A method for phase reconstruction in the optical three-dimensional shape measurement*

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In the optical three-dimensional shape measurement, a method of improving the measurement precision for phase reconstruction without phase unwrapping is analyzed in detail. Intensities of any five consecutive pixels that lie in the x-axis direction of the phase domain are given. Partial derivatives of the phase function in the x- and y-axis directions are obtained with a phase-shifting mechanism, the origin of which is analysed. Furthermore, to avoid the phase unwrapping in the phase reconstruction, we derive the gradient of the phase function and perform a two-dimensional integral along the x- and y-axis directions. The reconstructed phase can be obtained directly by performing the numerical integration, and thus it is of great convenience for phase reconstruction. Finally, the results of numerical simulations and practical experiments verify the correctness of the proposed method.

Keywords: phase reconstruction, optical three-dimensional shape measurement, measurement precision

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1. Introduction

Phase measurement of three-dimensional (3D) object shape plays a very important role in the automatic optical noncontact measurement of 3D object shape. It has been widely used in machine vision, solid modeling, industrial auto-measuring, etc.[1−14] and must undergo the phase reconstruction. However, in the process of phase reconstruction, the phase is difficult to measure because of the existence of a wrapped phase, which makes the object shape difficult to be reconstructed accurately. Thus, the wrapped phase will reduce the measurement precision, making the phase reconstruction become an important topic in the measurement of noncontact 3D object shape.[1−14]

In order to increase the measurement precision of three-dimensional object shape, many scholars have studied the phase reconstruction and acquired more and more satisfactory research achievements.[1−12] In 2010, Nelleri *et al.*[2] achieved phase reconstruction via lensless digital in-line holographic microscopy. In 2000, Yanez-Mendiola *et al.*[4] obtained the wrapped phase in an interferogram with a linear carrier using the iterative method. In 2008, Gao *et al.*[6] realized phase reconstruction from a single carrier-frequency interferogram without phase unwrapping. In 2009, Qiao *et al.*[7] achieved phase reconstruction in the measurement of discontinuous object based on dual-frequency grating.

The purpose of this paper is to propose a method of phase reconstruction without phase unwrapping in the phase measurement of 3D object shape. Firstly, the reason that phase cannot be unwrapped is analyzed in detail. Secondly, a method of phase reconstruction without phase unwrapping is proposed. Finally, numerical simulations and practical experiments are performed, respectively. Comparison between them supports the validity of the theory and the effectiveness of the proposed method.

2. Theoretical analysis

In a Cartesian coordinate system, the intensity distribution on a two-dimensional CCD output plane can be described as follows:

$$I(x,y) = I_{\rm b}(x,y) + I_{\rm m}(x,y)\cos[\phi(x,y) + \omega_0 x], \quad (1)$$

where $I_{\rm b}(x,y)$ denotes the background intensity function, $I_{\rm m}(x,y)$ denotes the intensity modulation function, $\phi(x,y)$ denotes the phase function, and ω_0 denotes the spatial carrier frequency (angular frequency)

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in the x-axis direction, which is formed by atilt between the reference and object waves.

According to Refs. [4] and [5], the three functions in Eq. (1) vary slowly compared with the spatial carrier frequency. Thus, the intensities of any five consecutive pixels that lie in the x-axis direction of the phase domain can be given as follows:

$$\begin{aligned} I(x-i,y) = {} & I_{\rm b}(x,y) + I_{\rm m}(x,y) \\ & \times \cos[\phi(x-i,y) + \omega_0(x-i)], \\ & i = -2,-1,\ldots,2. \end{aligned} \tag{2}$$

Combining Eq. (2) with phase-shifting mechanism,[8] the partial derivatives $\phi_x(x,y)$ in the x-axis direction can be obtained as follows:[6]

$$\begin{aligned} & \phi_x(x,y) \\ & = 2\arctan\sqrt{1-\frac{2(I_2-I_{-2})(2I_0-I_1-I_{-1})}{(I_1-I_{-1})(2I_0-I_2-I_{-2})}} - \omega_0, \end{aligned} \tag{3}$$

where I_{-2}, I_{-1}, I_0, I_1, and I_2 represent $I(x-2,y)$, $I(x-1,y)$, $I(x,y)$, $I(x+1,y)$, and $I(x+2,y)$, respectively.

It can be seen from Eq. (3) that $\phi_x(x,y)$ does not exist when the denominator of fraction equals zero, i.e., at a discontinuous point. The $\phi_x(x,y)$ cannot be unwrapped at all since it varies slowly in the carrier-frequency interferogram and is kept in the domain from $-\pi$ to π (rad per pixel), as shown in Fig. 1. In order to avoid the discontinuous point, namely, to avoid phase unwrapping, we can substitute the average of its adjacent three points by the value of $\phi_x(x,y)$.

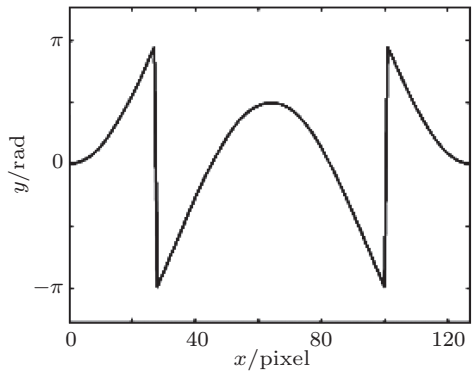


Fig. 1. The partial derivatives of the phase function.

Combining Eq. (2) with the phase-shifting mechanism, the partial derivative $\phi_y(x,y)$ of the phase function $\phi(x,y)$ in the y-axis direction can also be obtained as follows:[6]

$$\phi_y(x,y) = 2\tan(\frac{\phi_x+\omega_0}{2})\frac{I_{-1}I_{1y}-I_1I_{-1y}+I_1I_{0y}-I_0I_{1y}+I_0I_{-1y}-I_{-1}I_{0y}}{(2I_0-I_{-1}-I_1)^2+(I_{-1}-I_1)^2\tan^2[(\phi_x+\omega_0)/2]}, \tag{4}$$

where I_{-1y}, I_{0y}, and I_{1y} denote the derivatives of the intensity distributions I_{-1}, I_0, and I_1, respectively, in the y-axis direction.

According to the optical configuration of the measurement system (see Fig. 1 in Ref. [7]), when we carry out one-dimension Fourier transform to Eq. (1) along the x-axis direction, then separate the components of the two spectra by designing a suitable filter, finally perform reverse Fourier transform, we can obtain the relationship between the height information $h(x,y)$ of the measured object and the phase function $\phi(x,y)$ as follows:

$$h(x,y) = \frac{L_0[\phi(x,y)-\phi(x_0,y_0)]}{\phi(x,y)-\phi(x_0,y_0)-\omega_0 d}, \tag{5}$$

where $\phi(x_0,y_0)$ denotes the phase value of the initial point in the phase domain, d denotes the distance between P_2 and I_2, and L_0 denotes the distance between I_2 and O. All of these parameters are shown in Fig. 1 in Ref. [7].

The phase function $\phi(x,y)$ can also be obtained by performing the line integral of its gradient.[9] The gradient of the phase function $\phi(x,y)$ can be expressed as follows:

$$\nabla\phi(x,y) = \phi_x(x,y)i + \phi_y(x,y)j, \tag{6}$$

where i and j denote the unit vectors in the x- and y-axis directions, respectively.

We perform the two-dimensional integral along the x- and y-axis directions, and can obtain the phase function as follows:

$$\phi(x,y) = \phi(x_0,y_0) + \int_{x_0}^{x}\int_{y_0}^{y} \boldsymbol{\nabla}\phi(x,y)\,\mathrm{d}\boldsymbol{x}\,\mathrm{d}\boldsymbol{y}. \tag{7}$$

Then $\phi(x_0,y_0)$ that is known can be eliminated from Eqs. (5) and (7).

With the method of line integral of the phase gradient, the phase unwrapping is avoided. Thus, we can determine the phase for any domain shape.

Performing the numerical integration, we can obtain the reconstructed phase up to a constant offset,

which depends on the selection of the initial point (x_0, y_0). The reconstructed phase $\phi_{\rm r}(x, y)$ can be obtained as follows:

$$\begin{aligned}\phi_{\rm r}(x,y) &= \phi(x,y) - \phi(x_0,y_0) \\ &= \int_{x_0}^{x}\int_{y_0}^{y} \boldsymbol{\nabla}\phi(x,y)\cdot {\rm d}\boldsymbol{x}{\rm d}\boldsymbol{y}. \end{aligned} \tag{8}$$

This method reconstructs the phase by directly obtaining and integrating the derivatives of the desired phase. Therefore, it makes the phase reconstruction very convenient. It can also be seen from Eqs. (6)–(8) that $\phi_x(x, y)$, $\phi_y(x, y)$, and $\phi_r(x, y)$ do not dependent on the amplitudes of both the object wave and the reference wave. Hence, the method is insensitive to the non-uniformity of the intensity distribution of the interferogram.

The validity of the reconstruction method is decided by the value of phase error. The smaller the value is, the higher the precision of the reconstructed phase is.

The phase error arises mainly from the numerical integration given by Eqs. (5)–(7), which may be minimized by taking a few preventive steps. The phase error is defined as the difference between the reconstructed phase and the original phase, i.e.,

$$\Delta\phi(x,y) = \phi_{\rm r}(x,y) - \phi(x,y). \tag{9}$$

3. Results of simulation and experiment

3.1. Results of simulation

In order to verify the validity of the theoretical analyses, we numerically simulate an object with 256×256 pixels, and the object is $h(x, y) = 20[(x/80 - 12)^2 + (y/80 - 12)^2]$. The structure parameters in the measurement system are $d = 175$ mm and $L_0 = 1000$ mm. The benchmark period of grating is $T = 8$ s.

The interferogram of the simulated object is shown in Fig. 2(a). The simulated phase is shown in Fig. 2(b). The reconstructed phase with this method is shown in Fig. 2(c), and the phase reconstruction takes less than 1 s on a personal computer. The phase error is shown in Fig. 2(d), which further demonstrates the validity of the reconstruction method. It can be seen that the phase error is very small, and the average value is smaller than 0.05 rad per pixel.

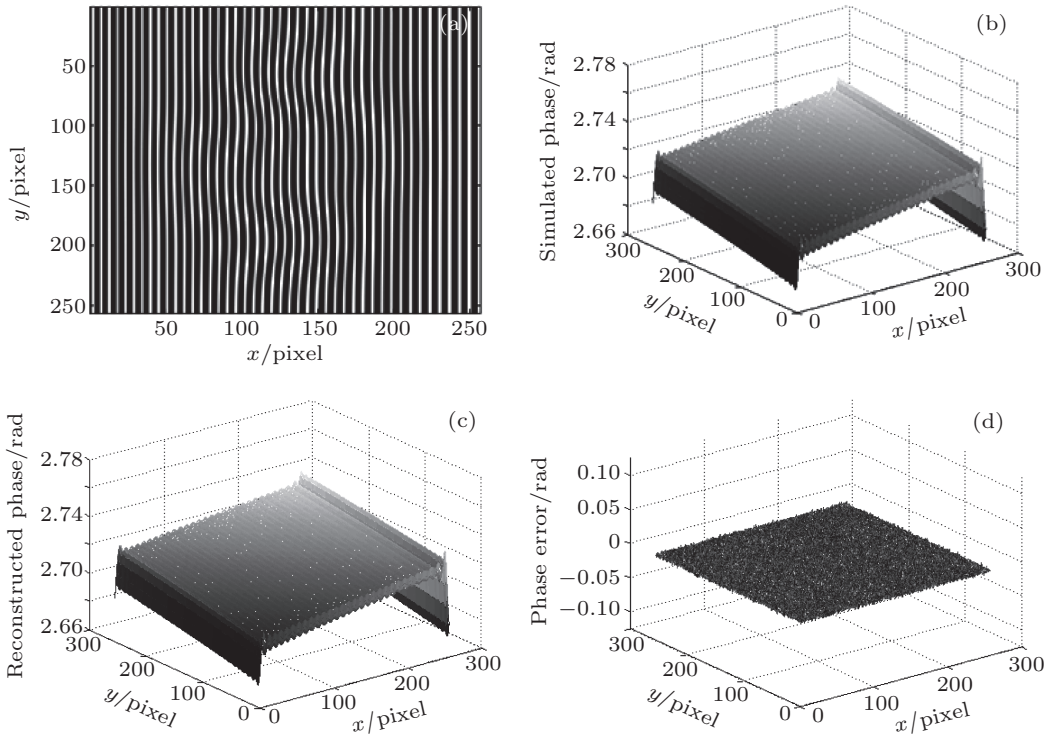


Fig. 2. (a) Interferogram with the simulated object, (b) simulated phase, (c) reconstructed phase, and (d) error in phase reconstruction.

3.2. Experimental Results

In order to further test the validity of the method, we carry out a practical experiment using a standard half spherical surface as a test object. The interferogram captured by a CCD with resolving power of 512×512 pixels is shown in Fig. 3(a). We can see from Fig. 3(a) that the intensity distribution of the interferogram is nonuniform.

We can obtain reconstructed phase of the half spherical surface with the method mentioned above. By performing the integral of the two partial derivatives, the reconstructed phase we obtain is shown in Fig. 3(b). Clearly, the phase is recovered very well.

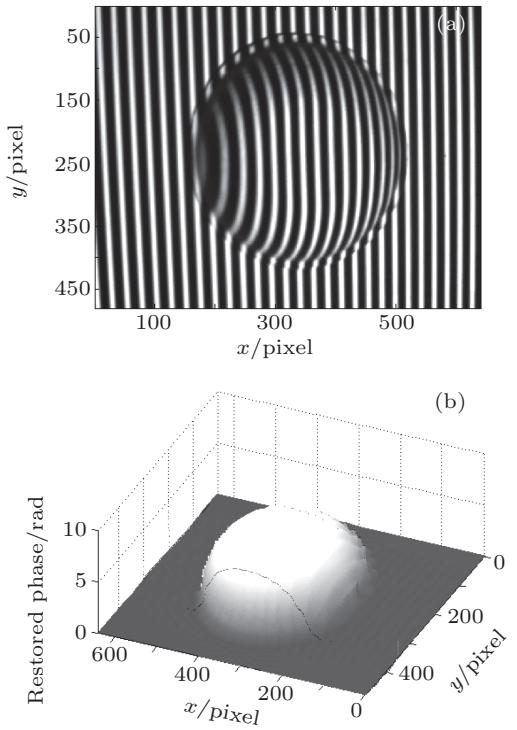


Fig. 3. (a) Interferogram captured by CCD; (b) reconstructed phase of the half spherical surface.

4. Conclusions

In this paper we propose a method for phase reconstruction without phase unwrapping. The method can avoid phase unwrapping and facilitate the phase reconstruction. Numerical simulations and practical experiments are implemented to test the proposed theory. Good match between the results supports the validity of the theoretical analyses.

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