Review

Asymmetric simple exclusion processes with complex lattice geometries: A review of models and phenomena*

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(Received 10 April 2012; revised manuscript received 3 June 2012)

We summarize the findings of a large number of researches concerning the totally asymmetric simple exclusion process (TASEP) with complex lattice geometries. The TASEP has been recognized as a paradigm in modeling and analyzing non-equilibrium traffic systems. The paper surveys both the observed physical phenomena and several popular mean-field approaches used to analyze the extended TASEP models. Several interesting physical phenomena, such as phase separation, spontaneous symmetry breaking, and finite-size effect, have been identified and explained. The future investigations of the extended TASEP with complex lattice geometries are also introduced. This paper may help to obtain a better understanding of non-equilibrium systems.

Keywords: non equilibrium process, spontaneous symmetry breaking, mean-field approach, Monte Carlo simulation

PACS: 05.70.Ln, 02.50.Ey, 05.60.Cd **DOI:** 10.1088/1674-1056/21/9/090510

1. Introduction

Traffic is a ubiquitous phenomenon and has been observed at almost all levels of natural and manmade systems, covering from macroscopic objects like cars, pedestrians, and ants to microscopic molecular motors. Such non-equilibrium systems can be regarded as complex systems and characterized by the non-zero continuous traffic currents.[1] Studies on such non-equilibrium systems are normally based on a paradigmatic model, which is known as the asymmetric simple exclusion process (ASEP). The ASEP is a one-dimensional lattice model, in which the particles hop to the nearest-neighbor sites with a preferred direction and interact through the hard-core exclusion, i.e., each site can be occupied by no more than one particle at any given time.[2] In the simplest ASEP, the particles can move along only one direction, which is called the totally ASEP (TASEP).

Recently, a large number of TASEP varieties have found their natural applications in biology, physics, and chemistry,[3] such as gel electrophoresis,[4] protein synthesis,[5,6] mRNA translation,[7] motion of molecular motors along cytoskeletal filaments,[8] the depolymerization of microtubules by special enzymes,[9] and the vehicular traffic.[10,11] Researches associated with these systems have motivated the development of the TASEP models. Those mathematical models have exhibited many interesting physical phenomena, such as phase separation, spontaneous symmetry breaking, finite-size effect, and jumping effect. In general, the extended TASEP models can be developed by changing the particle properties, the lattice geometries, the boundary conditions, and/or the updating procedures. Figure 1 illustrates some possible TASEP variations derived from the above-mentioned four aspects. This picture may show an integrated framework for understanding and developing the TASEP models and the non-equilibrium systems.

*Project supported by the State Key Laboratory of Geohazard Prevention and Geoenvironment Protection, China (Grant No. 2011Z006) and the Chengdu University of Technology, China (Grant Nos. HY0084 and HG0092).

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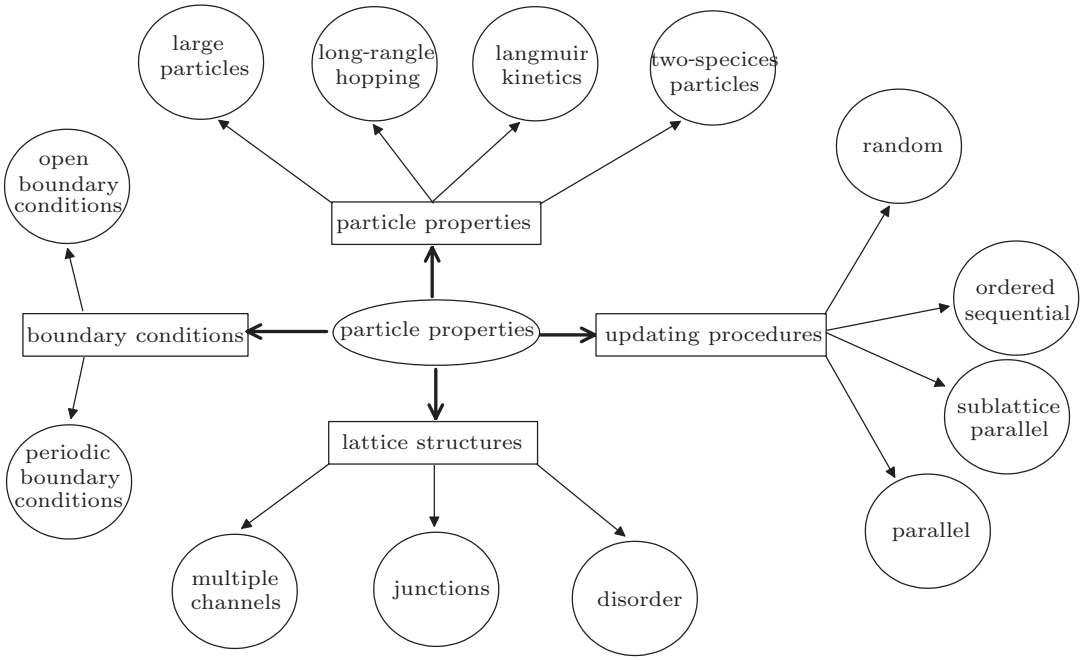


Fig. 1. Illustration of some possible TASEP extensions.

In this article, we provide a brief review to a special topic, the TASEP with complex lattice geometries. Complex geometries, e.g., multiple parallel lattices, junctions, and microtubule-like channels, make the TASEP models and the theoretical analysis much more complicated. We review some generalizations and extensions of the extended TASEP models on complex geometry lattices, and then summarize several popular mean-field approaches for the theoretical analysis of the TASEP models. The spontaneous symmetry breaking observed in the models is described and explained. Finally, we give the present status of the TASEP with complex lattice geometries and the future research directions.

2. TASEP with complex geometry lattices

2.1. TASEP on multiple parallel channels

In multiple-channel systems, the inter-channel coupling rules have a strong influence on the system properties, e.g., phase diagrams, currents, and bulk densities. In general, there are two kinds of basic inter-channel changing models, the positive channel-changing (PCC) model and the negative channel-changing (NCC) model (see Fig. 2). An occupation variable $\tau_{\ell,i}$ is introduced to denote the state of the i-th ($1 < i < N$) site in the ℓ-th channel ($\ell = 1, 2$), and $\tau_{\ell,i} = 1$ (0) corresponds to the occupied (unoccupied) state. The PCC model has the following channel changing rules (a two-channel system is used for example):

1) If $\tau_{\ell,i} = 1$, $\tau_{3-\ell,i} = 1$, and $\tau_{\ell,i+1} = 0$, a particle can move into site $(\ell, i+1)$ with probability 1.

2) If $\tau_{\ell,i} = 1$, $\tau_{3-\ell,i} = 0$, and $\tau_{\ell,i+1} = 0$, a particle can move into site $(\ell, i+1)$ with probability $1 - w_\ell$ or move into site $(3-\ell, i)$ with probability w_ℓ.

In other words, particles in the PCC model can hop to the corresponding site on the other channel with probability w_1 (or w_2) if the corresponding site on the other channel is empty. However, particles in the NCC model can hop to the corresponding site on the other channel with probability w_1 (or w_2) only when the immediately preceding site is occupied and the corresponding site on the other channel is empty.

A simple symmetric PCC model with $\alpha_1 = \alpha_2 = \alpha$, $\beta_1 = \beta_2 = \beta$, and $w_1 = w_2 = w$ was investigated in Ref. [12]. The computational results suggested that with the increase of w, the particle current of each channel decreases, while the particle density increases correspondingly. Following this line, Pronina and Kolomeisky then extended their work in Ref. [12] to a general case, in which the asymmetric coupling rules were applied, i.e., $w_1 \neq w_2$.[13] It was found that

the asymmetric coupling rules lead to a very complex phase diagram, which is quite different from that in the symmetric coupling case. There were seven phases in the TASEP with asymmetric coupling rules, in contrast to the three phases found in the system with symmetric coupling rules.[12]

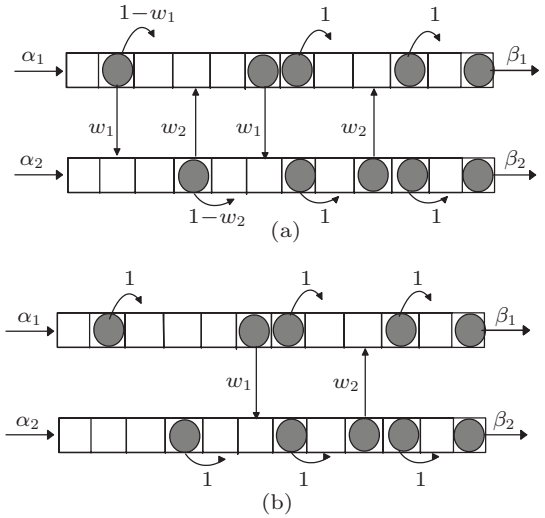


Fig. 2. General two-channel TASEP models, panels (a) and (b) show the positive and the negative coupling models, respectively.

In Refs. [12]–[15], the inter-channel symmetric or asymmetric coupling rules were applied to all sites. Tsekouras and Kolomeisky[16] investigated the case in which the symmetric/asymmetric coupling was only presented at one specific site far away from the boundaries. In the case of symmetric coupling, there were three stationary phases in the system. While the asymmetric coupling produced a more complex phase diagram with ten stationary-state regimes. More phase transitions were also observed. The TASEP with the coupling between two channels has been used to describe the obstruction behavior of molecular motors, which may occur in biological transport.[17] A rich phase behavior has been observed at the intermediate strength of the coupling.

A generic theoretical method for the PCC models is called the vertical cluster mean-field approximation. In this method, there are four possible states for a cluster of two vertical sites.[12,13,16] These states are shown in Fig. 3 and described as follows. (i) Both vertical sites are occupied, denoted as P_{11}. (ii) The vertical site on channel 1 is occupied, while the corresponding site on channel 2 is empty, denoted as P_{10}. (iii) The vertical site on channel 1 is empty, while the corresponding site on channel 2 is occupied, represented as P_{01}. (iv) Both vertical sites are empty, represented as P_{00}. Since these four states can be found in any vertical sites on both channels, the corresponding probabilities for these four states can be normalized to 1, that is

$$P_{11} + P_{10} + P_{01} + P_{00} = 1. \tag{1}$$

Thus, the bulk density in channel 1 can be written as $\rho_1 = P_{11} + P_{10}$. Similarly, $\rho_2 = P_{11} + P_{01}$. Combining the boundary conditions and the coupling rules, we can obtain the phase diagram, currents, and density profiles.

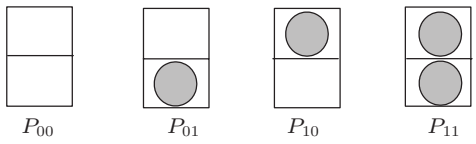


Fig. 3. Illustration of the four states of vertical sites.[12]

With regard to the NCC models, Mitsudo and Hayakawa investigated a two-channel TASEP with asymmetric coupling rules.[18] Different entry and exit rates of particles at the boundaries of two channels were considered. The positions of kinks (i.e., shocks) were confirmed to be synchronized, although the number of particles might be different on the two channels. The appearance of a kink meant the occurrence of a phase transition.

Jiang *et al.*[19] introduced Langmuir kinetics (LK for short) into one channel of a two-channel system with symmetric coupling rules. It was shown that the synchronization of shocks on both channels occurs when the coupling rate exceeds a threshold. A boundary layer was observed at the left boundary as the finite-size effect. The two-channel TASEP with both channels coupled with LK were investigated in Ref. [20]. The shock was found to move left first and then move towards the right with the increase of the channel-changing rate. This phenomenon was called the jumping effect.[20] It was also shown that increasing attachment and detachment rates would weaken the jumping effect.

Jiang *et al.* investigated a two-channel TASEP with asymmetric weak coupling.[21] The weak couplings mean that the coupling rates are inversely proportional to the system size. In the weak coupling case, the localized shock appeared on one channel, and the discontinuous phase transition was reproduced. Note that the NCC models are normally constructed in the hydrodynamic limit. The phase diagram, currents, and density profiles can be obtained by numerically solving the steady state equations.

The recent investigations have shown that the asymmetric coupling rules make the systems more complicated and increase the difficulty of theoretical analysis. More importantly, the asymmetric coupling rules can lead to more phases in the systems. These phases may make the system phase transition occur more frequently. Thus, the system may become unstable. This finding indicates that it should adopt fewer asymmetric channel coupling rules in the vehicular traffic.

Many researchers have attempted to propose generalized versions of the TASEP with coupled two channels.[22−24] Evans *et al.* considered a large class of two-channel TASEPs with reservoirs at their boundaries.[22] It was shown that the stability analysis can be understood as an extremal current principle for the total current in the two channels. Reichenbach *et al.* used the internal states to generalize various transport models ranging from molecular or vehicular traffic on parallel lanes to spintronics.[23] A striking polarization phenomenon accompanied by the domain wall motion and the delocalization was observed in the mesoscopic scale. Jiang *et al.* studied the asymmetric strong coupling effect in two parallel TASEPs, which can be viewed as a generalization of several two-channel TASEP models.[24] It was found that with different configurations of hopping rates, a diverse phase diagram and density profiles could be obtained. These efforts put a way to seek for a unified framework based on the normal TASEP so that we can understand better the non-equilibrium dynamic systems.

2.2. TASEP on multiple-input multiple-output junction

A junction can be seen as an extension of the multiple channels connected by junction points, which is one of the common used traffic facilities in nature. At junction points, several traffic flows are merged into one, or one flow is divided into several flows. In this sense, traffic points can be viewed as the local inhomogeneities that may lead to the traffic congestion.

The TASEP on the lattice with a Y junction (e.g., two-input-single-out junction) in random update has been investigated in Ref. [25]. In their model, three stationary phases (LD/LD, HD/HD, and HD/MC) were obtained, and two phase boundaries (LD/HD and LD/MC) were presented. A domain wall approach was proposed to predict the density profiles on the phase boundaries in Refs. [25] and [26]. A similar approach has been used to calculate the density profiles on the phase boundaries in synchronous TASEP.[27] The basic idea of the domain wall is that before a stationary state is reached, the left and the right parts of the system will form different domains of densities in the bulk due to the particle entrance and exit. The coexistence of both domains causes the appearance of a shock (or domain wall). The domain wall will move forward and backward along the channel according to the values of entrance rate and exit rate. For the normal TASEP in a stationary state, the domain wall will eventually move to the right (left) end for the LD (HD) phase. For the Y junction described in Ref. [25], the domain wall finally located at the junction points in a stationary state. In Ref. [27], the model in Ref. [25] was further extended to a general case in which unequal injection rates were considered. Wang *et al.*[28] investigated the dynamics of synchronous TASEP on the lattice with a multiple-input single-output (MISO) junction. They further extended the MISO junction[28] to the general case, an m-input n-output (MINO) junction, under the parallel update.[29] That generation showed an integrated picture of the dynamics of TASEP with junctions under the parallel update. Furthermore, the MINO junctions can be classified by a parameter $\lambda = m/n$. Junctions with the same λ possess the same traffic properties, e.g., phase diagrams, stationary currents, and density profiles. In the similar way, the dynamics of TASEP with MINO junctions under the random update can be obtained as well.

Recently, Cai *et al.*[30] introduced LK into the system with the Y junction described in Ref. [29]. Their model exhibited richer stationary phases depending on the ratio of ω_A to $\omega_{\rm D}$ and the attachment and detachment rates. Setting $K = \omega_A/\omega_{\rm D}$, the authors studied three cases, $K > 1$, $K = 1$, and $K < 1$. In the case of $K > 1$, there were four stationary phases. For $K \leq 1$, more phases could be observed. For a fixed K, the phase diagram structure changed with the increase of $\omega_{\rm D}$.

The TASEP with two consecutive junctions connected by a single channel in the middle was studied by Popkov *et al.*[31] Particles on junctions were governed by the TASEP, while they followed the rules of the Bridge model on the single channel. Two species of particles were involved in the system and moved along opposite directions. The model could be seen as a combination of the TASEP with junctions and the

Bridge model. The model exhibited the spontaneous symmetry breaking (SSB), i.e., the low-density-high-density phase. Moreover, there was a coexistence region of the SSB and the low-density symmetric phases.

The starting point for analyzing the TASEP with various junctions is the rule of current conservation at the junction points and in the system.[25,27,29−31] The junction geometry can be divided into two or more sections in terms of junction points. Each section can be treated as a TASEP or its extension. The overall phase diagram is thus a combination of possible phases in all sections. The weakness of this approach is that the correlation near the junction points is not considered, which leads to the deviations of theoretical calculations from computer simulations.

2.3. Spontaneous symmetry breaking

The spontaneous symmetry breaking means that a system may no longer keep its symmetry under certain conditions, that is, the symmetry of the system is spontaneously broken. In 2008, three physicists have been awarded the Nobel Prize in Physics for their contribution to the spontaneous symmetry breaking.[32]

In the TASEP, one can observe two different densities of the two species of particles in the SSB. The SSB normally involves two species of particles in non-equilibrium systems. The SSB has been observed in a single-channel system[33,34] as well as in a two-channel system. Here we focus on reviewing the SSB in two- or multi-channel systems. Popkov and Peschel[35] investigated a two-channel system, in which two species of particles moved in the same direction. The interactions between the two channels were considered via the hopping rate ϵ. The hopping rate in one channel was assumed to depend on the local status of the other channel. It was reported that the symmetry breaking phenomenon could weaken with the increase of ϵ.

Recently, the SSB has been investigated using the two-species two-channel TASEP with narrow entrances.[36]. The narrow entrances mean that particles cannot enter a channel if the exit site in the other channel is occupied. Two species of particles move along different channels and opposite directions. Two steady phases (LD/HD and LD/LD) exhibit the SSB phenomenon. It is suggested that the effective boundary defects (e.g., narrow entrances) can induce the SSB. Jiang *et al.*[37] studied the same model as that in Ref. [36] but with the parallel update. Two symmetric breaking phases (HD/LD and LD/LD) were obtained, however, the LD/LD phase just occupied a line in the phase diagram rather than a narrow area reported in Ref. [36]. The SSB can also be observed in a system in which the interactions for each species of particle happen at only one site in the two-channel TASEP with two species of particles.[38] It is shown that with the weakening of interaction, the SSB is suppressed. Authors in Ref. [38] reported an interesting phenomenon that the SSB disappears before the interaction is eliminated. However, the reason has not been given in Ref. [38]. The SSB was studied in the TASEP with two intersected channels in Ref. [39]. The effect of lane-changing probability p on the spontaneous symmetry breaking was investigated, and a threshold of p for the occurrence of symmetry breaking was confirmed.

More recently, Jiang *et al.* extended their work in Ref. [37] to an n-channel loop system ($n > 2$)[40] (see Fig. 4). The n-channel system was configured as a loop, the tail of channel 1 was next to the head of channel 2, the tail of channel 2 was next to the head of channel 3, and so on. The system exhibited more complicated properties. If n was an even number, the results reverted to the two-channel system,[37] i.e., the HD/LD and LD/LD phases were two symmetric breaking phases. In that case, the SSB was observed. When n was an odd number, a periodic structure was observed, and the period was related to n, the system size L, entry rate α, and exit rate β.

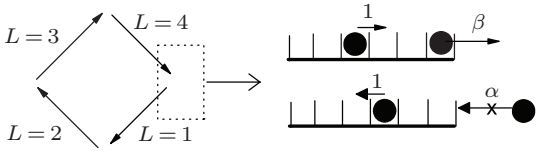


Fig. 4. Sketch of a four-channel TASEP model with narrow entrances.[40]

Wang *et al.*[41] and Liu *et al.*[42] investigated the TASEP on the microtubule-like geometry using the modified narrow entrance rules in random and parallel updates, respectively. Their theoretical analysis and computer simulations have also confirmed that the spontaneous symmetry breaking exists with two asymmetric phases, HD/LD and LD/LD phases. The flipping process of particles was also observed.

One of the remaining intriguing open questions related to the spontaneous symmetry breaking is the finite-size effect. The existence of the finite-size effect means that the asymmetric LD/LD phase cannot be observed in the hydrodynamic limit. For a single-channel TASEP system, Arndt *et al.*[43] have argued

that the asymmetric LD/LD phase did not exist based on numerical studies. Erickson *et al.*[44] also revisited the Bridge model via high-precision Monte Carlo data and associated their work with the study of traffic on a narrow bridge. Their simulation results indicated that the LD/LD phase would disappear if the system size was sufficiently large. Interestingly, the simulation results in Ref. [45] have shown that there existed two unequal LD phases. However, they did not correspond to the predicted asymmetric LD/LD phase. Differently, reference [46] suggested that the asymmetric LD/LD phase probably existed in the thermodynamic limit ($L \to \infty$). As for a two- or multi-channel system, the argument of the finite-size effect continues. Pronina and Kolomeisky suggested that the asymmetric LD/LD phase probably did not exist in the thermodynamic limit in the Monte Carlo simulation since the region of the asymmetric LD/LD phase seemed to shrink constantly with the increasing system size but without reaching a saturation.[36] However, Wang *et al.* showed that the region of the asymmetric LD/LD phase seemed to expand and then kept unchanged with the further increase of the system size. This suggested that the asymmetric LD/LD phase probably existed in the thermodynamic limit.[41] Similarly, Liu *et al.* also reported that the increase of the system size did not change the phase structure qualitatively.[42,47] Nevertheless, more careful numerical investigations of this asymmetric LD/LD phase are still needed in order to better understand the symmetry breaking phenomena in the non-equilibrium systems.

3. Summary

Some typical extensions of the TASEP with complex lattice geometries have been briefly reviewed in this paper. These TASEP variants are reminiscent of vehicular traffic and biological transport. The vertical cluster mean-field approximation can be used for the theoretical analysis of TASEP with multiple-channel coupling, while a rule of current conservation in the system is for the TASEP with junctions. Theoretical and numerical methods mentioned here and in the previous reviews could provide a basis for further developments.

The spontaneous symmetry breaking (SSB) has been of great importance in the study of particle interactions in statistical physics during the last decade. This paper reviews the SSB phenomena observed in the extended TASEP models. In these models, the lattice geometries, rules, and update procedures are symmetric. However, the systems may be spontaneous broken under the specific conditions. The SSB is mainly characterized by the differences of the density profiles, i.e., existing two phases, HD/LD and asymmetric LD/LD phases. The existence of the asymmetric LD/LD phase has been argued and remains an open question. More detailed Monte Carlo simulations are needed to clarify the finite-size effect of the asymmetric LD/LD phase.

More recently, the study on the dynamics of TASEP over networks has attracted much attention.[48−50] Such models take into account more complex traffic geometries and investigate under the framework of complex networks. These models are normally resolved via computer simulations, the complexity of the models precludes analytical solutions. Further studies along this line may give a better understanding of the spontaneous symmetry breaking and real-life traffic flows.

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