 periods.

In addition, because of the superposition of energy in phase near the Bragg reflection peak, almost zero group delay dispersion is found inside the Bragg reflection band, which has nearly the same width as the bandwidth of the Bragg reflection peak, as shown in Fig. 1. Besides, large oscillations of GDD shown at the band edges of the Bragg reflection may lead to serious pulse broadening during the attosecond pulse reflection.

Unfortunately, the 5.3 eV bandwidth may not be enough for reflecting the attosecond pulse with duration under 300 as according to the relation between the time domain and the frequency–energy domain in the Fourier transform theory[25]

$$\Delta\Gamma\Delta E \geq 1.82, \tag{12}$$

where $\Delta\Gamma$ is the length of the pulse in units of femtosecond, and ΔE is the full width at half-maximum (FWHM) bandwidth in units of eV.

One way to broaden the reflection bandwidth is to reduce the period number N of the periodic multilayer. Because the peak reflectivity $R_{\rm peak}$ and the spectral bandwidth ΔE of the periodic multilayer are mainly determined by the number of interfaces $N_{\rm eff}$ contributing to the reflectivity, which is defined as[17]

$$N_{\rm eff} = 2t_{\rm g}/T = \frac{\omega}{\pi}\frac{{\rm d}\phi(\omega)}{{\rm d}\omega}, \tag{13}$$

where $t_{\rm g}$, T, and $\phi(\omega)$ represent the time delay of the reflected pulse, the wave periods of the incident light, and the phase of the multilayer, respectively. Normally, the reflectivity of periodic multilayer is proportional to $N_{\rm eff}$, while the bandwidth is in inverse proportional to $N_{\rm eff}$.[26]

In Fig. 2, the reflectivity and the group delay dispersion of periodic Mo/Si multilayers with different periods are plotted in the spectral domain. All parameters of the periodic multilayers are the same as those used for Fig. 1, except the changing bi-layer number N.

Obviously, with the increasing bi-layer number N, the peak reflectivity of the periodic multilayer goes up firstly, while the band of reflection becomes narrower. And then, both of them remain unchanged after $N_{\rm eff}$ achieves the maximum value. In Fig. 2(a), when N reaches 30, the stable peak reflectivity and bandwidth are obtained. In this case, the broadband reflection can be achieved by reducing the layers of the periodic multilayer, however, at the expenses of reflectivity. So, properly choosing the layer number of the periodic multilayer to obtain a high-efficiency reflected pulse with a proper pulse duration is extremely important

for reflecting attoseocond pulse by the periodic multilayer. Meanwhile, the nearly zero-GDD regions of the periodic multilayers with different bi-layer numbers all exhibit the similar bandwidth with the reflection band shown in Fig. 2(b), proving the great potential for attosecond pulse reflection.

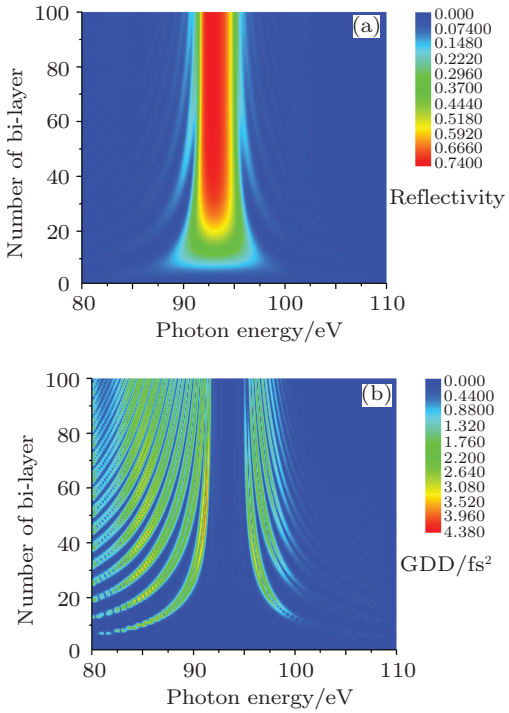


Fig. 2. (colour online) (a) Reflectivity and (b) group delay dispersion of periodic Mo/Si multilayers with different numbers of bi-layer in the spectral domain.

4. Numerical simulation of single attosecond pulse response from periodic multilayer

In order to evaluate the capabilities of reflecting attosecond pulse of the periodic Mo/Si multilayers with different bi-layer numbers, we use the single Gaussian pulse centered at 93 eV with 250 as duration given in Eq. (8),[4] and study the pulse responses of the periodic multilayers to such an incident pulse. In Fig. 3, the temporal and the spectral profiles of the incident pulse are displayed.

In addition, the same Mo/Si periodic multilayers with different bi-layer numbers showed in Fig. 2 will be employed in our temporal simulation. Based on Eqs. (5)–(7), the pulse responses of such periodic multilayers are easy to be calculated, which are shown in Fig. 4.

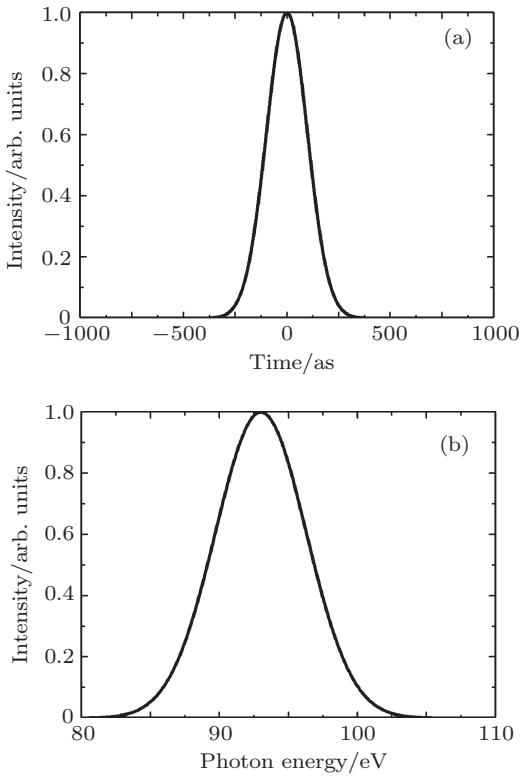


Fig. 3. (a) Temporal and (b) spectral profiles of the incident Gaussian pulse.

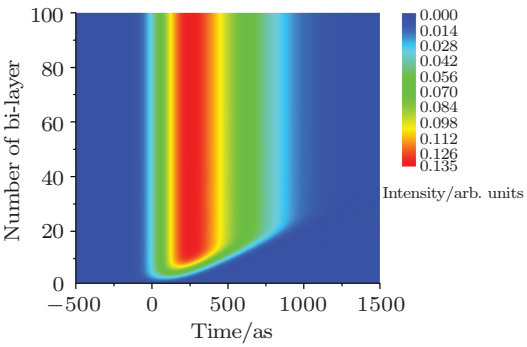


Fig. 4. (colour online) Pulse responses of periodic Mo/Si multilayers with different numbers of bi-layer.

From Fig. 4, we can see that the duration of the reflected pulse will be enlarged with the increasing bi-layer number N of the periodic multilayer until N approaches 30. With more than 30 periods, the reflected pulses will have a similar shape, and the durations will stay at the same value, which are in accordance with the spectral reflection responses of the periodic multilayers shown in Fig. 2. We also note that the high reflection region of pulse response always appears within a special time domain, independent of the bi-layer number. Such a high temporal synchronism comes from the flat group delay at the band of Bragg reflection peak, and shows the high temporal coherence of the periodic multilayer.

Furthermore, the duration τ, the pulse reflection coefficient R, and the power reflection coefficient $R_{\rm p}$, which have been defined in Eqs. (9), (10), and (11), respectively, are calculated for periodic multilayers with different N and shown in Fig. 5.

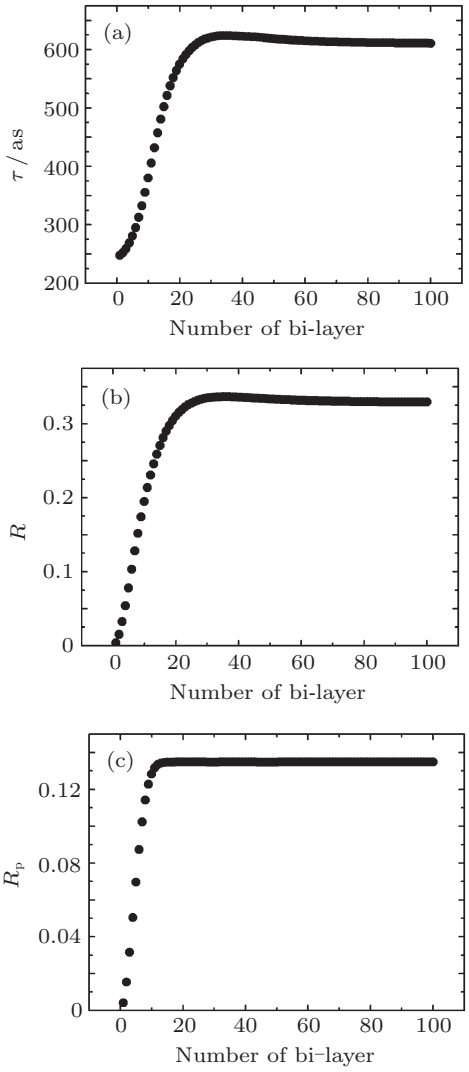


Fig. 5. (a) Duration τ, (b) pulse reflection coefficient R, and (c) power reflection coefficient $R_{\rm p}$ of reflected pulses each as a function of the number of bi-layer.

For duration τ and pulse reflection coefficient R, 30 bi-layers in the periodic multilayer are enough to get stable values for both parameters. For $N < 30$, there is a trade-off between reflection coefficient R and pulse duration τ, which means a high reflection coefficient can be obtained at expenses of a short pulse duration. Indeed, for the periodic multilayer with only several bi-layers, nearly transform limited reflected pulses can be obtain, with a pulse reflection coefficient nearly 5%.

In order to evaluate the capability of reflecting attosecond pulse of the periodic multilayer, power reflection coefficient $R_{\rm p}$ is shown in Fig. 5(c). According to its definition in Eq. (11), a higher $R_{\rm p}$ means a larger pulse reflection coefficient R together with a small duration τ, which is preferred in the attosecond pulse reflection.

In Fig. 5(c), although the curve of $R_{\rm p}$ exhibits the similar trend with that of R and τ, the maximum $R_{\rm p}$ appears when the bi-layer number N of the periodic multilayer approaches about 11, not 30, indicating that the coherent superposition of power for the periodic multilayer only exists when N is less than 11. However, for the periodic multilayer with more than 11 bi-layers, even the pulse reflection coefficient R keeps rising when N increases from 11 to 30, the power reflection coefficient $R_{\rm p}$ of the reflection system remains at the same value due to the incoherent power superposition caused by the group delay dispersion of the multilayer. In additional, when the maximum power reflection is first achieved, the reflected pulse with 406 as duration and nearly 21% pulse reflection is obtained.

In Fig. 6, the incident Gaussian pulse and the GDD of the periodic multilayer with 11 bi-layers are plotted. From this figure, we can see that the spectral components of the incident pulse just land in the low dispersion reflection band of the periodic multilayer, displaying the critical situation of nearly zero-GDD reflection in the periodic multilayer.

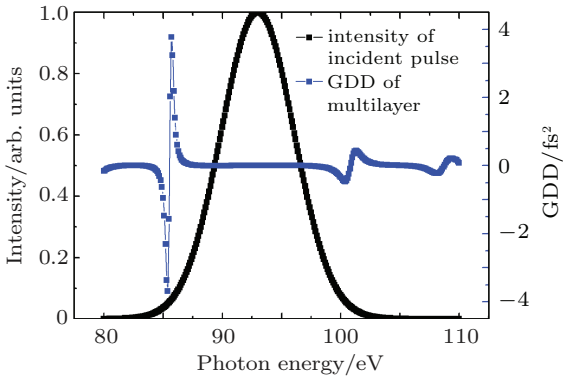


Fig. 6. (colour online) The incident Gaussian pulse and the group delay dispersion of the periodic multilayer with 11 bi-layers.

Furthermore, if we define the coherent length $L_{\rm c}$ of the incident pulse as

$$L_{\rm c} = \tau \times c, \tag{14}$$

where τ is the duration of the incident pulse, and c denotes the speed of light. The $L_{\rm c}$ is equal to 75 nm in our model. On the other hand, the critical length of

the periodic multilayer $L_{\rm m}$ with the maximum power reflection coefficient $R_{\rm p}$ can be calculated by

$$L_{\rm m} = N_l \times d, \tag{15}$$

where N_l is the least bi-layer number needed for the periodic multilayer to achieve the maximum power reflection, and d is the length of one periodic in the periodic multilayer. The result is 74.8 nm, nearly approaching the coherent length of the incident pulse, so we have

$$L_{\rm c} \approx N_l \times d. \tag{16}$$

In order to confirm the above hypothesize, a series of incident Gaussian pulses with different durations are designed to verify Eq. (16), the results are shown in Fig. 7 and Table 1.

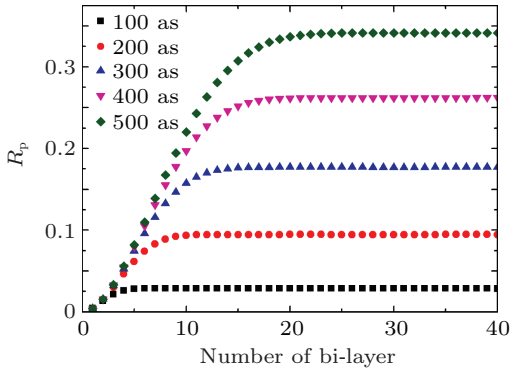


Fig. 7. (colour online) Power reflection coefficient $R_{\rm p}$ as a function of the number of bi-layers for incident Gaussian pulses with different durations.

Table 1. Coherent characteristics of periodic multilayers

τ/as	N_l	$L_{\rm c}$/nm	$L_{\rm m}$/nm
100	5	30	34.0
200	9	60	61.2
300	13	90	88.4
400	18	120	122.4
500	22	150	149.6

In Fig. 7, the power reflection coefficients $R_{\rm p}$ as a functions of the number of bi-layers have been shown for incident Gaussian pulses with different durations. The periodic multilayers used here share the same structure parameters with the former ones. The coherent length $L_{\rm c}$ of the incident pulse and the critical length $L_{\rm m}$ of the periodic multilayer with the maximum power reflection are listed in Table 1. The similar values of $L_{\rm c}$ and $L_{\rm m}$ under different incident pulses give a strong evident for the validity of Eq. (16), indicating a simple and effective way of choosing the periodic multilayer for high-efficiency attosecond pulse reflection.

5. Conclusion

In this paper, the spectral and the temporal responses of periodic Mo/Si multilayers with different layers for single attosecond pulse reflection have been shown. The spectral reflection and dispersion characteristics of periodic multilayers with different layers are displayed, showing the potential capability for attosecond pulse reflection. Then, pulse responses of different periodic multilayers for a designed incident Gaussian pulse with 250 as duration have been discussed. The maximum power reflection coefficient can be obtained when the bi-layer number of the reflective periodic multilayer reaches 11 due to the coherent superposition of power at each layer in the multilayer. Furthermore, an almost equivalent relation between the coherent length of the incident pulse and the length of the periodic multilayer with the least layers to achieve the maximum power reflection has been shown for different incident pulses, indicating an effective way to pick up the suitable periodic multilayer for high-efficiency attosecond pulse reflection.

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