Multipartite entanglement concentration of electron-spin states with CNOT gates*

Ren Bao-Cang(任宝藏), Hua Ming(华 明), Li Tao(李 涛),
Du Fang-Fang(杜芳芳), and Deng Fu-Guo(邓富国)†

Department of Physics, Applied Optics Beijing Area Major Laboratory, Beijing Normal University, Beijing 100875, *China*

(Received 30 March 2012; revised manuscript received 25 April 2012)

We propose a different entanglement concentration protocol (ECP) for nonlocal N-electron systems in a partially entangled Bell-type pure state using the CNOT gates and the projection measurements on an additional electron. For each nonlocal N-electron system, Alice first entangles it with the additional electron, and then she projects the additional electron onto an orthogonal basis for dividing the N-electron systems into two groups. In the first group, the N parties obtain a subset of N-electron systems in a maximally entangled state directly. In the second group, they obtain some less-entangled N-electron systems, which are the resource for the entanglement concentration in the next round. By iterating the entanglement concentration process several times, the present ECP has the maximal success probability, which is the theoretical limit of an ECP, equals to the entanglement of the partially entangled state, higher than the others. This ECP may be useful in quantum computers based on electron-spin systems in the future.

Keywords: entanglement concentration, electron-spin states, decoherence, quantum communication

PACS: 03.67.Bg, 03.65.Yz, 03.67.Hk **DOI:** 10.1088/1674-1056/21/9/090303

1. Introduction

Entanglement is a very important quantum resource in quantum information processing.[1] In a long-distance quantum communication, the legitimate users should distribute entangled photon systems acting as their quantum channels, such as quantum key distribution,[2−8] quantum teleportation,[9−14] quantum dense coding,[15,16] quantum secret sharing,[17−24] quantum state sharing,[25−30] controlled teleportation,[31,32] and quantum secure direct communication.[33−38] As the entanglement can only be produced locally, an entangled photon system should be transmitted over a noisy channel, which means that it will inevitably interact with its environment. On the other hand, in the storage of the entangled quantum state in a quantum repeater, the decoherence will decrease the entanglement of the entangled photon system. The decoherence of the entangled quantum system will decrease the fidelity and the security of the quantum communication protocol.

The entanglement purification and the entanglement concentration are two useful ways for depressing the noise effect on entangled photon systems. In general, the entanglement purification is used to extract a subset of high-fidelity entangled systems from a set of photon systems in a mixed entangled state with less entanglement. Since Bennett *et al.*[39] proposed the first entanglement purification protocol (EPP) in 1996, many works were focused on it, and many important EPPs have been proposed for different physical systems.[39−49] Compared with the EPP, the entanglement concentration protocol (ECP) is usually more efficient for two parties in quantum communication, say Alice and Bob, to distill some maximally entangled systems from an ensemble in a less-entangled pure state. Up to now, there have been some interesting ECPs.[50−58] In 1996, Bennett *et al.*[50] proposed the first ECP for two-photon systems based on the Schmidt projection method. In 2001, an ECP for photon pairs based on linear optical elements was proposed.[53,54] In 2008, Sheng *et al.*[55] proposed an ECP for photon systems based on the cross-Kerr non-

*Project supported by the National Natural Science Foundation of China (Grant Nos. 10974020 and 11174039), the Program for New Century Excellent Talents in University of Ministry of Education of China (Grant No. NCET-11-0031), and the Fundamental Research Funds for the Central Universities, China.

†Corresponding author. E-mail: fgdeng@bnu.edu.cn

© 2012 Chinese Physical Society and IOP Publishing Ltd

linearities. By iterating the entanglement concentration process, it has far higher efficiency and yield than those in Refs. [54] and [53]. Certainly, those three ECPs have lower efficiencies compared to those for known entangled pure states shown in Refs. [51] and [52]. In 2010, the first single-photon ECP[56] was discussed with the cross-Kerr nonlinearity. In 2012, Sheng *et al.*[57] proposed an interesting ECP for partially entangled photon pairs assisted by single photons. And an optimal ECP for multipartite photon systems was proposed by resorting to the projection measurement on an additional photon.[58]

An electron-spin system is an interesting qubit in quantum computation and quantum communication. It can be used to construct the elementary logical gate and quantum repeater, generate the entanglement, store quantum states, and so on. For example, in 2004, Beenakker *et al.*[59] exploited the charge detection[60] to construct a controlled-not (CNOT) gate based on both the charge and the spin degrees of freedom of electrons. In 2007, Ionicioiu[61] used the charge detection to complete the entanglement generation of electron spins. In 2006, Zhang *et al.*[62] presented a scheme for the multipartite entanglement analyzer. Some other interesting EPPs and ECPs for electron-spin systems were proposed in Refs. [63]–[66]. In 2005, Feng *et al.*[63] proposed an EPP for two-electron systems based on the charge detection. In 2011, an EPP for multi-electron systems[64] was proposed. Two ECPs for multi-electron systems were discussed by Sheng *et al.*[65] and Wang *et al.*[66] by resorting to the charge detection and the quantum dots in micro-wave cavities, respectively.

In this paper, we propose a different ECP for nonlocal N-electron systems in a partially entangled Bell-type pure state, resorting to the CNOT gates and the projection measurements on an additional electron. In the present ECP, Alice first entangles each nonlocal N-electron system with an additional electron a with a CNOT gate, and then she projects electron a into an orthogonal basis $\{|\varphi\rangle, |\varphi^{\perp}\rangle\}$ for dividing the outcomes into two groups. In the first group, the N-electron system is in the maximally entangled state. In the second group, the parties obtain the N-electron system in another partially entangled state with less entanglement, which can be used as the resource for entanglement concentration in the next round. By iterating the process several times, the present ECP has an optimal success probability, the theoretical limit, which is just the entanglement of the partially entangled state E, twice of those based on the entanglement swapping and a collective unitary evolution,[51,52] and is far higher than that of the other typical ECPs.[53–55,65,66] This ECP may be useful with quantum computers based on electron-spin systems in the future.

2. Entanglement concentration of two-electron spin states with CNOT gates

The quantum circuit for the present ECP is shown in Fig. 1. Let us assume that the partially entangled pure state before the entanglement concentration process is

$$|\Phi_1\rangle_{\rm AB} = \alpha|\uparrow\rangle_{\rm A}|\uparrow\rangle_{\rm B} + \beta|\downarrow\rangle_{\rm A}|\downarrow\rangle_{\rm B}, \tag{1}$$

where subscripts A and B represent the two electrons shared by two remote parties in quantum communication, say Alice and Bob, $|\uparrow\rangle$ and $|\downarrow\rangle$ represent the up-spin and the down-spin states, respectively, α and β are two coefficients and satisfy the relation

$$|\alpha|^2 + |\beta|^2 = 1. \tag{2}$$

Alice and Bob know these two parameters before they distill a subset of maximally entangled electron pairs from a set of nonlocal electron pairs in state $|\Phi_1\rangle_{\rm AB}$, which is the same as that in the ECPs with the entanglement swapping and a collective unitary evolution.[51,52] In fact, it is not difficult to obtain this information about the partially entangled pure state by measuring some samples in a practical quantum communication.

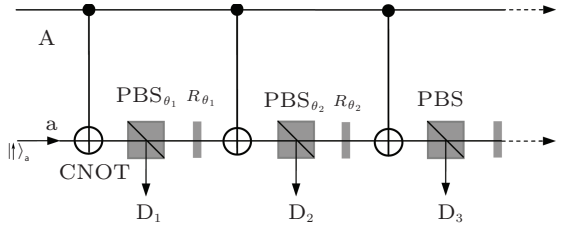


Fig. 1. Circuit for the present ECP with CNOT gates, where PBS represents the polarizing beam splitter for electron spins, R is a unitary rotation for electron spins, and a represents the additional electron in spin state $|\uparrow\rangle$.

For distilling a subset of maximally entangled electron pairs from a set of pairs in a partially entangled pure state $|\Phi_1\rangle_{\rm AB}$, Alice prepares an additional electron a in the spin state $|\Phi\rangle_{\rm a} = |\uparrow\rangle_{\rm a}$ for each nonlocal two-electron system AB and then performs a CNOT operation on her electrons A and a by using the electron A as the controlling qubit. After the

CNOT gate operation, the three-electron system ABa is in the state

$$|\Psi_1\rangle_{\rm ABa} = \alpha|\uparrow\rangle_{\rm A}|\uparrow\rangle_{\rm B}|\uparrow\rangle_{\rm a} + \beta|\downarrow\rangle_{\rm A}|\downarrow\rangle_{\rm B}|\downarrow\rangle_{\rm a}. \quad (3)$$

We can rewrite state $|\Psi_1\rangle_{\rm ABa}$ under the orthogonal basis $\{|\varphi_1\rangle_{\rm a} = \alpha|\uparrow\rangle - \beta|\downarrow\rangle, |\varphi_1^{\perp}\rangle_{\rm a} = \beta|\uparrow\rangle + \alpha|\downarrow\rangle\}$, i.e.,

$$\begin{aligned}|\Psi_1\rangle_{\rm ABa} =\ & (\alpha^2|\uparrow\rangle_{\rm A}|\uparrow\rangle_{\rm B} - \beta^2|\downarrow\rangle_{\rm A}|\downarrow\rangle_{\rm B})|\varphi_1\rangle_{\rm a} \\ & + \sqrt{2}\alpha\beta\frac{|\uparrow\rangle_{\rm A}|\uparrow\rangle_{\rm B} + |\downarrow\rangle_{\rm A}|\downarrow\rangle_{\rm B}}{\sqrt{2}}|\varphi_1^{\perp}\rangle_{\rm a}. \quad (4)\end{aligned}$$

The $\rm PBS_\theta$, which is a polarizing beam splitter with its axial direction placed at angle θ along the incidence electron, is used to transmit the electron in the spin state $\cos\theta|\uparrow\rangle + \sin\theta|\downarrow\rangle$ and reflect the electron in the spin state $-\sin\theta|\uparrow\rangle + \cos\theta|\downarrow\rangle$. With the $\rm PBS_{\theta_1}$ in the setup shown in Fig. 1, Alice can filter out the additional electron in state $|\varphi_1^{\perp}\rangle_{\rm a}$. Here $\cos\theta = \alpha$ and $\sin\theta = -\beta$.

When the additional electron a is reflected by $\rm PBS_{\theta_1}$ and emitted out from outport $\rm D_1$, the electron pair AB is in the maximally entangled state $|\phi^+\rangle_{\rm AB} = (|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)_{\rm AB}/\sqrt{2}$, which takes place with the probability of

$$P_{\rm s1} = 2|\alpha\beta|^2. \quad (5)$$

If there is no electron emitted from outport $\rm D_1$ and the additional electron passes through $\rm PBS_{\theta_1}$, the electron pair AB is in another partially entangled pure state

$$\begin{aligned}|\Phi_2\rangle_{\rm AB} =\ & \frac{1}{\sqrt{|\alpha|^4 + |\beta|^4}} \\ & \times (\alpha^2|\uparrow\rangle_{\rm A}|\uparrow\rangle_{\rm B} - \beta^2|\downarrow\rangle_{\rm A}|\downarrow\rangle_{\rm B}), \quad (6)\end{aligned}$$

which takes place with the probability of

$$P_{\rm f_1} = |\alpha|^4 + |\beta|^4 = 1 - 2|\alpha\beta|^2. \quad (7)$$

The unitary rotation R_{θ_1} is used to rotate the spin state of the additional electron from the state $\alpha|\uparrow\rangle_{\rm a} - \beta|\downarrow\rangle_{\rm a}$ to state $|\uparrow\rangle_{\rm a}$. Subsequently, Alice performs another CNOT gate on her electron and the additional electron transmitted. The composite system is in the state

$$\begin{aligned}|\Psi_2\rangle_{\rm ABa} =\ & \frac{1}{\sqrt{|\alpha|^4 + |\beta|^4}}(\alpha^2|\uparrow\rangle_{\rm A}|\uparrow\rangle_{\rm B}|\uparrow\rangle_{\rm a} \\ & - \beta^2|\downarrow\rangle_{\rm A}|\downarrow\rangle_{\rm B}|\downarrow\rangle_{\rm a}). \quad (8)\end{aligned}$$

The $\rm PBS_{\theta_2}$ is used to transmit the spin state

$$|\varphi_2\rangle = \frac{1}{\sqrt{\alpha^4 + \beta^4}}(\alpha^2|\uparrow\rangle + \beta^2|\downarrow\rangle)_{\rm a} \quad (9)$$

and reflect the spin state

$$|\varphi_2\rangle^{\perp} = \frac{1}{\sqrt{\alpha^4 + \beta^4}}(\beta^2|\uparrow\rangle - \alpha^2|\downarrow\rangle)_{\rm a}. \quad (10)$$

We can rewrite state $|\Psi_2\rangle_{\rm ABa}$ under the basis of $\{|\varphi_2\rangle_{\rm a}, |\varphi_2^{\perp}\rangle_{\rm a}\}$, that is,

$$\begin{aligned}|\Psi_2\rangle_{\rm ABa} =\ & \frac{1}{|\alpha|^4 + |\beta|^4}\Bigg[(\alpha^4|\uparrow\rangle_{\rm A}|\uparrow\rangle_{\rm B} \\ & - \beta^4|\downarrow\rangle_{\rm A}|\downarrow\rangle_{\rm B})|\varphi_2\rangle_{\rm a} + \sqrt{2}\alpha^2\beta^2 \\ & \times \frac{|\uparrow\rangle_{\rm A}|\uparrow\rangle_{\rm B} + |\downarrow\rangle_{\rm A}|\downarrow\rangle_{\rm B}}{\sqrt{2}}|\varphi_2^{\perp}\rangle_{\rm a}\Bigg]. \quad (11)\end{aligned}$$

If the additional electron a is in state $|\varphi_2^{\perp}\rangle_{\rm a}$, it emits from output $\rm D_2$ of $\rm PBS_{\theta_2}$, and the electron pair AB is in the standard Bell state $|\phi^+\rangle_{\rm AB}$. Otherwise, it transmits through $\rm PBS_{\theta_2}$ and enters the next round of entanglement concentration. The success probability that the two parties obtain the electron pair in the standard Bell state becomes (considering the probability that the additional electron transmits through $\rm PBS_{\theta_1}$)

$$\begin{aligned}P_{\rm s2} &= P_{\rm f_1}\frac{1}{(|\alpha|^4 + |\beta|^4)^2}2|\alpha\beta|^4 \\ &= \frac{2|\alpha\beta|^4}{|\alpha|^4 + |\beta|^4}. \quad (12)\end{aligned}$$

The probability that the addition electron transmits through $\rm PBS_{\theta_2}$ is

$$\begin{aligned}P_{\rm f2} &= P_{\rm f_1}\frac{1}{(|\alpha|^4 + |\beta|^4)^2}(|\alpha|^8 + |\beta|^8) \\ &= \frac{|\alpha|^8 + |\beta|^8}{|\alpha|^4 + |\beta|^4}. \quad (13)\end{aligned}$$

The R_{θ_2} is used to rotate the spin state of the additional electron from state $\frac{(\beta^2|\uparrow\rangle + \alpha^2|\downarrow\rangle)_{\rm a}}{\sqrt{\alpha^4+\beta^4}}$ to state $|\uparrow\rangle_{\rm a}$. The $\rm PBS_{\theta_n}$ transmits spin state $\frac{(\alpha^{2^{n-1}}|\uparrow\rangle + \beta^{2^{n-1}}|\downarrow\rangle)_{\rm a}}{\sqrt{\alpha^{2^n}+\beta^{2^n}}}$ and reflects spin state $\frac{(\beta^{2^{n-1}}|\uparrow\rangle - \alpha^{2^{n-1}}|\downarrow\rangle)_{\rm a}}{\sqrt{\alpha^{2^n}+\beta^{2^n}}}$. The R_{θ_n} is used to rotate the spin state of the additional electron $\frac{(\alpha^{2^{n-1}}|\uparrow\rangle + \beta^{2^{n-1}}|\downarrow\rangle)_{\rm a}}{\sqrt{\alpha^{2^n}+\beta^{2^n}}}$ to be $|\uparrow\rangle_{\rm a}$ ($n \geq 2$). When the additional electron is detected at output $\rm D_1$, $\rm D_2$, ..., or $\rm D_n$, the two-electron system AB will be in the maximally entangled state $|\phi^+\rangle_{\rm AB}$. That is, by iterating the entanglement concentration process n times, the total success probability of this ECP is

$$\begin{aligned}P_n =\ & 2\Bigg[\alpha^2\beta^2 + \frac{\alpha^4\beta^4}{\alpha^4+\beta^4} + \frac{\alpha^8\beta^8}{(\alpha^4+\beta^4)(\alpha^8+\beta^8)} \\ & + \frac{\alpha^{16}\beta^{16}}{(\alpha^4+\beta^4)(\alpha^8+\beta^8)(\alpha^{16}+\beta^{16})} + \cdots\end{aligned}$$

$$+\frac{\alpha^{2^n}\beta^{2^n}}{(\alpha^4+\beta^4)(\alpha^8+\beta^8)\cdots(\alpha^{2^n}+\beta^{2^n})}\Bigg]. \quad (14)$$

The relation between the success probability P, with which the two parties obtain a nonlocal two-electron system AB in the maximally entangled state $|\phi^+\rangle_{\rm AB}$ from a system in the partially entangled state $|\Phi_1\rangle_{\rm AB}$, and the entanglement E is shown in Fig. 2. If $|\alpha| \le |\beta|$, the entanglement of the state $|\Phi_1\rangle_{\rm AB} = \alpha|\uparrow\rangle_{\rm A}|\uparrow\rangle_{\rm B} + \beta|\downarrow\rangle_{\rm A}|\downarrow\rangle_{\rm B}$ is $E = 2|\alpha|^2$.[51,52] When $E < 0.4$, Alice and Bob only need to perform the entanglement concentration process twice ($n = 2$) for obtaining a success probability P nearly equivalent to the entanglement E. When $0.4 < E < 0.72$ ($0.72 < E < 0.88$), they should perform the process 3 (4) times for obtaining an optimal success probability. From Fig. 2, we can see that 5 iterations of the entanglement concentration process are usually enough to obtain an optimal success probability (i.e., the entanglement E).

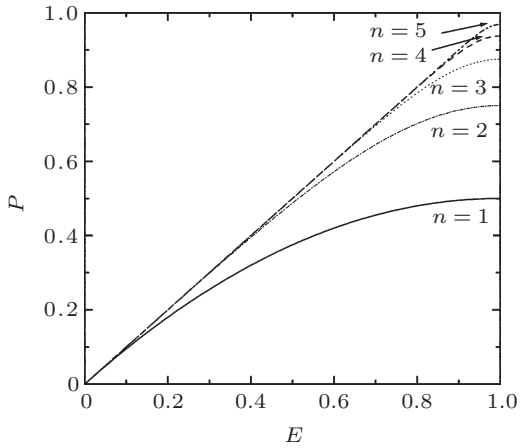


Fig. 2. Relation between the success probability of the present ECP P and the entanglement of the partially entangled state E for different iteration numbers n of entanglement concentration process.

3. Optimal multipartite entanglement concentration of N-electron spin states

It is straightforward to generalize our ECP to reconstruct maximally entangled N-electron Greenberger–Horne–Zeilinger (GHZ) states from the partially entangled GHZ-class states. Suppose the partially entangled N-electron GHZ-class states are described as

$$|\Phi_N\rangle = \alpha|\uparrow\uparrow\cdots\uparrow\rangle_{\rm AB\cdots Z} + \beta|\downarrow\downarrow\cdots\downarrow\rangle_{\rm AB\cdots Z}, \quad (15)$$

where $|\alpha|^2+|\beta|^2 = 1$, subscripts A, B, ..., Z represent the electrons hold by Alice, Bob, ..., Zach, respectively. For obtaining a subset of nonlocal N-electron systems in a maximally entangled state, Alice prepares an additional electron a′ in the state $|\uparrow\rangle_{\rm a'}$ and performs a CNOT operation on her electron A and the additional electron a′. Then the $(N+1)$-electron system is in the state

$$|\Phi_{N+1}\rangle_1 = \alpha|\uparrow\rangle_{\rm A}|\uparrow\rangle_{\rm a'}|\uparrow\cdots\uparrow\rangle_{\rm B\cdots Z} + \beta|\downarrow\rangle_{\rm A}|\downarrow\rangle_{\rm a'}|\downarrow\cdots\downarrow\rangle_{\rm B\cdots Z}. \quad (16)$$

By projecting the state of the additional electron a′ into the orthogonal basis $\{|\varphi\rangle_{\rm a'} = \alpha|\uparrow\rangle - \beta|\downarrow\rangle$, $|\varphi^\perp\rangle_{\rm a'} = \beta|\uparrow\rangle + \alpha|\downarrow\rangle\}$, the N parties obtain the maximally entangled state $|\phi^+\rangle = \frac{(|\uparrow\uparrow\cdots\uparrow\rangle+|\downarrow\downarrow\cdots\downarrow\rangle)_{\rm AB\cdots Z}}{\sqrt{2}}$ directly if Alice obtains the state of $|\varphi^\perp\rangle_{\rm a'}$ for the additional electron, which takes place with the probability of $2|\alpha\beta|^2$. If Alice obtains the state $|\varphi\rangle_{\rm a'}$, the parties obtain a partially less-entangled pure state

$$|\Phi'_N\rangle = \frac{1}{\sqrt{|\alpha|^4+|\beta|^4}} \times(\alpha^2|\uparrow\uparrow\cdots\uparrow\rangle_{\rm AB\cdots Z} - \beta^2|\downarrow\downarrow\cdots\downarrow\rangle_{\rm AB\cdots Z}).$$

The parties can also distill some maximally entangled states from this partially less-entangled state, similar to the case with nonlocal two-electron systems in a partially entangled pure state. That is, this ECP works for N-electron systems in a partially entangled pure state.

4. Discussion and summary

Let us compare the efficiency of the present ECP for electron systems with those in the literature.[50−52,65] Of course, there are some differences between the present ECP and the others. The present ECP requires that the parties know the information about the initial state of the nonlocal N-electron systems, which is the same as those in Refs. [51] and [52]. However, in Refs. [50] and [65], the parties did not require to know the information. In essence, all existing ECPs[50−52,65] for electron systems are based on the Schmidt projection method, in which the parties exploit the combination of a pair of systems with the same parameters to obtain a system in a maximally entangled state with an average success probability of $\alpha^2\beta^2$. The ECP in Ref. [52] exploited an additional collective unitary evolution on

one qubit in the system and an additional qubit to improve the success probability to be α^2. The ECP for electron systems in Ref. [65] simplified the implementation by sacrificing the efficiency compared with those in Refs. [51] and [52]. However, the present ECP distills an N-electron system from a system in a partially entangled pure state and an additional electron, not from a pair of N-electron systems, which is very different from all existing ECPs.[50−52,65] Moreover, the success probability of the present ECP is $2\alpha^2$ ($|\alpha| \leq |\beta|$), which is just the entanglement of the partially entangled pure state. That is, the success probability of the present ECP is the theoretical limit of ECPs for a partially entangled pure state. It is an optimal one. Moreover, the present ECP is based on CNOT gates and may work in quantum computers based on electron spins in the future.

In summary, we have proposed an optimal ECP for nonlocal N-electron systems in a partially entangled pure state. One of the N parties, say Alice, exploits a CNOT gate to extend the partially entangled N-electron system to an $(N+1)$-electron system first and then projects the additional electron onto an orthogonal basis. By detecting the output of the additional electron from a PBS, the N parties in quantum communication can divide their N-electron systems into two groups. One is in the maximally entangled state. The other is in another partially entangled state with less entanglement, which is just the resource for the entanglement concentration in the next round. By iterating the entanglement concentration process several times, the N parties can obtain a subset of N-electron systems in the maximally entangled state with the maximal success probability just equivalent to the entanglement of the partially entangled state. Compared with the other ECPs for electron spins,[50−52,65] the present ECP has the optimal success probability.

References

[1] Nielsen M A and Chuang I L 2000 *Quantum Computation and Quantum Information* (Cambridge: Cambridge University Press)

[2] Ekert A K 1991 *Phys. Rev. Lett.* **67** 661

[3] Deng F G and Long G L 2003 *Phys. Rev.* A **68** 042315

[4] Li X H, Deng F G and Zhou H Y 2008 *Phys. Rev.* A **78** 022321

[5] Zhong P P, Zhang H N, Wang J D, Qin X J, Wei Z J, Chen S and Liu S H 2011 *Chin. Phys.* B **20** 050307

[6] Zou L, Feng Y, Yang Y B, Wang A B, Yang L Z and Zhang J Z 2011 *Chin. Phys.* B **20** 094209

[7] Chen M J and Liu X 2011 *Chin. Phys.* B **20** 100305

[8] Li H W, Yin Z Q, Wang S, Bao W S, Guo G C and Han Z F 2011 *Chin. Phys.* B **20** 100306

[9] Bennett C H, Brassard G, Crepeau C, Jozsa R, Peres A and Wootters W K 1993 *Phys. Rev. Lett.* **70** 1895

[10] Wang M Y and Yan F L 2011 *Chin. Phys.* B **20** 120309

[11] Zhang J T, He G Q, Ren L J and Zeng G H 2011 *Chin. Phys.* B **20** 050311

[12] Tang J W, Zhao G X and He X H 2011 *Chin. Phys.* B **20** 050312

[13] Wang Z J, Zhang K and Fan C Y 2010 *Chin. Phys.* B **19** 110502

[14] Gao D, Zhao Z S, Zhu A D, Wang H F, Shao X Q and Zhang S 2010 *Chin. Phys.* B **19** 090313

[15] Bennett C H and Wiesner S J 1992 *Phys. Rev. Lett.* **69** 2881

[16] Liu X S, Long G L, Tong D M and Feng L 2002 *Phys. Rev.* A **65** 022304

[17] Xiao L, Long G L, Deng F G and Pan J W 2004 *Phys. Rev.* A **69** 052307

[18] Yan F L and Gao T 2005 *Phys. Rev.* A **72** 012304

[19] Zhang Z J, Li Y and Man Z X 2005 *Phys. Rev.* A **71** 044301

[20] Deng F G, Li X H, Zhou H Y and Zhang Z J 2005 *Phys. Rev.* A **72** 044302

[21] Zhang Z R, Liu W T and Li C Z 2011 *Chin. Phys.* B **20** 050309

[22] Zhu Z C, Zhang Y Q and Fu A M 2011 *Chin. Phys.* B **20** 040306

[23] Gu B, Li C Q, Xu F and Chen Y L 2009 *Chin. Phys.* B **18** 4690

[24] Wang C and Zhang Y 2009 *Chin. Phys.* B **18** 3238

[25] Deng F G, Li X H, Li C Y, Zhou P and Zhou H Y 2005 *Phys. Rev.* A **72** 044301

[26] Deng F G, Li X H, Li C Y, Zhou P and Zhou H Y 2006 *Eur. Phys. J.* D **39** 459

[27] Li X H, Zhou P, Li C Y, Zhou H Y and Deng F G 2006 *J. Phys.* B **39** 1975

[28] Man Z X, Xia Y J and An N B 2007 *Eur. Phys. J.* D **42** 333

[29] Wang Z Y, Yuan H, Shi S H and Zhang Z J 2007 *Eur. Phys. J.* D **41** 371

[30] Wang D, Zha X W, Lan Q, Li N and Wei J 2011 *Chin. Phys.* B **20** 090305

[31] Deng F G, Li C Y, Li Y S, Zhou H Y and Wang Y 2005 *Phys. Rev.* A **72** 022338

[32] Zhou P, Li X H, Deng F G and Zhou H Y 2007 *J. Phys.* A **40** 13121

[33] Long G L and Liu X S 2002 *Phys. Rev.* A **65** 032302

[34] Deng F G, Long G L and Liu X S 2003 *Phys. Rev.* A **68** 042317

[35] Deng F G and Long G L 2004 *Phys. Rev.* A **69** 052319

[36] Wang C, Deng F G, Li Y S, Liu X S and Long G L 2005 *Phys. Rev.* A **71** 044305

[37] Yang J, Wang C and Zhang R 2010 *Chin. Phys.* B **19** 110311

[38] Gu B, Huang Y G, Fang X and Zhang C Y 2011 *Chin. Phys.* B **20** 100309

[39] Bennett C H, Brassard G, Popescu S, Schumacher B, Smolin J A and Wootters W K 1996 *Phys. Rev. Lett.* **76** 722

[40] Pan J W, Simon C and Zellinger A 2001 *Nature* **410** 1067

[41] Sheng Y B, Deng F G and Zhou H Y 2008 *Phys. Rev.* A **77** 042308
[42] Sheng Y B and Deng F G 2010 *Phys. Rev.* A **81** 032307
[43] Sheng Y B and Deng F G 2010 *Phys. Rev.* A **82** 044305
[44] Li X H 2010 *Phys. Rev.* A **82** 044304
[45] Deng F G 2011 *Phys. Rev.* A **83** 062316
[46] Deng F G 2011 *Phys. Rev.* A **84** 052312
[47] Wang C, Zhang Y and Jin G S 2011 *Quantum Inform. Comput.* **11** 988
[48] Wang C, Zhang Y and Zhang R 2011 *Opt. Express* **19** 25685
[49] Gu B, Chen Y L, Zhang C Y and Huang Y G 2010 *Chin. Phys. Lett.* **27** 100304
[50] Bennett C H, Bernstein H J, Popescu S and Schumacher B 1996 *Phys. Rev.* A **53** 2046
[51] Bose S, Vedral V and Knight P L 1999 *Phys. Rev.* A **60** 194
[52] Shi B S, Jiang Y K and Guo G C 2000 *Phys. Rev.* A **62** 054301
[53] Yamamoto T, Koashi M and Imoto N 2001 *Phys. Rev.* A **64** 012304
[54] Zhao Z, Pan J W and Zhan M S 2001 *Phys. Rev.* A **64** 014301
[55] Sheng Y B, Deng F G and Zhou H Y 2008 *Phys. Rev.* A **77** 062325
[56] Sheng Y B, Deng F G and Zhou H Y 2010 *Quantum Inform. Comput.* **10** 272
[57] Sheng Y B, Zhou L, Zhao S M and Zheng B Y 2012 *Phys. Rev.* A **85** 012307
[58] Deng F G 2012 *Phys. Rev.* A **85** 022311
[59] Beenakker C W J, Divincenzo D P, Emary C and Kindermann M 2004 *Phys. Rev. Lett.* **93** 020501
[60] Field M, Smith C G, Pepper M, Ritchie D A, Frost J E F, Jones G A C and Hasko D G 1993 *Phys. Rev. Lett.* **70** 1411
[61] Ionicioiu R 2007 *Phys. Rev.* A **75** 032339
[62] Zhang X L, Feng M and Gao K L 2006 *Phys. Rev.* A **73** 014301
[63] Feng X L, Kwek L C and Oh C H 2005 *Phys. Rev.* A **71** 064301
[64] Sheng Y B, Deng F G and Long G L 2011 *Phys. Lett.* A **375** 396
[65] Sheng Y B, Deng F G and Zhou H Y 2009 *Phys. Lett.* A **373** 1823
[66] Wang C, Zhang Y and Jin G S 2011 *Phys. Rev.* A **84** 032307