Temperature dependence of the energy-level shift induced by the Bose–Einstein condensation of photons*

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We investigate the energy-level shift of a hydrogen atom in a two-dimensional optical microcavity, where there exists a Bose–Einstein condensation of photons. It is found that below the critical temperature T_c, the energy-level shift of the bound electron is dependent on temperature, and it is a monotonically increasing function of the absolute temperature T. Especially, at the absolute zero temperature, the energy-level shift entirely comes from the Lamb shift, and the atom can be treated approximately, that is, in vacuum.

Keywords: Bose–Einstein condensation, energy-level shift, two-level atom

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1. Introduction

Bose–Einstein condensate (BEC) is a remarkable state of matter that spontaneously emerges when a system of bosons becomes cold enough that a significant fraction of them condenses into a single quantum state to minimize the system's free energy. Particles in that state then act collectively as a coherent wave. The phase transition for an atomic gas was first predicted by Einstein in 1924 and experimentally confirmed by the discovery of superfluid helium-4 in 1938. Obviously, atoms are not the only option for a BEC. In recent years, with the development of techniques, the phenomena of BEC have been observed in several physical systems,[1−11] including exciton polaritions and solid-state quasi-particles. We have known that photons are the simplest bosons that are expected in principle to undergo this kind of condensation. The difficulty is that in the usual blackbody configuration consisting of an empty three-dimensional (3D) cavity, the photon is massless and its chemical potential is zero, making the BEC of photons under these circumstances impossible. However, Klaers *et al.* have overcome both obstacles using a simple approach.[12] By confining a laser light within a thin cavity filled with dye and bounded by two concave mirrors, they established the conditions required for the light to thermally equilibrate as a gas of conserved particles rather than as an ordinary blackbody radiation.

Furthermore, it is well known that the discovery[13] and explanation[14] of the Lamb shift mark the beginning of modern quantum electrodynamics. The $2S$–$2P_{1/2}$ level difference arises from the coupling of the bound electron to the vacuum electromagnetic modes.[15] Since then the energy-level shift has become one of the central themes in quantum optics. People have explored various modifications of the mode structure to obtain different shifts. Essentially, these modifications come from some confined environments or periodic structures, for example, near a conducting plane,[16] inside a waveguide,[17] inside a cavity,[18] or inside a quantum well.[19] A variety of large, small, and negative energy shifts have been observed. Despite of the excellent agreement between measured and calculated energy shifts of the atom, there remain small differences between theory and experiment. Some scholars also pointed out that the differences mainly come from thermal correction in radiation field,[20,21] i.e., the differences are dependent on the temperature. However, for usual electromagnetic environments, the photons have a vanishing chemical potential, meaning that their number is not

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conserved when the temperature of the photon gas is varied. In that case, it is difficult to determine their relation. In this letter, however, we restrict the object atom in a new electromagnetic environment: the BEC of photons. We find that in the new environment an explicit temperature correction factor can be obtained, which is useful for people to determine the fine structure of an atom. Furthermore, the investigation into the fine structure of the atom is important for its connection with basic quantum physics, and for its practical applications to ultrahigh-precision spectroscopy and metrology.

This paper is organized as follows. In Section 2, we theoretically investigate the phenomenon of BEC of photons in a two-dimensional (2D) optical microcavity. The expression of the energy-level shift of a hydrogen atom is derived in Section 3. Finally, we present a further discussion and make a simple conclusion.

2. Bose–Einstein condensation of photons

In this section, we will theoretically investigate the BEC of photons. Here we introduce such a 2D optical microcavity consisting of two curved dielectric mirrors with high reflectivity (about 99.9) which is shown in Fig. 1. The microcavity is filled with a dye solution. An external radiation field is injected into the microcavity, where photons are repeatedly absorbed and re-emitted by the dye molecules.

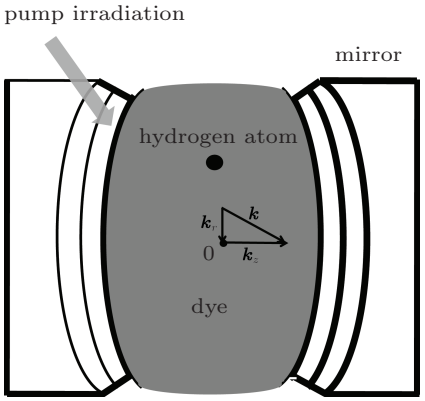


Fig. 1. Schematic of the optical microcavity. The microcavity consists of two high-reflectivity curved mirrors which are filled with a dye solution. A hydrogen atom is immersed inside the microcavity.

Generally, the coupling of the cavity photon and the atoms in a solution can lead to the formation of new eigenstates called polaritons, which are half-light, half-matter bosonic quasi-particles. However, in the present work, due to the frequent collisions of dye molecules with the solvent, the phase coherence time of photons does not match with the solvent. That is, no transient coherent effects occur, i.e., there is no enough time to form ploaritons. We can therefore assume the relevant particles to be well described as photons instead of ploaritons.

2.1. Free-photon dispersion relation inside the optical microcavity

We know that for a free photon, its frequency as a function of transversal (k_r) and longitudinal (k_z) wavenumber is $\omega_0 = c\left[k_z^2 + k_r^2\right]^{1/2}$. However, in the case of photons confined inside the microcavity, the vanishing of the electric field on the reflecting surfaces of the curved mirrors imposes a quantization condition on the allowed value of the z component of the photon wave vector $k_z(r) = n\pi/D(r)$, where $D(r) = D_0 - 2(R - \sqrt{R^2 - r^2})$ is the separation between the two curved mirrors at distance r from the optical axis, with D_0 being the mirror separation at distance $r = 0$ and R the radius of the curvature. Thus, the common frequency–wavenumber relation becomes the energy–momentum relation for the photon after multiplying $\hbar$

$$E = \hbar c\left[k_r^2 + n^2\pi^2/D^2(r)\right]^{1/2}. \tag{1}$$

In the limit of small-angle propagation ($R \gg r$), where the small transverse momentum of the photon satisfies the inequality $k_r \ll k_z$, we obtain from a Taylor expansion of the relativistic relation a nonrelativistic energy–momentum relation for the 2D noninteracting photons inside the microcavity

$$E \cong m_{\rm ph}c^2 + \frac{(p_r)^2}{2m_{\rm ph}}, \tag{2}$$

where $m_{\rm ph} = p_z/c = \hbar w_{\rm eff}/c^2$ is the effective mass of the confined photons for mixed longitudinal wavenumber n, with $p_z \approx n\pi\hbar/D_0$. At low temperatures, it is convenient to redefine the zero of energy to ensure that only the effective kinetic energy remains as

$$E \cong \frac{(p_r)^2}{2m_{\rm ph}}. \tag{3}$$

The above analysis demonstrates that in the limit of small-angle propagation, the photon confined inside the 2D microcavity is formally equivalent to a boson having an effective mass $m = \hbar w_{\rm eff}/c^2$ that is moving

in the transverse resonator plane. Furthermore, thermalization is achieved in a photon-number-conserving way by photon scattering off the dye molecules.[22] As a result, the system of photons can be treated as a grand canonical ensemble with nonvanishing chemical potential. This is the key prerequisite for the BEC of photons. Given the fact that there exist repulsive interactions between the photons, and the interactions are always weak (see appendix for details), then for the case of 2D photon gas inside the microcavity, we can connect it with an earlier problem solved by Bogoliubov for the weakly-interacting Bose gas. All the approximations involved in the Bogoliubov theory should be valid for the weakly-interacting photon gas.

2.2. Bogoliubov dispersion relation for the photon gas

In virtue of the above analysis, for the weakly-interacting photon gas inside the microcavity, we can start with the Hamiltonian (H). Here, let $\boldsymbol{k} = \boldsymbol{k}_r$ for simplicity.

$$H = H_{\text{nonint}} + H_{\text{int}}, \tag{4}$$

with

$$H_{\text{nonint}} = \sum_{\boldsymbol{k}} \varepsilon(\boldsymbol{k}) a_{\boldsymbol{k}}^{+} a_{\boldsymbol{k}}, \tag{5}$$

$$H_{\text{int}} = \frac{1}{2} \sum_{\boldsymbol{pkq}} V(\boldsymbol{k}) a_{\boldsymbol{p+k}}^{+} a_{\boldsymbol{q-k}}^{+} a_{\boldsymbol{p}} a_{\boldsymbol{q}}, \tag{6}$$

where $a_{\boldsymbol{k}}^{+}$ and $a_{\boldsymbol{k}}$ are the creation and annihilation operators, respectively, for photons with transverse wave vector $\boldsymbol{k}$ and effective mass $m_{\text{ph}} = \hbar w_{\text{eff}}/c^2$. The first term H_{nonint} in the Hamiltonian represents the energy of the noninteracting photon gas in the microcavity, where $\varepsilon(\boldsymbol{k})$ satisfies the relation given by Eq. (3). The second term H_{int} represents the interaction energy between the photons. The interactions mean a scattering process with a wave-vector transfer $\boldsymbol{k}$ between a pair of photons with initial wave vectors $\boldsymbol{p}$ and $\boldsymbol{q}$, along with the assignment of energy $V(\boldsymbol{k})$ to this scattering process.

We know that in a weakly-interacting Bose gas at absolute zero temperature, there exists a Bose–Einstein condensate consisting of a great number (N_0) of particles occupying the zero-momentum state. The feature should also survive in the case of the weakly interacting photon gas. For the ground state $|\varphi_0(N_0)\rangle$ with N_0 particles in Bose–Einstein condensate in the $p = 0$ state, the zero-momentum operators a_0^{+} and a_0 operating on the ground state obey the following relations:

$$\begin{aligned} a_0 |\varphi_0(N_0)\rangle &= \sqrt{N_0}\, |\varphi_0(N_0)\rangle\,, \\ a_0^{+} |\varphi_0(N_0)\rangle &= \sqrt{N_0+1}\, |\varphi_0(N_0)\rangle\,. \end{aligned} \tag{7}$$

Since $N_0 \gg 1$, we can neglect the difference between the factors $\sqrt{N_0+1}$ and $\sqrt{N_0}$. Thus, one can replace all occurrences of a_0^{+} and a_0 by the c number $\sqrt{N_0}$. Moreover, the fact that only small momenta are involved in the solution of the weakly-interacting many-body problem allows us to only consider the $k = 0$ value of the two-body potential $V(\boldsymbol{k})$, and finally we obtain (see Ref. (23) and appendix)

$$V(\boldsymbol{k}) \approx V_0 = \frac{4\pi\hbar^2 a}{m_{\text{ph}}}, \tag{8}$$

where a is the s-wave scattering length. As a result, at the absolute zero temperature, by separating out the condensate state from the sums in the Hamiltonian, and doing corresponding replacement, we find that the Hamiltonian of the weakly-interacting photon gas has the following form:

$$\begin{aligned} H \approx\ & \varepsilon'_{\text{eff}}(0) + \sum_{\boldsymbol{k}\neq 0} \varepsilon'(\boldsymbol{k}) a_{\boldsymbol{k}}^{+} a_{\boldsymbol{k}} \\ & + \frac{1}{2} N_0 V_0 \sum_{\boldsymbol{k}\neq 0} (a_{\boldsymbol{k}}^{+} a_{-\boldsymbol{k}}^{+} + a_{\boldsymbol{k}} a_{-\boldsymbol{k}}), \end{aligned} \tag{9}$$

where $\varepsilon'_{\text{eff}}(0)$ is given by

$$\varepsilon'_{\text{eff}}(0) = N_0 \left[\varepsilon(0) + \frac{1}{2} V_0 N_0 \right], \tag{10}$$

and

$$\varepsilon'(\boldsymbol{k}) = \varepsilon(\boldsymbol{k}) + N_0 V_0 \tag{11}$$

is the modified photon energy.

Based on the Bogolubov theory,[24,25] by introducing the following canonical transformation:

$$\begin{aligned} b_{\boldsymbol{k}} &= \mu_{\boldsymbol{k}} a_{\boldsymbol{k}} + \upsilon_{\boldsymbol{k}} a_{-\boldsymbol{k}}^{+}, \\ b_{\boldsymbol{k}}^{+} &= \mu_{\boldsymbol{k}} a_{\boldsymbol{k}}^{+} + \upsilon_{\boldsymbol{k}} a_{-\boldsymbol{k}}, \end{aligned} \tag{12}$$

we can diagonalize the Hamiltonian as

$$H_{\text{BEC}} = E_0 + \sum_{\boldsymbol{k}\neq \boldsymbol{0}} \tilde{\varepsilon}(\boldsymbol{k}) \left(b_{\boldsymbol{k}}^{+} b_{\boldsymbol{k}} + \frac{1}{2} \right), \tag{13}$$

where E_0 is the ground-state energy. The transformation introduces a new set of operators, $b_{\boldsymbol{k}}$ and $b_{\boldsymbol{k}}^{+}$, to which we impose the same Bose commutation relation $b_{\boldsymbol{k}} b_{\boldsymbol{k}'}^{+} - b_{\boldsymbol{k}'}^{+} b_{\boldsymbol{k}} = \delta_{\boldsymbol{k},\boldsymbol{k}'}$. It can be easily verified that the commutation relation is satisfied if

$$|\mu_{\boldsymbol{k}}|^2 - |\upsilon_{\boldsymbol{k}}|^2 = 1, \tag{14}$$

or if

$$\mu_{\boldsymbol{k}} = \cosh\alpha_{\boldsymbol{k}}, \quad \upsilon_{\boldsymbol{k}} = \sinh\alpha_{\boldsymbol{k}}. \tag{15}$$

Equation (13) shows that the original system of interacting particles can be described in terms of a Hamiltonian of independent simple harmonic oscillators, i.e., quasi-particles. The operators $b_{\boldsymbol{k}}^{+}$ and $b_{\boldsymbol{k}}$ represent the creation and annihilation of the quasi-particles having energy $\tilde{\varepsilon}(\boldsymbol{k})$, and they satisfy the following equation of motion in the Heisenberg picture:

$$i\hbar\dot{b}_{\boldsymbol{k}}(\dot{b}_{\boldsymbol{k}}^{+}) = [b_{\boldsymbol{k}}(b_{\boldsymbol{k}}^{+}), H] = \tilde{\varepsilon}(\boldsymbol{k})b_{\boldsymbol{k}}(b_{\boldsymbol{k}}^{+}). \tag{16}$$

According to Eqs. (11) and (16), we can obtain the relation

$$\begin{aligned}\tilde{\varepsilon}(\boldsymbol{k})^2 &= \varepsilon'(\boldsymbol{k})^2 - N_0^2V_0^2 \\ &= \varepsilon(\boldsymbol{k})^2 + 2\varepsilon(\boldsymbol{k})N_0V_0.\end{aligned} \tag{17}$$

Substituting Eq. (3) into Eq. (17), we can finally obtain

$$\tilde{\varepsilon}(\boldsymbol{k}) = \left(\frac{(\hbar\boldsymbol{k})^2N_0V_0}{m_{\rm ph}} + \frac{(\hbar\boldsymbol{k})^4}{4m_{\rm ph}^2}\right)^{1/2}, \tag{18}$$

which is the famous Bogoliubov dispersion law. At low temperatures, the long-wavelength approximation can be used, and the dispersion law of quasi-particle takes the phonon-like form

$$\tilde{\varepsilon}(\boldsymbol{k}) = \hbar\tilde{w}_{\boldsymbol{k}} = \hbar\tilde{c}\,|\boldsymbol{k}|\,, \tag{19}$$

where

$$\tilde{c} = \sqrt{N_0V_0/m_{\rm ph}} \tag{20}$$

is the sound velocity. This feature, together with the fact that the operator $b_{\boldsymbol{k}}^{+}b_{\boldsymbol{k}}$ in Eq. (13) describes a density fluctuation in the photon fluid, indicates that the nature of the elementary excitations here is that of phonons. For phonons,[26] we also find that the effective velocity of phonons has the form

$$\tilde{c} = \left(\frac{V_0}{m_{\rm ph}}\right)^{1/2}(N-\alpha T)^{1/2}, \tag{21}$$

where N is the total number of photons in the microcavity, and $\alpha = m_{\rm ph}k_{\rm B}\hbar^{-2}\sum\limits_{\boldsymbol{k}\neq 0}\boldsymbol{k}^{-2}$.

3. Energy-level shift

We consider a hydrogen atom which is embedded inside the microcavity, as shown in Fig. 1. The energy-level shift (S) of the bound electron consists of two parts. The first part, referred to as the usual Lamb shift ($S_{\rm Lamb}$), comes from the coupling of the bound electron with the vacuum electromagnetic modes. The other part comes from the interactions between the bound electron and the weakly-interacting photon field, and is named the normal shift ($S_{\rm Norm}$). According to the above analysis, the energy-level shift of the atom inside the microcavity can be phenomenologically expressed as $S = S_{\rm Lamb} + S_{\rm Norm}$. The Lamb shift is studied by many scholars.[27−29] In the present mode, we are only interested in the novel effects that are caused by the BEC of photons and will derive the expression of the normal shift of the atom using a non-relativistic QED theory. The relativistic correction is neglected because it is much smaller than the nonrelativistic contribution.[30]

According to Eq. (13) and the above analysis, we can find that in the Bose–Einstein condensate state, the total Hamiltonian of the field and the atom can be given by

$$H = H_0 + H_{\rm BEC} + H_{\rm I}, \tag{22}$$

where H_0 is the unperturbed atomic Hamiltonian which obeys the eigenvalue equation

$$H_0\,|n\rangle = E_n\,|n\rangle\,, \tag{23}$$

where $|n\rangle$ $(n = 1, 2, \ldots)$ is the eigenstate of the atom with a corresponding eigenvalue E_n, and $H_{\rm I}$ is the interaction Hamiltonian which is given by

$$H_{\rm I} = -\frac{e}{m}\boldsymbol{p}\cdot\boldsymbol{A}, \tag{24}$$

where $\boldsymbol{p}$ is the momentum of the atom, and $\boldsymbol{A}$ is the vector potential of the electromagnetic field

$$\boldsymbol{A} = \sum_{\boldsymbol{k}}\left(\frac{\hbar}{2V\varepsilon_0w_{\boldsymbol{k}}}\right)^{1/2}\left(a_{\boldsymbol{k}} + a_{\boldsymbol{k}}^{+}\right)\boldsymbol{e}_{\boldsymbol{k}}.$$

We have already shown that in a Bose–Einstein condensate state, the original system of interacting photons can be described as a collection of noninteracting phonons. This implies that at low temperatures, due to the BEC of photons, the interactions between the atom and the weakly-interacting photon field turn into the interactions between the atom and the noninteracting phonon field. As a result, the vector potential of the electromagnetic should be changed. In order to reveal the characteristics of the present system, we can express the vector potential in terms of the annihilation and creation operators of phonons through Eq. (12). In addition, in a BEC state we can approximatively replace the Bohr frequency $\omega_{\boldsymbol{k}}$ of photons by

ω_0 ($\omega_0 = m_{\rm ph}c^2/\hbar$). Since $b_{\boldsymbol{k}}^+$ is equivalent to $b_{\boldsymbol{k}}$,[31] the vector potential can be reexpressed as

$$\boldsymbol{A} = \sum_{\boldsymbol{k}} \left(\frac{\hbar}{2V\varepsilon_0 w_0}\right)^{1/2} \exp(\alpha_{\boldsymbol{k}}) \left(b_{\boldsymbol{k}} + b_{\boldsymbol{k}}^+\right) \boldsymbol{e}_{\boldsymbol{k}}, \quad (25)$$

where $\exp(\alpha_{\boldsymbol{k}}) = (c/n\tilde{c})^{1/2}$, and n is the refractive index of the dye solution. Using the second-order perturbation theory, the energy-level shift for a reference state $|n_0\rangle$ is given by

$$\Delta E_{n_0} = \sum_n \sum_{\boldsymbol{k}} \frac{|\langle n, 1_{\boldsymbol{k}}| H_{\rm I} |n_0, G\rangle}{E_{n_0} - E_n - \tilde{\varepsilon}(\boldsymbol{k})}, \quad (26)$$

where the summation over n includes both discrete and continuous spectra of the hydrogen atom. As we all know, the energy-level shift is infinite in all existing theories. However, it is possible to identify the most divergent term in the energy-level shift due to an electromagnetic mass effect which must exist for both bound and free electron. The effect is reasonably thought to be already included in the observed mass of the electron, and is incorporated into the mass renormalization term.[14] Consequently, it should be subtracted from Eq. (26). We can obtain the mass renormalization term by setting $E_n = E_{n_0}$ on the right hand of Eq. (26)

$$\Delta E_{\rm mass} = \sum_n \sum_{\boldsymbol{k}} \frac{|\langle n, 1_{\boldsymbol{k}}| H_{\rm I} |n_0, G\rangle}{-\tilde{\varepsilon}(\boldsymbol{k})}. \quad (27)$$

After some direct calculations, the normal shift of the hydrogen atom inside the microcavity is derived to be

$$S_{\rm Norm} = S_{\rm r} S_0, \quad (28)$$

where

$$S_{\rm r} = \frac{1}{w_0}\frac{1}{n}\left(\frac{m_{\rm ph}c^2}{V_0}\right)^{3/2} (N - \alpha T)^{-3/2} \quad (29)$$

is the relative shift and

$$S_0 = \frac{e^2}{2\pi\hbar\varepsilon_0 c^2 m^2} \sum_n |\boldsymbol{p}_{n,n_0}|^2 (E_n - E_{n_0}) \times \ln\frac{K}{|E_n - E_{n_0}|} \quad (30)$$

is the usual energy-level shift for a hydrogen atom in 2D free photon field. The nonrelativistic limit requires an effective upper limit of the order of $K = mc^2$ to the frequencies of light which effectively contribute to the energy-level shift of a bound electron.

4. Results and discussion

From Eq. (28), we can see that in a Bose–Einstein condensate state the energy-level shift of the atom inside the microcavity is modified by a factor $S_{\rm r}$ compared with that in free photon field. In order to know the physical origin of $S_{\rm r}$, we can rewrite it in another form. Substituting Eq. (21) into Eq. (29), we obtain the relative shift

$$S_{\rm r} = S_{\rm L} S_{\rm N}, \quad (31)$$

where

$$S_{\rm L} = \frac{1}{w_0}\frac{1}{n}\left(\frac{m_{\rm ph}c^2}{V_0}\right)^{3/2} \quad (32)$$

is the linear contribution part depending on the geometry of the microcavity and the interactions between photons, and $S_{\rm N}$ is the nonlinear contribution part and reads

$$S_{\rm N} = (N - \alpha T)^{-3/2}, \quad (33)$$

which comes from the BEC of photons.

In this paper, we mainly focus on the nonlinear effect on the energy shift induced by the BEC of photons. From Eq. (33), it is obvious that the nonlinear contribution factor $S_{\rm N}$ is an increasing function of the absolute temperature T. In order to better illustrate the temperature dependence relation, we introduce the notation of effective temperature: $T^* = T/(N - \alpha T)$. Subsequently, a simple relation $S_{\rm N} = (T^*/T)^{3/2}$ can be obtained (see Fig. 2). From Fig. 2, it is obvious that $S_{\rm N}$ increases monotonically from zero to unity as the ratio T^*/T rises. At absolute zero temperature, almost all photons are in the condensate state, allowing the relations $N_0 \gg 1$ and $T^*/T = 1/N_0 \approx 0$ to be satisfied. As a result, at absolute zero temperature, the nonlinear factor is equal to zero, and the normal shift of the bound electron can be ignored. That is to say, at absolute zero temperature the energy-level shift entirely comes from the Lamb shift, and the atom can be approximately regarded to be in vacuum. With the increase in the absolute temperature T, the ratio T^*/T also increases, leading to the enhancement of the nonlinear effect. Meanwhile, the energy-level shift of the bound electron enlarges. However, as the temperature becomes higher than the critical temperature $T_{\rm c}$, the BEC of photons disappears and the relation $S_{\rm N} = 1$ holds. The result means that above $T_{\rm c}$, the photon system is in a normal thermal radiation state and the temperature dependence relation of the normal shift disappears.

That is to say, above T_c the normal shift only depends on the linear contribution factor S_L: $S_{Norm} = S_L S_0$. Here, if we connect Eq. (25) with Eq. (28), we also find that in the BEC state, the vector potential of the electromagnetic field can be reexpressed as $\boldsymbol{A} = \beta \boldsymbol{A'}$, with $\beta = (c/n\tilde{c})^{1/2} = n^{-1}V_0^{-1}m_{ph}c^2 S_N$ and $\boldsymbol{A'} = \sum_k (\hbar/2V\varepsilon_0\omega_k)^{1/2} \boldsymbol{e}_k \left(b_k + b_k^+\right)$. The term $\boldsymbol{A'}$ with the identical form compared with the original vector potential of the field determines the usual energy-level shift S_0, and this part is closely related to the structure of the atom. A prototypic example is atomic Helium and Hydrogen. However, the nonlinear correction factor S_N comes from the parameter β, which is independent of the structure of the atom. According to above analysis, we therefore can predict that the 3/2 principle should also survive for other atoms.

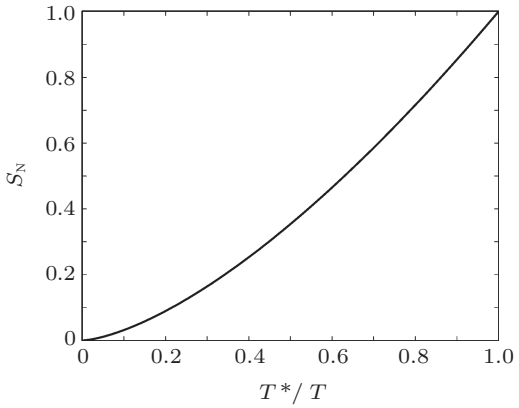


Fig. 2. Variation of the nonlinear contribute factor S_N with relative temperature T^*/T.

Up to now, there has always been a question why the normal shift increases monotonally from zero to $S_L S_0$, as the absolute temperature rises from zero to T_c. We have already known that at low temperatures, due to the BEC of photons, the normal shift mainly comes from the interactions between the bound electron and the noninteracting phonon field. At absolute zero temperature, the number of phonons is equal to zero, thus the interactions between the bound electron and the phonons disappear. As a result, there is no normal shift. With the increase in the absolute temperature T, the phonon number also increases. This result means that more phonons are involved in the interactions between the bound electron and the phonon field. Thus, when the absolute temperature rises, the normal shift of the bound electron enlarges. However, above T_c, the BEC of photons disappears and the interactions between the bound electron and the phonon field turn into the interactions between the bound electron and the photon field. In the present model, the photon number is conserved by photon scattering off the dye molecules. Thus, above T_c, the normal energy shift only depends on the geometry of the microcavity and the interactions between photons. In other words, above T_c, the normal energy shift only depends on the linear contribution factor S_L.

5. Conclusion

To sum up, the BEC of photons in a 2D optical microcavity is investigated. The effective mass and the chemical potential for a photon inside the microcavity are nonvanishing. This implies the existence of a Bogoliubov dispersion relation for the elementary excitations of the photon gas, and in particular, that sound waves exists for long-wavelength disturbances of the system. The energy-level shift of the hydrogen atom immersed in the microcavity is also investigated. It is found that below the critical temperature T_c, the energy-level shift of the bound electron is temperature-dependent, and it is a monotonically increasing function of the absolute temperature T. Especially, at absolute zero temperature, the energy-level shift of the bound electron entirely comes from the Lamb shift, and the atom can be approximately considered to be in vacuum.

Appendix A

According to the experimental results given by Klaers *et al.*,[10] we have known that the diameter of the transversal ground state is a function of N_0/N_{ph}. Slightly above criticality the measured diameter of (14 ± 2) μm is well consistent with the expected ground-state diameter

$$d_0 = 2\sqrt{\hbar \ln 2/m_{ph}\Omega}, \tag{A1}$$

for non-interacting photons. However, for a higher ground-state occupation N_0/N_{ph}, this does not hold anymore. This observation suggests the presence of a repelling photon–photon interaction mediated by the dye solution.

The interaction energy between photons can be determined by $E_{int} = m_{ph}c^2 n_2 I(r)/n_0$, where $I(r)$ is the optical intensity, and n_0 and n_2 are the linear and nonlinear indices of refraction, respectively. For BEC, using a dimensionless interaction parameter $g = -\left(m_{ph}^4 c^6 n_2\right) / \left(2\pi\hbar^3 n_0 q\right)$ and the

Gross–Pitaevskii order parameter $\psi(r)$ with $I(r) = \left(m_{\rm ph}c^2\right)^2 (\hbar q)^{-1} N_0 |\psi(r)|^2$, the interaction energy can be written in a more familiar form

$$E_{\rm int} = \left(\hbar^2/m_{\rm ph}\right) g N_0 |\psi(r)|^2 . \qquad \text{(A2)}$$

From the increase in the mode diameter, Klaers *et al.* have estimated the magnitude of the repulsive interaction with $g \approx (7 \pm 3) \times 10^{-4}$. This is far below the value g (in the order of 10^{-2} to 10^{-1}), which is reported for 2D atomic physics quantum gas experiments. The result means that the interactions between photon are always weak. Furthermore, for BEC, the parameter $|\psi(r)|^2$ can be replaced by the density of the photon gas n.[32] As a result, the interaction between photons can be rewritten in the s-wave interaction form $E_{\rm int} = nV_0N_0/2$, where $V_0 = 4\pi\hbar^2 a/m_{\rm ph}$ is the two-body potential for $k = 0$, and $a = g/2\pi$ is the s-wave scattering length.

References

[1] Anderson M H, Ensher J R, Matthews M R, Wieman C E and Cornell E A 1995 *Science* **269** 198

[2] Davis K B, Mewes M O, Andrews M R, van Druten N J, Kurn D M and Ketterle W 1995 *Phys. Rev. Lett.* **75** 3969

[3] Bradley C C, Sackett C A and Hulet R G 1997 *Phys. Rev. Lett.* **78** 985

[4] Jochim S, Bartenstein M, Altmeyer A, Hendl G, Riedl S, Chin C, Hecker Denschlag J and Grimm R 2003 *Science* **302** 2101

[5] Deng H, Weihs G, Bloch J and Yamamoto Y 2002 *Science* **298** 199

[6] Wang Y Z, Zhou S Y, Long Q, Zhou S Y and Fu H X 2003 *Chin. Phys. Lett.* **20** 799

[7] Balili R, Hartwell V, Snoke D, Pfeiffer L and West K 2007 *Science* **316** 1007

[8] Demokritov S O, Demidov V E, Dzyapko O, Melkov G A, Serga A A, Hillebrands B and Slavin A N 2006 *Nature* **443** 430

[9] Ortiz G and Dukelsky J 2005 *Phys. Rev.* A **72** 043611

[10] Li S C and Duan W S 2009 *Chin. Phys.* B **18** 4177

[11] Zhou L, Kong L B and Zhan M S 2008 *Chin. Phys.* B **17** 1601

[12] Klaers J, Schmitt J, Vewinger F and Weitz M 2010 *Nature* **468** 545

[13] Lamb W E and Retherford R C 1947 *Phys. Rev.* **72** 241

[14] Bethe H A 1947 *Phys. Rev.* **72** 339

[15] Lundeen S R and Pipkin F M 1981 *Phys. Rev. Lett.* **46** 232

[16] Barton G 1974 *J. Phys.* B **7** 2134

[17] Horak P, Domokos P and Ritsch H 2003 *Euro. Phys. Lett.* **61** 459

[18] Kleppner D 1981 *Phys. Rev. Lett.* **47** 233

[19] Ivanov A L and Haug H 1995 *Jpn. J. Appl. Phys. Suppl.* **34** 143

[20] Walsh J E 1971 *Phys. Rev. Lett.* **27** 208

[21] Łach G, Jeziorski B and Szalewicz K 2004 *Phys. Rev. Lett.* **92** 233001

[22] Klaers J, Vewinger F and Weitz M 2010 *Nature Phys.* **6** 512

[23] Pitaevskii L and Stringari S 2003 *Bose–Einstein Condensation* (New York: Oxford University Press) pp. 27–30

[24] Bogoliubov N 1947 *J. Phys., U.S.S.R* **11** 23

[25] Pines D 1961 *The Many-Body Problem* (New York: Benjamin)

[26] Pitaevskii L and Stringari S 2003 *Bose-Einstein Condensation* (New York: Oxford University Press) pp. 36,37

[27] Welton T A 1948 *Phys. Rev.* **74** 1157

[28] Milonni P W 1982 *Phys. Rev.* A **25** 1315

[29] Drake G W F and Swainson R A 1990 *Phys. Rev.* A **41** 1243

[30] Grotch H 1988 *Phys. Scr.* **T21** 86

[31] Cheng Z 2002 *J. Opt. Soc. Am.* B **19** 1962

[32] Leggett A J 2001 *Rev. Mod. Phys.* **73** 307