Projective synchronization of hyperchaotic system via periodically intermittent control*

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We further study the projective synchronization of a new hyperchaotic system. Different from the most existing methods, intermittent control is applied to chaotic synchronization in the present paper. We formulate the intermittent control system that governs the dynamics of the projective synchronization error, then derive the sufficient conditions for the exponential stability of intermittent control system by using Lyapunov stability theory, and finally establish the periodically intermittent controller according to the stability criterion by which the projective synchronization is expected to be achieved. The analytical results are also demonstrated by several numerical simulations.

Keywords: intermittent control, hyperchaotic system, projective synchronization

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1. Introduction

Since the seminal work of Pecora and Carroll,[1] the idea of synchronization of chaotic systems has aroused a great deal of interest among researchers from various fields. Over the past decades, several different regimes of chaos synchronization have been investigated, such as, complete synchronization,[1,2] generalized synchronization,[3] projective synchronization,[4−6] phase synchronization,[7] lag synchronization,[8] and anticipating synchronization.[9,10] In the complete synchronization, the master state $x(t)$ and the slave state $y(t)$ are identical, i.e., $y(t) \to x(t)$ (which means that $y(t)$ approaches $x(t)$ as t approaches infinity, throughout the paper). Generalized synchronization is defined as the presence of some function relation between the slave state and the master state, i.e., there is a function g such that $y(t) \to g(x(t))$. In the projective synchronization, there exists such a scale factor that $y(t) \to \alpha x(t)$. Phase synchronization, which means the difference in phase between master state and slave state, is constant during interaction, whereas their amplitudes remain chaotic and uncorrelated, i.e., $n\phi_x - m\phi_y = \text{const}$ (n and m are integers). Lag synchronization appears as a coincidence of shifted-in-time states of two systems, $y(t) \approx x_\tau(t) \equiv x(t-\tau)$ with positive τ, and has been studied between symmetrically coupled non-identical oscillators.[8] Moreover, for anticipating synchronization, the synchronization manifold is $y(t) \to x(t+\tau)$, where τ is the time delay of the driven system.

Some important control methods including continuous control and discontinuous control have been developed for stabilizing and synchronizing chaotic systems, such as state feedback control,[11,12] adaptive control,[13] switching control,[14,15] impulsive control,[16−20] and intermittent control.[21−24] Recently, intermittent control of nonlinear system has drawn increasing interests in process control, ecosystem management, synchronizing chaotic systems, and communication. Compared with continuous control method, intermittent control method is advantageous in its efficiency. In Ref. [25], the synchronization of coupled chaotic systems with time delay by using intermittent linear state feedback control was investigated. In Ref. [26], a centralized adaptive intermittent control approach was proposed to realize the cluster synchronization. To the best of our knowledge, there

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is no more report on the program of projective synchronization of hyperchaotic systems using intermittent control.

Motivated by the above discussion, in this paper we first investigate projective synchronization of hyperchaotic systems by periodically intermittent control. By using Lyapunov stability theory and intermittent control technique, the intermittent controllers and the corresponding parameter update rules are designed to obtain projective synchronization of hyperchaotic systems. Numerical simulations are conducted to compare with the theoretical results.

The rest of the paper is organized as follows. In Section 2, the projective synchronization problem to be considered is formulated. In Section 3, we will study the projective synchronization of coupled hyperchaotic systems. Accordingly, we obtain the control laws according to the rigorous theoretical analysis. In Section 4, numerical examples are given to show the theoretical results. Some conclusions are drawn from the present study in Section 5.

2. Problem formulation and preliminaries

Now we consider a novel hyperchaotic system described as[27]

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1), \\ \dot{x}_2 = bx_1 - x_2 - x_1x_3 + x_4, \\ \dot{x}_3 = cx_3 + x_1x_2, \\ \dot{x}_4 = d_1x_1 + d_2x_2, \end{cases} \tag{1}$$

where x_1, x_2, x_3, and x_4 are state variables, and a, b, c, d_1, and d_2 are real parameters. It is shown that this system is hyperchaotic when the parameters are chosen to be $a = 10$, $b = 28$, $c = -8/3$, and $(d_1,d_2) = (-9.3, 1)$.

We rewrite Eq. (1) as follows:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{M}(x_1)\boldsymbol{x}(t), \tag{2}$$

where $\boldsymbol{x}(t) = (x_1(t), x_2(t), x_3(t), x_4(t))^{\mathrm{T}} \in R^n$, and

$$\boldsymbol{M}(x_1) = \begin{pmatrix} -a & a & 0 & 0 \\ b & -1 & -x_1 & 1 \\ 0 & x_1 & c & 0 \\ d_1 & d_2 & 0 & 0 \end{pmatrix}.$$

Hence, it is easy to see that $\boldsymbol{M}(x_1) + \boldsymbol{M}^{\mathrm{T}}(x_1)$ is independent of x_1. In order to achieve the projective synchronization, we construct the coupled response system as follows:

$$\dot{\boldsymbol{y}}(t) = \boldsymbol{M}(x_1)\boldsymbol{y}(t) + \boldsymbol{u}(t), \tag{3}$$

where $\boldsymbol{y}(t) = (y_1(t), y_2(t), y_3(t), y_4(t))^{\mathrm{T}}$ is the response state, $\boldsymbol{u}(t)$ is the intermittent control signal defined by

$$\boldsymbol{u}(t) = \begin{cases} -k(\alpha\boldsymbol{x}(t) - \boldsymbol{y}(t)), & nT \le t \le nT + \sigma T, \\ 0, & nT + \sigma T \le t \le (n+1)T, \end{cases} \tag{4}$$

where k denotes control strength, $0 < \sigma < 1$ represents the switching rate, T refers to the control period, and α is a desired value. Let $\boldsymbol{e}(t) = \alpha\boldsymbol{x}(t) - \boldsymbol{y}(t)$ be the projective synchronization error between systems (2) and (3), then we will obtain the error system as follows:

$$\begin{aligned} \dot{\boldsymbol{e}}(t) &= \alpha\dot{\boldsymbol{x}}(t) - \dot{\boldsymbol{y}}(t) \\ &= \alpha\boldsymbol{M}(x_1)\boldsymbol{x}(t) - \boldsymbol{M}(x_1)\boldsymbol{y}(t) - \boldsymbol{u}(t). \end{aligned} \tag{5}$$

Under the control of formula (4), projective synchronization error system (5) can be rewritten as

$$\begin{cases} \dot{\boldsymbol{e}}(t) = \boldsymbol{M}(x_1)\boldsymbol{e}(t) + k\boldsymbol{e}(t), nT \le t \le nT + \sigma T, \\ \dot{\boldsymbol{e}}(t) = \boldsymbol{M}(x_1)\boldsymbol{e}(t), \ nT + \sigma T \le t \le (n+1)T. \end{cases} \tag{6}$$

3. Projective synchronization of hyperchaotic systems via intermittent control

In this section, we will derive a sufficient condition for projective synchronization. The main result is stated as follows:

Theorem 1 Suppose that there exist positive constants $g_1 > 0$ and $g_2 > 0$ such that the following inequalities hold:

$$\begin{aligned} &2k\boldsymbol{I} + \boldsymbol{M}(x_1) + \boldsymbol{M}(x_1)^{\mathrm{T}} + g_1\boldsymbol{I} \le 0, \\ &\boldsymbol{M}(x_1) + \boldsymbol{M}(x_1)^{\mathrm{T}} - g_2\boldsymbol{I} \le 0, \\ &g_1\sigma - g_2(1-\sigma) > 0. \end{aligned}$$

Then, projective synchronization error system (6) is globally exponentially stable, and

$$\|\boldsymbol{e}(t)\| = \|\boldsymbol{e}(t_0)\| \exp\left(-\frac{1}{2}(g_1\sigma - g_2(1-\sigma))(t-\sigma T)\right).$$

This implies that two systems (1) and (3) are globally exponentially projectively synchronized.

Proof Consider the following Lyapunov function:

$$V(\boldsymbol{e}(t)) = \boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{e}(t), \tag{7}$$

which implies that $V(\boldsymbol{e}(t)) = \|\boldsymbol{e}(t)\|^2$.

When $nT \le t \le nT+\sigma T$, the derivative of Eq. (7) with respect to time t along the trajectory of the first subsystem of system (6) is calculated and estimated as follows:

$$\begin{aligned}\dot{V}(\boldsymbol{e}(t)) &= \boldsymbol{e}^{\mathrm{T}}(t)[\boldsymbol{M}(x_1)\boldsymbol{e}(t) + k\boldsymbol{e}(t)] \\ &\quad + [\boldsymbol{M}(x_1)\boldsymbol{e}(t) + k\boldsymbol{e}(t)]^{\mathrm{T}}\boldsymbol{e}(t) \\ &\le \boldsymbol{e}^{\mathrm{T}}(t)[2k\boldsymbol{I} + \boldsymbol{M}(x_1) + \boldsymbol{M}(x_1)^{\mathrm{T}}]\boldsymbol{e}(t) \\ &\le \boldsymbol{e}^{\mathrm{T}}(t)[2k\boldsymbol{I} + \boldsymbol{M}(x_1) + \boldsymbol{M}(x_1)^{\mathrm{T}} \\ &\quad + g_1\boldsymbol{I}]\boldsymbol{e}(t) - g_1\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{e}(t) \\ &\le -g_1 V(\boldsymbol{e}(t)).\end{aligned}$$

Thus, we have

$$\dot{V}(\boldsymbol{e}(t)) \le -g_1 V(\boldsymbol{e}(t)), nT \le t \le nT + \sigma T,$$

which implies that when $nT \le t \le nT + \sigma T$, we have

$$V(\boldsymbol{e}(t)) \le V(\boldsymbol{e}(nT)) \exp(-g_1(t - nT)).$$

Similarly, when $nT + \sigma T \le t \le (n+1)T$, we have

$$\begin{aligned}\dot{V}(\boldsymbol{e}(t)) &= \boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{M}(x_1)\boldsymbol{e}(t) + [\boldsymbol{M}(x_1)\boldsymbol{e}(t)]^{\mathrm{T}}\boldsymbol{e}(t) \\ &\le \boldsymbol{e}^{\mathrm{T}}(t)[\boldsymbol{M}(x_1) + \boldsymbol{M}(x_1)^{\mathrm{T}} \\ &\quad - g_2\boldsymbol{I}]\boldsymbol{e}(t) + g_2\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{e}(t) \\ &\le g_2 V(\boldsymbol{e}(t)).\end{aligned}$$

Therefore, we derive that when $nT + \sigma T \le t \le (n+1)T$, we have

$$\dot{V}(\boldsymbol{e}(t)) \le g_2 V(\boldsymbol{e}(t)),$$

and

$$V(\boldsymbol{e}(t)) \le V(\boldsymbol{e}(nT + \sigma T)) \exp(g_2(t - nT - \sigma T)).$$

Hence,

$$\begin{cases} V(\boldsymbol{e}(t)) \le V(\boldsymbol{e}(nT)) \exp(-g_1(t - nT)), & nT \le t \le nT + \sigma T, \\ V(\boldsymbol{e}(t)) \le V(\boldsymbol{e}(nT + \sigma T)) \exp(g_2(t - nT - \sigma T)), & nT + \sigma T \le t \le (n+1)T. \end{cases}$$

By simple computation, we can derive that

(i) for $nT \le t \le nT + \sigma T$, i.e., $(t - \sigma T)/T \le n \le t/T$,

$$\begin{aligned}V(\boldsymbol{e}(t)) &\le V(\boldsymbol{e}(t_0)) \exp(-ng_1\sigma T + ng_2(T - \sigma T)) \\ &\le V(\boldsymbol{e}(t_0)) \exp(-(g_1\sigma - g_2(1-\sigma))(t - \sigma T)).\end{aligned}$$

(ii) for $nT+\sigma T \le t \le (n+1)T$, i.e., $t/T \le n+1 \le (t + T - \sigma T)/T$,

$$\begin{aligned}V(\boldsymbol{e}(t)) &\le V(\boldsymbol{e}(t_0)) \exp(-(n+1)g_1\sigma T \\ &\quad + (n+1)g_2(T - \sigma T)) \\ &\le V(\boldsymbol{e}(t_0)) \exp(-(g_1\sigma - g_2(1-\sigma))(t - \sigma T)).\end{aligned}$$

Therefore, for any $t \ge 0$,

$$\begin{aligned}\|\boldsymbol{e}(t)\|^2 &= V(\boldsymbol{e}(t)) \le V(\boldsymbol{e}(t_0)) \\ &\quad \times \exp(-(g_1\sigma - g_2(1-\sigma))(t - \sigma T)).\end{aligned} \tag{8}$$

This implies that the origin of system (6) is globally exponentially stable, and the following estimation holds:

$$\begin{aligned}\|\boldsymbol{e}(t)\| &= \|\boldsymbol{e}(t_0)\| \exp(-\frac{1}{2}(g_1\sigma - g_2(1-\sigma))(t - \sigma T)), \\ &\quad t \ge 0.\end{aligned} \tag{9}$$

Hence, the two systems (2) and (3) are globally exponentially projectively synchronized. The proof is thus completed.

Remark 1 Let μ be the largest eigenvalue of $\boldsymbol{M}(x_1) + \boldsymbol{M}^{\mathrm{T}}(x_1)$. If we replace the first two conditions in Theorem 1 by the scalar equalities $g_1^* = -2k - \mu$ and $g_2^* = \mu$, where $g_1^* \ge g_1$ and $g_2^* \le g_2$, it is easy to see that Theorem 1 also holds. This result is restated as the following corollary.

Corollary 1 If there exist constant k and $0 < \sigma < 1$, such that $g_1^*\sigma - g_2^*(1-\sigma) > 0$, where $g_1^* = -2k - \mu$ and $g_2^* = \mu$, then projective synchronization error system (6) is globally exponentially stable, and the projective synchronization between systems (2) and (3) is achieved.

4. Numerical examples

In this section, we will present some numerical simulations for projective synchronization of the hyperchaotic system to verify and illustrate the effectiveness of the theoretical analysis in Sections 3. In all these simulations, we set $a = 10$, $b = 28$, $c = -8/3$, and $(d_1, d_2) = (-9.3, 1)$. In this case, system (1) exhibits hyperchaotic behaviours without control as shown in Fig. 1.

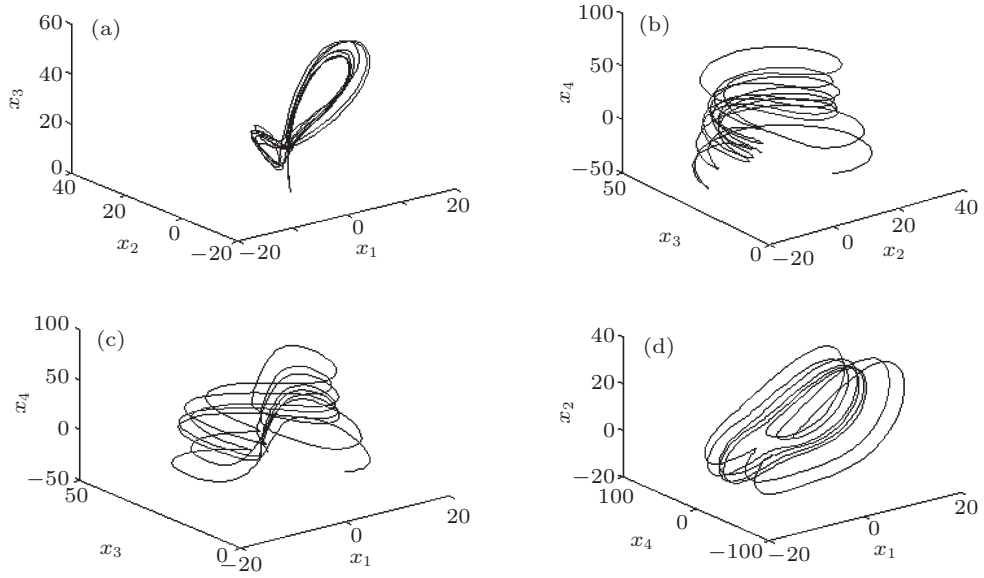


Fig. 1. Several projections of Hyperchaotic attractors in (a) x_1–x_2–x_3, (b) x_2–x_3–x_4, (c) x_1–x_3–x_4, and (d) x_1–x_2–x_4 3D views.

From Corollary 1, one observes that the control strength k can be estimated as follows:

$$k < -\mu/2\sigma < 0. \tag{10}$$

From inequality (10), one then estimate the feasible region D of control parameters (k, σ) as

$$D = \{(k, \sigma) \,|k < -\mu/2\sigma < 0, 0 < \sigma < 1\}. \tag{11}$$

By simple calculation, we have

$$\mu = \sup[\boldsymbol{M}^{\mathrm{T}}(x_1) + \boldsymbol{M}(x_1)] = 28.6866.$$

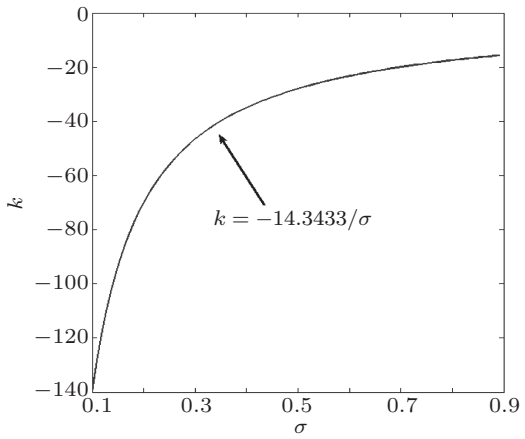


Fig. 2. Feasible region D (the region above the curve) of the control parameters (k, σ).

Therefore, the following is obtained from expression (11):

$$D = \{(k, \sigma) \,|k < -14.3433/\sigma < 0, 0 < \sigma < 1\}\ . \tag{12}$$

In the light of expression (12), we then can plot the possible region of control parameters as shown in Fig. 2, where the region below the curve is just the feasible region of control parameters (k, σ).

For numerical simulations, we select $T = 4$, $\sigma = 0.8$, and $k = -20$, which leads to projective synchronization between systems (2) and (3) as shown in Fig. 3.

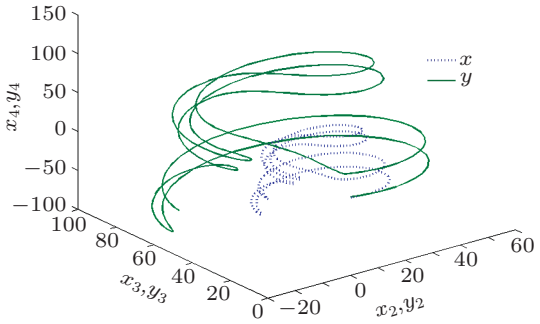


Fig. 3. (colour online) Projective synchronization of systems (2) and (3) with $\alpha = 2$.

5. Conclusions

In this paper, we formulate the projective synchronization problem for hyperchaotic systems by means of periodically intermittent control and design a general periodically-intermittent controller for a new hyperchaotic system. Projective synchronization criteria are established based on Lyapunov stability theory and linear matrix inequality techniques. Numerical simulations show the validity of theoretical results.

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