# Gain enhancement in two-stream free electron laser with planar wiggler and axial guide magnetic field

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The theory for the two-stream free electron laser (FEL) consisting of a relativistic electron beam transported along the axis of a planar wiggler in the presence of an axial guiding magnetic field is proposed and investigated. The electron trajectories and the small signal gain are derived. The characteristic of the linear gain and the normalized maximum gain are studied numerically. The result shows that the normalized maximum gain is considerably enhanced in comparison with that of the single stream. The effect of the difference between the energies of the two beams in this configuration of FEL is also considered, and we find that the gain is affected by the energy differences between groups 1 and 2.

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## 1. Introduction

In the free electron laser (FEL), the wiggler field is responsible for the instability of the relativistic electron beam. The large-amplitude short-wavelength field generated is due to the interaction of a radiation field with this unstable beam. The planar wiggler field must be approximately 71% higher than the helical wiggler field to have a comparable effect, but the planar wiggler can more easily adjusted in the direction of the axis of symmetry. The use of the guide field can enhance the transverse wiggler-induced velocity and the gain of FEL.[1−3]

The two-stream instability has been studied extensively in FELs during the last few decades.[4−10] Botton and Ron[11] calculated the gain of an electromagnetic wave coupled with the unstable electrostatic mode of a two-cold-stream beam. Bekefi and Jacobs[10] have shown that the injection of a prebunched electron beam results in the two-stream instability and greatly enhances the radiation intensity of the wave in the Raman regime. Seo and Park[4] have studied a FEL with a dual beam in the Raman regime, compared the two-stream instability with the Raman instability, and showed that in a limited weak wiggler strength, the peak growth rate of the two-stream mode might be much higher than that of the Raman mode. The effects of the two-stream instability on the growth rate of the FEL with the planar wiggler and an axial magnetic field have been investigated by Mehdian *et al.*[12].

In this paper, a two-stream free electron laser in the presence of the axial guiding magnetic field is investigated. In Section 2, the analysis of single-particle trajectories of the two streams in the presence of the axial guide magnetic field and the planar wiggler are given. Section 3 is devoted to the derivation of the small-signal gain. The numerical results and the conclusion are given in Sections 4 and 5, respectively.

## 2. Single particle orbits

The system studied is a two-stream relativistic electron beam propagating through a planar wiggler in the presence of an axial magnetic field. The planar wiggler and the axial guide magnetic fields can be expressed as

$$\boldsymbol{B}_{\rm w} = B_{\rm w} \sin\left(k_{\rm w} z\right) \hat{\boldsymbol{e}}_y, \tag{1}$$

$$\boldsymbol{B}_0 = B_0 \hat{\boldsymbol{e}}_k, \tag{2}$$

where $B_{\rm w}$ and $k_{\rm w}$ are the wiggler amplitude and the wave number, respectively.

The relativistic equations of motion for an electron with rest mass $m$ and charge $-e$ moving with

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velocity $v$ and energy $\gamma$ in these combined fields are

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t}=-\frac{e}{m}\left[\boldsymbol{E}-\frac{1}{c^2}\boldsymbol{v}\left(\boldsymbol{v}\cdot\boldsymbol{E}\right)+\frac{1}{c}(\boldsymbol{v}\times\boldsymbol{B})\right], \quad (3)$$

$$\frac{\mathrm{d}\gamma}{\mathrm{d}t}=-\frac{e}{mc^2}\boldsymbol{v}\cdot\boldsymbol{E}, \quad (4)$$

where $\boldsymbol{E}$ is the electric component of the electromagnetic wave.

With the aid of Eqs. (3) and (4) and the field Eqs. (1) and (2), the constant transverse velocity of each beam is given by

$$v_{0x}=\frac{k_{\mathrm{w}}v_{\parallel}^2\Omega_{\mathrm{w}}}{\Omega_0^2-k_{\mathrm{w}}^2v_{\parallel}^2}\cos\left(k_{\mathrm{w}}z\right), \quad (5)$$

$$v_{0y}=\frac{\Omega_{\mathrm{w}}\Omega_0v_{\parallel}}{\Omega_0^2-k_{\mathrm{w}}^2v_{\parallel}{}^2}\sin\left(k_{\mathrm{w}}z\right), \quad (6)$$

where $\Omega_{\mathrm{w}}=eB_{\mathrm{w}}/\gamma mc$ and $\Omega_0=eB_0/\gamma mc$ denote the wiggler and the cyclotron frequencies, respectively.

Since the energy is a constant of motion, the average axial velocity over a wiggler period for this trajectory is obtained using the root-mean-square value of the transverse velocity component, and we find

$$\frac{v_{\parallel i}^2}{c^2}\left[1+\frac{\Omega_{\mathrm{w}i}^2\left(\Omega_{0i}^2+k_{\mathrm{w}}^2v_{\parallel i}^2\right)}{2\left(\Omega_{0i}^2-k_{\mathrm{w}}^2v_{\parallel i}^2\right)^2}\right]=1-\frac{1}{\gamma_i^2},\quad i=1,2, \quad (7)$$

where $i$ represents each beam of the two stream beams.

## 3. Small signal gain

Now we consider a small-amplitude plane electromagnetic wave propagating parallel to the axial-guide field and the direction of the two electron beams. The electric component of this radiation field is in the form of

$$\delta\boldsymbol{E}=-\delta E\sin\left(kz-\omega t+\varphi\right)\hat{\boldsymbol{e}}_x, \quad (8)$$

where $\delta E$ is the amplitude of wave, $c$ is the speed of light, $\omega$ and $k$ are the frequency and the wave number, respectively, and $\varphi$ is the initial position of the electron with respect to this wave. So the equation of motion is given by

$$\frac{\mathrm{d}(\gamma_i m\boldsymbol{v}_i)}{\mathrm{d}t}=-e\left(\delta\boldsymbol{E}+\frac{1}{c}\boldsymbol{v}_i\times\boldsymbol{B}\right), \quad (9)$$

$$\frac{\mathrm{d}\delta\gamma_i}{\mathrm{d}t}=-\frac{e}{mc^2}(\boldsymbol{v}_{0i}+\delta\boldsymbol{v}_i)\cdot\delta\boldsymbol{E}. \quad (10)$$

Substituting Eqs. (5), (6), and (7) into Eq. (10) leads to

$$\begin{aligned}\frac{\mathrm{d}\delta\gamma_i}{\mathrm{d}t}=&\frac{e}{2mc^2}\frac{k_{\mathrm{w}}v_{\parallel i}^2\Omega_{\mathrm{w}i}}{\Omega_{0i}^2-k_{\mathrm{w}}^2v_{\parallel i}^2}\delta E\\&\times\{\sin\left[(k+k_{\mathrm{w}})z-\omega t+\varphi\right]\\&+\sin\left[(k-k_{\mathrm{w}})z-\omega t+\varphi\right]\}.\end{aligned} \quad (11)$$

By defining $z=v_{\parallel i}t$ and $\psi_i=(k+k_{\mathrm{w}})v_{\parallel i}-\omega$, the synchronism condition is met when $\psi=0$. Since near synchronism, $\psi\approx0$, the frequency in the second term of Eq. (11) is $-2k_{\mathrm{w}}z$, the average of this term is zero in a few wiggler periods. Thus, we are left with the first term of Eq. (11),

$$\frac{\mathrm{d}\delta\gamma}{\mathrm{d}t}=\frac{e}{2mc^2}\frac{k_{\mathrm{w}}v_{\parallel}^2\Omega_{\mathrm{w}}}{\Omega_0^2-k_{\mathrm{w}}^2v_{\parallel}^2}\delta E\sin\left(\psi t+\varphi\right). \quad (12)$$

Integrating Eq. (12) with respect to time and averaging it over all phases yield

$$\left\langle\delta\gamma_i\right\rangle_{\varphi}=0. \quad (13)$$

So we replace the first-order $z=v_{\parallel}t$ position with

$$\begin{aligned}z(t)=&\int v\,\mathrm{d}t=\int(v_{0z}+\delta v_z)\,\mathrm{d}t\\=&\,v_{\parallel i}t+\int c\delta\beta_{zi}\,\mathrm{d}t,\end{aligned} \quad (14)$$

where $\delta\beta_{zi}=\delta v_{zi}/c$. Using

$$\bar{\beta}_{zi}^2=1-\bar{\beta}_{xi}^2-\bar{\beta}_{yi}^2-\gamma_i^{-2}, \quad (15)$$

where $\beta_{x,y}=v_{x,y}/c$, and substituting the root-mean-square values of Eqs. (5) and (6) into Eq. (15), we obtain

$$\bar{\beta}_{zi}^2=1-\frac{1}{2c^2}\frac{v_{\parallel i}^2\Omega_{\mathrm{w}i}^2}{\Omega_{0i}^2-k_{\mathrm{w}}^2v_{\parallel i}^2}\left(k_{\mathrm{w}}^2v_{\parallel i}^2+\Omega_{0i}^2\right)-\gamma_i^{-2}, \quad (16)$$

which is the root-mean square value of the axial relative velocity for the electrons of each beam.

Differentiating Eq. (16) with respect to time, using Eq. (12), and substituting them into Eq. (14), we obtain

$$\begin{aligned}z\left(t\right)=&\,v_{\parallel i}t+\frac{cD_i}{\psi_i}[-\sin\left(\psi_it+\varphi\right)+\sin\left(\varphi\right)\\&+\psi_it\cos\left(\varphi\right)],\end{aligned} \quad (17)$$

where

$$\begin{aligned}D_i=&-\frac{e\delta E}{4mc^2}\left(\frac{k_{\mathrm{w}}v_{\parallel i}^2\Omega_{\mathrm{w}i}}{\Omega_{0i}^2-k_{\mathrm{w}}^2v_{\parallel i}^2}\right)^3\frac{1}{c^2\gamma_i\psi_i}\\&\times\left[\left(\frac{k_{\mathrm{w}}v_{\parallel i}^2\Omega_{\mathrm{w}i}}{\Omega_{0i}^2-k_{\mathrm{w}}^2v_{\parallel i}^2}\right)^2\right.\\&\left.\times\left(1+2\frac{k_{\mathrm{w}}^2v_{\parallel i}^2+\Omega_{0i}^2}{\Omega_{0i}^2-k_{\mathrm{w}}^2v_{\parallel i}^2}\right)-2\frac{c^2}{\gamma_i^2}\right].\end{aligned}$$

Substituting Eq. (17) into Eq. (8), using Eqs. (5), (6), and (10), and discarding the non synchronous term lead to

$$\frac{\mathrm{d}\delta\gamma_i}{\mathrm{d}t} = \frac{e}{2mc^2}\frac{k_{\mathrm{w}}v_{\parallel i}^2\Omega_{\mathrm{w}i}}{\Omega_{0i}^2-k_{\mathrm{w}}^2v_{\parallel i}^2}\delta E \times \sin\big((\psi_i t+\varphi)+\frac{kcD_i}{\psi_i}[-\sin(\psi_i t+\varphi) + \sin(\varphi)+\psi_i t\cos(\varphi)]\big). \quad (18)$$

By comparing Eq. (18) with Eq. (12), we can define

$$\Delta\phi_i = \frac{kcD_i}{\psi_i}[-\sin(\psi_i t+\varphi)+\sin(\varphi) + \psi_i t\cos(\varphi)]. \quad (19)$$

Since $D_i \propto \delta E$, $\Delta\phi_i$ can be made arbitrary small. So for $\Delta\phi_i \ll \pi$, equation (18) becomes

$$\frac{\mathrm{d}\delta\gamma_i}{\mathrm{d}t} = \frac{e}{2mc^2}\frac{k_{\mathrm{w}}v_{\parallel i}^2\Omega_{\mathrm{w}i}}{\Omega_{0i}^2-k_{\mathrm{w}}^2v_{\parallel i}{}^2}\delta E \times \big(\sin(\psi_i t+\varphi) + \cos(\psi_i t+\varphi)\frac{kcD_i}{\psi_i}[-\sin(\psi_i t+\varphi) + \sin(\varphi)+\psi_i t\cos(\varphi)]\big). \quad (20)$$

Integrating Eq. (20) with respect to the electron transit time through the wiggler and then averaging it over all phases yield the average change in $\delta\gamma_i$ per electron beam

$$\langle\delta\gamma_i\rangle_\varphi = \frac{e}{4mc^2}\frac{k_{\mathrm{w}}v_{\parallel i}^2\Omega_{\mathrm{w}i}}{\Omega_{0i}^2-k_{\mathrm{w}}^2v_{\parallel i}^2}\delta E\frac{\omega D_i}{\psi_i^2} \times[\psi_i t\sin(\psi_i t)+2\cos(\psi_i t)-2], \quad (21)$$

where $T_i = L/v_{\parallel i}$, and $L$ is the interaction length of FEL. The change of the electromagnetic power due to the two-stream beam in one transit is

$$\delta p = -mc^2\frac{I_1}{e}\langle\delta\gamma_1\rangle_\varphi - mc^2\frac{I_2}{e}\langle\delta\gamma_2\rangle_\varphi, \quad (22)$$

where $I_{1,2}$ are the average current of each beam, and $\delta\gamma_{1,2}$ is the change in the energy of each beam due to the reaction with electromagnetic wave. Defining the electromagnetic power as $p = c\frac{\delta E^2}{8\pi}\mathrm{Area}$ and using Eq. (22) lead to

$$G = \frac{\delta p}{p} = \frac{8\pi}{c\delta E^2\mathrm{Area}}\frac{-mc^2}{e} \times\big[I_1\langle\delta\gamma_1\rangle_\varphi + I_2\langle\delta\gamma_2\rangle_\varphi\big]. \quad (23)$$

Substituting Eq. (21) into Eq. (23) and considering the synchronism condition, we finally obtain the gain equation as

$$G = \frac{\bar\beta_{z1}^5\hat\omega_{\mathrm{b}1}^2\hat\Omega_{\mathrm{w}1}^2}{8\big(\hat\Omega_{01}^2-\bar\beta_{z1}^2\big)^2\hat\psi_1^3(1-\bar\beta_{z1})} \times\left[\frac{-\bar\beta_{z1}^4\hat\Omega_{\mathrm{w}1}^2}{\big(\hat\Omega_{01}^2-\bar\beta_{z1}^2\big)^2}\left(1+2\frac{\big(\bar\beta_{z1}^2+\hat\Omega_{01}^2\big)}{\big(\hat\Omega_{01}^2-\bar\beta_{z1}^2\big)^2}\right)+\frac{2}{\gamma_1^2}\right] \times g(\hat\psi)+\frac{\bar\beta_{z2}^5\hat\omega_{\mathrm{b}2}^2\hat\Omega_{\mathrm{w}2}^2}{8\big(\hat\Omega_{02}^2-\bar\beta_{z2}^2\big)^2\hat\psi_2^3(1-\bar\beta_{z2})} \times\left[\frac{-\bar\beta_{z2}^4\hat\Omega_{\mathrm{w}2}^2}{\big(\hat\Omega_{02}^2-\bar\beta_{z2}^2\big)^2}\left(1+2\frac{\big(\bar\beta_{z2}^2+\hat\Omega_{02}^2\big)}{\big(\hat\Omega_{02}^2-\bar\beta_{z2}^2\big)^2}\right)+\frac{2}{\gamma_2^2}\right] \times g(\hat\psi), \quad (24)$$

where $\hat\omega_{\mathrm{b}1,2} = \omega_{\mathrm{b}1,2}/ck_{\mathrm{w}}$, $\omega_{\mathrm{b}1,2} = 4\pi n_{1,2}e^2/m$, $n_{1,2}$ are the densities of electron beams, $\hat\Omega_{\mathrm{w}1,2} = \Omega_{\mathrm{w}1,2}/ck_{\mathrm{w}}$, $\hat\Omega_{01,2} = \Omega_{01,2}/ck_{\mathrm{w}}$, $\hat\psi_{1,2} = \psi/ck_{\mathrm{w}}$, $\hat L = Lk_{\mathrm{w}}$, and

$$g(\hat\psi) = \left[2-2\cos\frac{\hat L\hat\psi}{\bar\beta_{z1}}-\frac{\hat L\hat\psi}{\bar\beta_{z1}}\sin\frac{\hat L\hat\psi}{\bar\beta_{z1}}\right].$$

# 4. Numerical results

A numerical study of the relativistic two-stream electron trajectories in the presence of planar wiggler and axial magnetic field has been taken. The typical dependence of the axial velocity on the cyclotron frequency is obtained in the solution of Eqs. (5)–(7). The electron energy factor is taken to be $\gamma_1 = 4$ and $\gamma_2 = 4.2$, the normalized magnetic wiggler and the plasma frequencies of the two beams are chosen to be $\hat\Omega_{\mathrm{w}1} = 0.05\gamma_1$, $\hat\Omega_{\mathrm{w}2} = 0.05\gamma_1/\gamma_2$, $\hat\omega_{\mathrm{b}1}^2 = 0.1\gamma_1$, and $\hat\omega_{\mathrm{b}2}^2 = 0.1$, respectively. Figure 1 shows the normalized axial velocities of the two beams, $\beta_{\parallel 1}$ (solid line) and $\beta_{\parallel 2}$ (dashed line), each as a function of the normalized cyclotron frequency $\hat\Omega_0 = \Omega_0/ck_{\mathrm{w}}$.

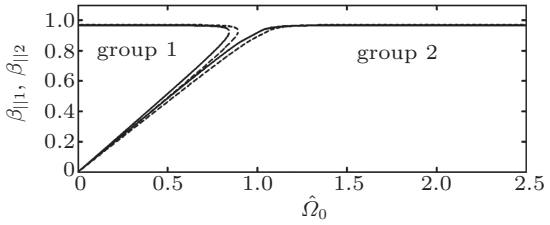



**Fig. 1.** The normalized axial velocities $\beta_{\parallel 1}$ (solid line) and $\beta_{\parallel 2}$ (dashed line) each as a function of the normalized cyclotron frequency $\hat\Omega_0$ in the presence of two streams, axial guide field, and planar wiggler. The chosen parameters are $\gamma_1 = 4$, $\gamma_2 = 4.2$, $\hat\Omega_{\mathrm{w}1} = 0.05\gamma_1$, $\hat\Omega_{\mathrm{w}2} = 0.05\gamma_1/\gamma_2$, $\hat\omega_{\mathrm{b}1}^2 = 0.1\gamma_1$, and $\hat\omega_{\mathrm{b}2}^2 = 0.1$.

The normalized maximum gain, $\hat G = G_{\max}/G_0$ (the maximum gain normalized by the maximum gain without the cyclotron frequency), as a function of the normalized cyclotron frequency is shown in Fig. 2. It is found that the gain of group 2 decreases with the increasing $\hat\Omega_0$ until a minimum is reached. A further

increase in frequency leads to an asymptotic behavior. Figure 2 also shows that in contrary to group 2, the normalized gain of group 1 follows an increasing trend with the increasing $\hat{\Omega}_0$ until it reaches its asymptotic region. To compare the maximum gains of single-stream and two-stream, we plot the maximum gains in Fig. 3. As shown in this figure, the two streams increase the maximum gain in comparison with maximum gain of single-stream in both groups. In the presence of $B_0$, we illustrate the maximum gain as a function of $\hat{\Omega}_0$ and $\delta\gamma$ ($\delta\gamma = \gamma_1 - \gamma_2$) in Figs. 4(a) (for group 1) and 4(b) (for group 2). As seen in these figures, the maximum of $\hat{G} = G_{\max}/G_0$ occurs at a suitable range of $\delta\gamma$.

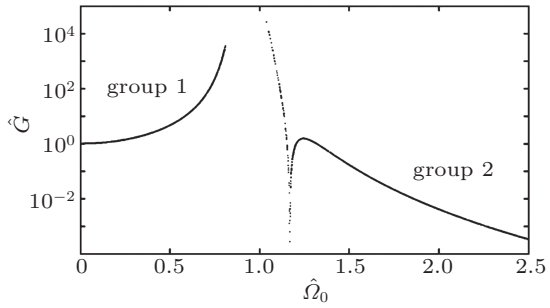



**Fig. 2.** The normalized maximum gain $\hat{G}$ as a function of the normalized axial magnetic field $\hat{\Omega}_0$ for groups 1 and group 2 in the presence of two streams, axial guide field, and planar wiggler. The parameters are the same as those in Fig. 1.

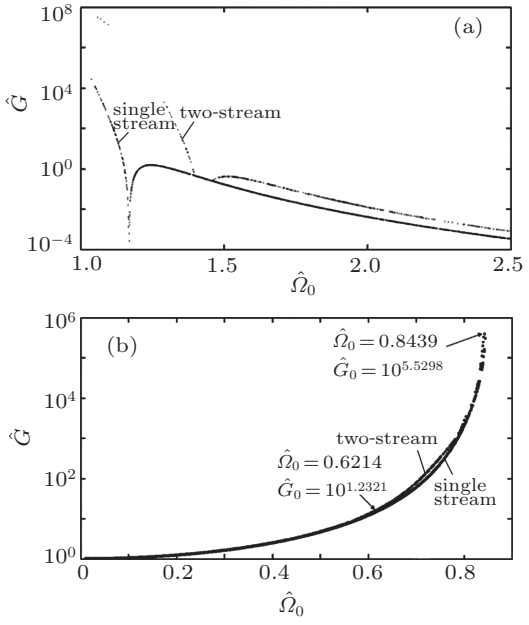



**Fig. 3.** The normalized maximum gain $\hat{G}$ as a function of the normalized cyclotron frequency $\hat{\Omega}_0$ for two streams and single beam. Panel (a) shows the results of group 1, the parameters are $\gamma_1 = 4$, $\gamma_2 = 4.0156$, $\hat{\Omega}_{w1} = 0.05\gamma_1$, $\hat{\Omega}_{w2} = 0.05\gamma_1/\gamma_2$, and $\hat{\omega}^2_{b1} = 0.1\gamma_1$, $\hat{\omega}^2_{b2} = 0.1$. Panel (b) shows the results of group 2, the parameters are $\gamma_1 = 4$, $\gamma_2 = 4.8$, $\hat{\Omega}_{w1} = 0.05\gamma_1$, $\hat{\Omega}_{w2} = 0.05\gamma_1/\gamma_2$, and $\hat{\omega}^2_{b1} = 0.1\gamma_1$, $\hat{\omega}^2_{b2} = 0.1$.

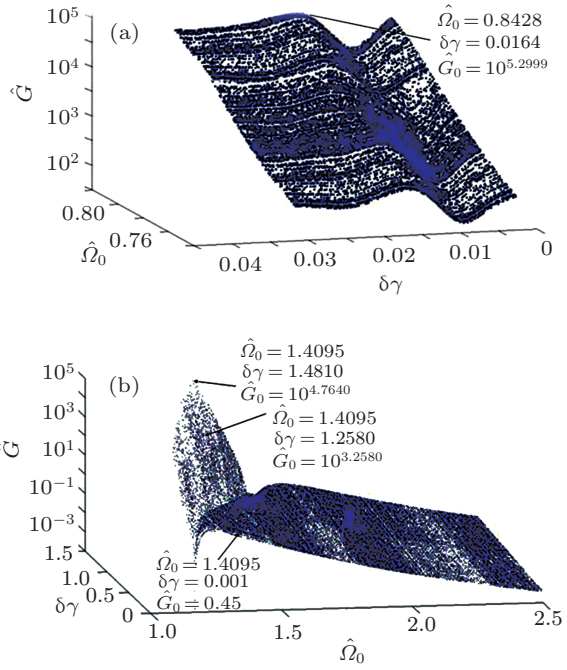



**Fig. 4.** (colour online) The normalized maximum gains of (a) group1 and (b) group 2 each as a function of the normalized cyclotron frequency $\hat{\Omega}_0$ and $\delta\gamma$.

## 5. Conclusion

We summarize the specific results of this project as:

1) The gain of group 2 decreases with the increasing cyclotron frequency, and the normalized gain of group 1 follows an increasing trend with the increasing cyclotron frequency.

2) The normalized maximum gain is considerably enhanced in comparison with that of the single stream.

3) The maximum of $\hat{G} = G_{\max}/G_0$ occurs at a suitable range of $\delta\gamma$.

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