Non-equilibrium Landauer transport model for Hawking radiation from a Reissner–Nordstrom black hole*

Zeng Xiao-Xiong(曾晓雄)[a], Zhou Shi-Wei(周史薇)[b], and Liu Wen-Biao(刘文彪)[a]†

[a] *Department of Physics, Institute of Theoretical Physics, Beijing Normal University, Beijing* 100875, *China*
[b] *Department of Foundation, Academy of Armored Forces Engineering, Beijing* 100072, *China*

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The recent work of Nation *et al.*, in which the Hawking radiation energy and entropy flow from a black hole is considered to be produced in a one-dimensional Landauer transport process, is extended to the case of a Reissner–Nordstrom black hole. The energy flow contains not only the contribution of the thermal flux but also that of the particle flux. It is found that the charge can also be transported via the one-dimensional quantum tunnel. Because of the existence of the electrostatic potential, the entropy production rate is shown to be smaller than that of the Schwarzschild black hole.

Keywords: Hawking radiation, entropy, Landauer transport model, black hole

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1. Introduction

The Hawking radiation from a black hole is an interesting topic in theoretical physics, because it provides not only a clue to detect black holes but also a platform to research the quantum gravity. Since the proposal of Hawking that a black hole can emit radiation in the curved space time background,[1,2] there have been many methods to derive it.[3−9] Now it is believed that the Hawking radiation arises from the spontaneous production of virtual particle pairs near the horizon due to the vacuum fluctuation. When the negative energy virtual particle tunnels inwards, the positive energy virtual particle materializes as a real particle and escapes to infinity.[10−21] But, how does the positive energy particle get away? Recently, Nation *et al.* gave a model, in which the positive energy particle escapes to infinity via a one-dimensional (1D) quantum channel.[22] The key idea is that the black hole and the vacuum can be viewed as two thermal reservoirs connected by a single 1D quantum channel. When the scattering effect is ignored, the Hawking radiation flow rate is obtained, which is shown to be equal to the energy–momentum tensor expectation value of an infinite observer.

Now, we extend the Landauer transport model to the Reissner–Nordstrom (RN) black hole. We intend to explore the effect of the chemical potential on the energy flux. For the spacetime with chemical potential, when the near horizon conformal symmetry is considered, the expectation value of charge flow is considered besides the flow of energy–momentum tensor, which contains contributions from both the thermal and the particle fluxes. For the charged black hole, the electric charge is the gauge charge, and the electrostatic potential is the gauge potential. Thus, when the Landauer transport model is used, we should also explore the charge flux besides the energy flux. As far as the energy flux is concerned, we expect it to contain not only the thermal flux but also the particle flux. Throughout this paper, we use the units $G = \hbar = c = k_{\rm B} = 1$.

2. Conformal symmetry and Hawking radiation flux

The line element of a RN black hole is

$$\begin{aligned} {\rm d}s^2 = &-\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right){\rm d}t^2 \\ &+\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)^{-1}{\rm d}r^2 \\ &+r^2{\rm d}\theta^2+r^2\sin^2\theta{\rm d}\varphi^2, \end{aligned} \tag{1}$$

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†Corresponding author. E-mail: wbliu@bnu.edu.cn

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where M and Q are the mass and the charge of the black hole, respectively. The electromagnetic four-vector is

$$A_\mu = \left(-\frac{Q}{r},\ \ 0,\ \ 0,\ \ 0\right). \tag{2}$$

From Eq. (1), we can obtain the event horizon $r_{\rm h} = M + \sqrt{M^2 - Q^2}$ immediately.

According to the dimensional reduction technique, equation (1) can be expressed in the effective two-dimensional form[9]

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + f^{-1}(r)\mathrm{d}r^2, \tag{3}$$

where $f(r) = 1-2M/r+Q^2/r^2$. Under the tortoise coordinate transformation defined as $\mathrm{d}r^* = [1/f(r)]\mathrm{d}r$, we can define the null coordinates $u = t+r^*$, $v = t-r^*$, and the Kruskal coordinates $U = -(1/\kappa)/\,\mathrm{e}^{-\kappa u}$, $V = (1/\kappa)\,\mathrm{e}^{\kappa v}$. Then, the corresponding conformal form for Eq. (3) is

$$\mathrm{d}s^2 = -f(r)\,\mathrm{e}^{-2\kappa r^*}\,\mathrm{d}U\,\mathrm{d}V, \tag{4}$$

where $\kappa = (r_{\rm h} - M)/r_{\rm h}^2$ is the surface gravity.

In the following, we intend to find the expectation values of the energy–momentum tensor and the gauge current. It is well known that the classical Einstein field equation can be derived from the classical action by using the minimal variational principle. In the gravitational field with an electric field background, the action consists of a gravitational part[23–28]

$$\begin{aligned} \Gamma_{\rm grav} \ =\ & \frac{1}{96\pi}\int \mathrm{d}^2x\,\mathrm{d}^2y\sqrt{-g}R(x)\frac{1}{\Delta_g}(x,y) \\ & \times \sqrt{-g}R(y), \end{aligned} \tag{5}$$

and a gauge part

$$\begin{aligned} \Gamma_{U(1)} \ =\ & \frac{e^2}{2\pi}\int \mathrm{d}^2x\,\mathrm{d}^2y\epsilon^{\mu\nu}\partial_\mu A_\nu(x)\frac{1}{\Delta_g}(x,y) \\ & \times \epsilon^{\rho\sigma}\partial_\rho A_\sigma(y), \end{aligned} \tag{6}$$

where R is the two-dimensional scalar curvature, g is the metric determinant, e is the gauge charge, Δ_g is the Laplacian, and $\epsilon^{\mu\nu}$ represents the two-dimensional Levi–Civita tensor. From these actions, we can obtain the expectation values of the gauge current and the energy–momentum tensor with proper boundary conditions[25–28]

$$\langle J_u\rangle \ =\ \frac{e^2}{2\pi}[A_t(r) - A_t(r_{\rm h})], \tag{7}$$

$$\langle J_v\rangle \ =\ \frac{e^2}{2\pi}A_t(r), \tag{8}$$

$$\begin{aligned} \langle T_{uu}\rangle \ =\ & \frac{1}{24\pi}\left(\frac{\kappa_{\rm h}^2}{2} - \frac{M}{r^3} + \frac{3M^2}{2r^4} + \frac{3e^2}{2r^4} - \frac{3Me^2}{r^5}\right. \\ & \left. + \frac{e^4}{r^6}\right) + \frac{e^2}{4\pi}[A_t(r) - A_t(r_{\rm h})]^2, \end{aligned} \tag{9}$$

$$\begin{aligned} \langle T_{vv}\rangle \ =\ & \frac{1}{24\pi}\left(-\frac{M}{r^3} + \frac{3M^2}{2r^4} + \frac{3e^2}{2r^4} - \frac{3Me^2}{r^5} + \frac{e^4}{r^6}\right) \\ & + \frac{e^2}{4\pi}A_t^2(r). \end{aligned} \tag{10}$$

Obviously, from Eq. (9), we can find that the infinite observers will see a bunch of flow $\pi T_{\rm h}^2/12 + e^2A_t^2(r_{\rm h})/4\pi$, where $T_{\rm h} = \kappa/2\pi$ is the Hawking temperature. Correspondingly, according to Eq. (10), a near horizon observer may find that a bunch of flow $-\pi T_{\rm h}^2/12 + e^2A_t^2(r_{\rm h})/4\pi$ will drop into the horizon. From Eqs. (7) and (8), we also know that a negative flow $e^2A_t(r_{\rm h})/2\pi$ at the event horizon is responsible for the flow of the charge current observed by an infinite observer.

3. Hawking radiation in Landauer transport model

The Landauer transport model was first proposed to study the electrical transport in mesoscopic circuits.[29] Subsequently, it was used to describe the thermal transport.[30] In 2000, the phonetic quantized thermal conductance counterpart was measured for the first time.[31] For a 1D quantum channel of thermal conductance, it is supposed that there are two thermal reservoirs characterized by temperatures $T_{\rm L}$, $T_{\rm R}$ and chemical potentials $u_{\rm L}$, $u_{\rm R}$, where subscripts L and R denote the left and the right thermal reservoirs, respectively. They are coupled adiabatically through an effective 1D connection. Because of the temperature difference, the thermal transportation occurs. Typically, a wire can provide several available parallel channels for given values of chemical potential and temperature. For the sake of simplicity in the present investigation, we only consider the ballistic transport, which means that the channel currents do not interfere with each other.[22] Universally, there are several distribution functions, here we adopt the Haldane's statistics[32]

$$f_g(E) = \left[\omega\left(\frac{E-u}{T}\right) + g\right]^{-1}, \tag{11}$$

where function $\omega(x)$ satisfies the relation

$$\omega(x)^g[1+\omega(x)]^{1-g} = \mathrm{e}^x, \tag{12}$$

in which $g = 0$ and $g = 1$ correspond to bosons and fermions, respectively.

The left (right) component of the single channel energy current is

$$\dot{E}_{\mathrm{L(R)}} = \frac{T^2_{\mathrm{L(R)}}}{2\pi}\int_{x^0_{\mathrm{L(R)}}}^{\infty} \mathrm{d}x \left(x + \frac{u_{\mathrm{L(R)}}}{T_{\mathrm{L(R)}}}\right) f_g(x), \quad (13)$$

where $x^0_{\mathrm{L(R)}} = -u_{\mathrm{L(R)}}/T_{\mathrm{L(R)}}$. Following Ref. [22], we define the zero level of energy with respect to the longitudinal component of the kinetic energy. The total energy current is then $\dot{E} = \dot{E}_{\mathrm{L}} - \dot{E}_{\mathrm{R}}$. Here, we consider the energy flow of bosons. It has been pointed out that in a (1+1)-dimensional curved space time, the fermionic field describing a massless particle plus its antiparticle is equivalent to a single massless bosonic field.[33] Hence, the maximum energy flow of bosons can be treated as the combination of the fermionic particle flux with its antiparticle flux. We define $x = (E-u)/T$ and $y = (E+u)/T$, then the flow of energy can be expressed as

$$\begin{aligned}\dot{E}_{\mathrm{L(R)}} = {} & \frac{T^2_{\mathrm{L(R)}}}{2\pi}\Bigg[\int_{\frac{-u_{\mathrm{L(R)}}}{T_{\mathrm{L(R)}}}}^{\infty} \mathrm{d}x \left(x + \frac{u_{\mathrm{L(R)}}}{T_{\mathrm{L(R)}}}\right) \frac{1}{\mathrm{e}^x + 1} \\ & + \int_{\frac{u_{\mathrm{L(R)}}}{T_{\mathrm{L(R)}}}}^{\infty} \mathrm{d}y \left(y - \frac{u_{\mathrm{L(R)}}}{T_{\mathrm{L(R)}}}\right) \frac{1}{\mathrm{e}^y + 1}\Bigg]. \quad (14)\end{aligned}$$

To obtain the final value, we first vary the lower limit of the integral, that is

$$\begin{aligned}\dot{E}_{\mathrm{L(R)}} = {} & \frac{T^2_{\mathrm{L(R)}}}{2\pi}\Bigg[\int_{0}^{\infty} \mathrm{d}x \left(x + \frac{u_{\mathrm{L(R)}}}{T_{\mathrm{L(R)}}}\right) \frac{1}{\mathrm{e}^x + 1} \\ & + \int_{0}^{\frac{u_{\mathrm{L(R)}}}{T_{\mathrm{L(R)}}}} \mathrm{d}x \left(-x + \frac{u_{\mathrm{L(R)}}}{T_{\mathrm{L(R)}}}\right) \frac{1}{\mathrm{e}^{-x} + 1} \\ & + \int_{0}^{\infty} \mathrm{d}y \left(y - \frac{u_{\mathrm{L(R)}}}{T_{\mathrm{L(R)}}}\right) \frac{1}{\mathrm{e}^y + 1} \\ & - \int_{0}^{\frac{u_{\mathrm{L(R)}}}{T_{\mathrm{L(R)}}}} \mathrm{d}y \left(y - \frac{u_{\mathrm{L(R)}}}{T_{\mathrm{L(R)}}}\right) \frac{1}{\mathrm{e}^y + 1}\Bigg]. \quad (15)\end{aligned}$$

As $1/(\mathrm{e}^{-x}+1) = 1 - 1/(\mathrm{e}^x+1)$, equation (15) can take the form

$$\begin{aligned}\dot{E}_{\mathrm{L(R)}} = {} & \frac{T^2_{\mathrm{L(R)}}}{2\pi}\Bigg(\int_{0}^{\infty} \frac{x}{\mathrm{e}^x + 1}\mathrm{d}x \\ & + \int_{0}^{\infty} \frac{y}{\mathrm{e}^y + 1}\mathrm{d}y + \frac{u^2_{\mathrm{L(R)}}}{2T^2_{\mathrm{L(R)}}}\Bigg). \quad (16)\end{aligned}$$

The energy current of bosons hence can be expressed as

$$\dot{E} = \dot{E}_{\mathrm{L}} - \dot{E}_{\mathrm{R}} = \frac{\pi}{12}(T^2_{\mathrm{L}} - T^2_{\mathrm{R}}) + \frac{1}{4\pi}(u^2_{\mathrm{L}} - u^2_{\mathrm{R}}), \quad (17)$$

where we have used the integration

$$\int_0^{\infty} \frac{x}{\mathrm{e}^x + 1}\mathrm{d}x = \frac{\pi^2}{12}. \quad (18)$$

As the left and the right thermal systems are the RN black hole and the thermal environment with absolute zero temperature, respectively, we find

$$\dot{E} = \frac{e^2}{4\pi}A^2_t(r_{\mathrm{h}}) + \frac{\pi T^2_{\mathrm{h}}}{12}, \quad (19)$$

in which $u_{\mathrm{R}} = 0$ and $u_{\mathrm{L}} = eQ/r_{\mathrm{h}}$ are used. Different from the result in Ref. [22], our result shows that the energy current flowing through the 1D system can be divided into two independent components: one due exclusively to the flux of particles and carrying no heat, and the other entirely determined by the temperature of the emitting reservoir irrespective of the number of particles. This result agrees with the energy–momentum tensor flux in Eq. (9) obtained by using the conformal symmetry.

For the transportation of charge via the 1D quantum channel, the charge current flow from left (with higher chemical potential u) to right without scattering can be expressed as[32]

$$\dot{I} = \frac{e}{2\pi}\int_0^{\infty} f(\omega)\mathrm{d}\omega, \quad (20)$$

where we have considered the left and the right thermal systems as the RN black hole and the thermal environment, respectively. For the case of bosons, similarly, we also should take into account the fermionic charge flux and its antiparticle charge flux, namely, equation (20) should become

$$\begin{aligned}\dot{I} = {} & \frac{e}{2\pi}\int_0^{\infty} \Bigg[\frac{1}{\exp[(\omega - u_{\mathrm{h}})/T_{\mathrm{h}}] + 1} \\ & - \frac{1}{\exp[(\omega - u_{\mathrm{h}})/T_{\mathrm{h}}] + 1}\Bigg]\mathrm{d}\omega, \quad (21)\end{aligned}$$

where $f(\omega) = 1/\{\exp[(\omega - u_{\mathrm{h}})/T_{\mathrm{h}}] + 1\}$ is the radiation spectrum of fermions at the event horizon, and u_{h} is the chemical potential at the event horizon. With the help of the integration

$$\int \frac{1}{b + c\,\mathrm{e}^{ax}}\mathrm{d}x = \frac{x}{b} - \frac{1}{ab}\ln(b + c\,\mathrm{e}^{ax}), \quad (22)$$

equation (21) is changed into

$$\dot{I} = -\frac{e^2}{2\pi}A_t(r_{\mathrm{h}}), \quad (23)$$

which is consistent with the charge current flow at the event horizon given in Eq. (7), namely, the gauge flux with respect to the electric charge of the RN space time also can be transported via the 1D quantum channel.

4. Discussion and conclusion

The flows of the Hawking radiation energy and the charge current from an RN black hole have been investigated using a 1D quantum transport model, which are shown to be consistent with the vacuum expectation values of the energy–momentum tensor and the gauge flux. For the energy flow of bosons, we find that it contains not only the contribution of the thermal flux but also that of the particle flux. In addition, we also obtain the charge current, which is shown to be independent of the black hole temperature.

As an application of the non-equilibrium Landauer transport model, we can obtain the net entropy production rate defined as $R = \mathrm{d}S/\mathrm{d}S_{\mathrm{BH}}$ in the two-dimensional space time.[34] In the Landauer transport model, the entropy flow is given by

$$\begin{aligned}&\dot{S}_{\mathrm{L(R)}}\\ &= \frac{T_{\mathrm{L(R)}}}{2\pi}\int_{x^0_{\mathrm{L(R)}}}^{\infty}\mathrm{d}x\{f_g\ln f_g+(1-gf_g)\ln(1-gf_g)\\ &\quad -[1+(1-gf_g)]\ln[1+(1-g)f_g]\}. \end{aligned}\tag{24}$$

Considering the contributions of the fermionic particle and its antiparticle, we can integrate Eq. (24) as

$$\dot{S} = \dot{S}_{\mathrm{L}} - \dot{S}_{\mathrm{R}} = \frac{\pi}{6}(T_{\mathrm{L}} - T_{\mathrm{R}}). \tag{25}$$

When the left and the right are regarded as the black hole and its surrounding environment with temperature zero respectively, the entropy flow for the RN black hole can be expressed as $\dot{S} = \pi T_{\mathrm{h}}/6$. So, the entropy production rate for the RN black hole is

$$\begin{aligned} R &= \frac{\mathrm{d}S}{\mathrm{d}S_{\mathrm{BH}}}\\ &= \frac{\pi}{6}\frac{T_{\mathrm{h}}^2}{\dot{E}-V_0\dot{I}}\\ &= \frac{2}{1-(3/\pi^2)(e^2A_t^2/T_{\mathrm{h}}^2)}, \end{aligned}\tag{26}$$

where we have used the first law of black hole thermodynamics and considered the energy conservation and the charge conservation. Because of the existence of electromagnetic field, we find that the entropy production rate of the RN black hole is bigger than that of the Schwarzschild black hole with $R = 2$.

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