Effects of photon addition on quantum nonlocality of squeezed entangled coherent states*

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Effects of photon addition on quantum nonlocality of squeezed entangled coherent states for Bell-inequality tests are studied theoretically. By utilizing the method of photon-parity measurement, it is found that photon addition can always increase the degrees of Bell violations within a certain parameter range. A possible scheme for generating photon-added squeezed entangled coherent states is proposed.

Keywords: quantum nonlocality, photon addition, squeezed entangled coherent states

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1. Introduction

It is well known that quantum nonlocality is one of the most outstanding features of quantum mechanics. The paradox suggested by Einstein, Podolsky, and Rosen aroused controversy about nonlocality of quantum states.[1] Bell's theorem[2−4] provides an effective way to test the predictions of quantum mechanics against those of classical (local realistic) theories.[1] Numerous theoretical studies and experimental demonstrations have been performed to understand nonlocal properties of quantum states. A couple of refined versions of Bell's inequality have been proposed based on the original one.[2]

Considering that the Wigner function for a field can be expressed as an expectation value of the displaced parity operator, Banaszek and Wodkiewicz[5] proved that the photon detection probabilities can be directly related to certain phase-space densities, and that for the EPR state a Bell-type inequality can be violated, showing quantum nonlocality.[6] Chen *et al.* studied Bell's inequality of continuous-variable states[7] using the pseudospin operator related to Bell operator,[8] and found that the EPR state can completely violate Bell's inequality. Gisin and Peres proposed pairs of observables whose correlations violate Bell's inequality for a discrete N-dimensional entangled state.[9] Meanwhile, Kim *et al.*[10] proposed a scheme to test quantum nonlocality for the field prepared in the two spatially separated cavities, by sending two independent two-level atoms through respective cavities to interact with cavity fields, or sending a single atom sequentially to interact with the cavities. Xie *et al.*[11] have studied the nonlocal behaviour of a three-qubit system homogeneously coupled to a general XY spin chain in a three-site interaction environment by calculating the extent of violation of Bell inequality. Nogues *et al.*[12] reported experimentally the measurement of a negative value of the Wigner function of the microwave field stored in a superconducting cavity. Lee *et al.*[13] have analyzed how squeezing influences the extent that several different beam-split-entangled superpositions of coherent states (SCSs) violate Bell's inequality. It has been found that external squeezing can always increase the violation degree of Bell's inequality through photon-parity measurements where the squeezing direction is properly chosen. Therefore, the parity measurement is an effective way to test quantum nonlocality for a field.

Moreover, there has been increasing interest in the study of various nonclassical states because of their applications in the engineering of quantum

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states, and quantum information processing. In 1991, Agarwal and Tara[14] introduced an interesting class of photon-added coherent states $a^{+m}|\alpha\rangle$ which are intermediate between the Fock states and the coherent states. They also suggested a scheme to generate such states by passing excited atoms through a cavity containing pre-existing coherent states. If the atom is detected to be in its ground state, then the cavity field collapses into a photon-added coherent state. Dakna *et al.*[15] found the addition of photons to a squeezed vacuum through the conditional measurement on a beam splitter yields a photon-added Jacobi polynomial squeezed vacuum state, which is a Schrodinger-cat-like state. Li *et al.* have investigated the entanglement properties and nonclassicality of non-Gaussian states generated by photon addition and subtraction.[16,17]

Recently, it is possible to generate and manipulate various nonclassical optical fields by adding photons to traditional quantum states or Gaussian states. For this reason, the photon addition (a non-Gaussian operation) has been widely applied in quantum information communication. Photon-added states have received more attention from both experimentalists and theoreticians.[18,19] Enhanced Bell violation by a coherent superposition of photon subtraction and addition has been investigated by Park *et al.*[20]

Superposition of coherent states in free-traveling fields[21] has attracted special attention due to their remarkable applications. The SCSs with large amplitude show typical properties of macroscopic quantum superpositions. Marek *et al.*[22] have studied how photon addition can be used to generate squeezed SCSs in free-traveling field with high fidelity and large amplitude. It has been clearly pointed out that the generated squeezed SCSs can be used for proof-of-principle experiments such as quantum teleportation and single qubit gates without any modifications.[23] This strongly motivates us to study the effects of photon addition on quantum nonlocality of squeezed entangled coherent states (SECS).

In this paper, we study the effects of photon addition on quantum nonlocality of SECS for the purpose of testing Bell's inequality via photon-parity measurements. From the photon-parity measurements, we show that the photon addition can increase the degree of Bell violations. We also discuss how to generate photon-added SECS experimentally. This paper is organized as follows. In Section 2, two different approaches to create SECS and photon-added SECS are briefly presented. In Section 3, we explore the effects of two-mode photon addition on the test of Bell's inequality, and simply analyze the physics behind these effects from two different aspects. In Section 4, we propose a scheme for dealing with how to generate photon-added SECS experimentally. Summary is given in Section 5.

2. Photon-added squeezed superposition of coherent states

The SECS is an important continuous-variable entangled state. Many possible applications to quantum information processing have been studied utilizing such state. We present one particular type of SECS, which can be defined as

$$|\Phi_{\varphi}^{\mathrm{S}}\rangle = N_{\varphi} S(r) \left(|\gamma\rangle_{a_1}|\gamma\rangle_{a_2} + \mathrm{e}^{\mathrm{i}\varphi}|-\gamma\rangle_{a_1}|-\gamma\rangle_{a_2}\right), \quad (1)$$

where N_{φ} is a normalization factor, $|\pm\gamma\rangle$ is a coherent state with an amplitude of $\pm\gamma$, γ is assumed to be real for simplicity, and φ is a real local phase factor. Meanwhile, the $S(r) = \exp[r\left(a_1 a_2 - a_1^{+} a_2^{+}\right)]$ is the two-mode squeezing operator, a_i^{+} (a_i) represents the bosonic creation (annihilation) operators of photons in mode i $(i = 1, 2)$. We assume that the squeezing parameter r is real for $S(r)$. It is worth mentioning that the SECS becomes two-mode even or odd squeezed coherent state for $\varphi = \pm\pi$, respectively. That is,

$$|\Phi_{\varphi}^{\mathrm{S}}\rangle_{\pm} = N_{\varphi} S(r) \left(|\gamma\rangle_{a_1}|\gamma\rangle_{a_2} \pm |-\gamma\rangle_{a_1}|-\gamma\rangle_{a_2}\right). \quad (2)$$

The reason why the two-mode squeezed coherent state with plus or minus sign is called even or odd coherent state is that it contains an even or odd number of photons regardless of the value of γ.

However, in this paper, we introduce photon-added SECS, which are obtained by applying repeatedly the photon creation operator on the SECS. In other words, they can be obtained by adding m photons to the two modes of SECS simultaneously. Namely,

$$\begin{aligned} |\Phi_{\varphi}^{\mathrm{S}}\rangle_m^a = {} & \left(a_1^{+} a_2^{+}\right)^m S(r) \left(|\gamma\rangle_{a_1}|\gamma\rangle_{a_2}\right. \\ & \left. + \mathrm{e}^{\mathrm{i}\varphi}|-\gamma\rangle_{a_1}|-\gamma\rangle_{a_2}\right), \end{aligned} \quad (3)$$

where m is a positive integer. Here, the photon-added SECS is not normalized because single-photon addition may be realized in experiment now. In this paper, we focus on the simplest photon-added SECS, namely $m = 1$ in Eq. (3). It can be expressed as

$$|\Phi_{\varphi}^{\mathrm{S}}\rangle^a = N_{\varphi}^a \left(a_1^{+} a_2^{+}\right) S(r) \left(|\gamma\rangle_{a_1}|\gamma\rangle_{a_2}\right.$$

$$+ \mathrm{e}^{\mathrm{i}\varphi}|-\gamma\rangle_{a_1}|-\gamma\rangle_{a_2}), \quad (4)$$

where N_φ^a is a normalization constant.

3. Violation of Bell's inequality with photon-parity measurement schemes

The type of quantum observable to be measured is a crucial factor in the nonlocality test. Banaszek and Wódkiewicz studied Bell's inequality for continuous-variable systems in the phase space by means of the Wigner function based upon photon number parity measurements and the displacement operation.[24] The displaced parity operator used for the Bell–CHSH inequality is defined as

$$\begin{aligned}\hat{\Pi}(\alpha) &= \hat{\Pi}^+(\alpha) - \hat{\Pi}^-(\alpha)\\ &= D(\alpha)\sum_{k=0}^{+\infty}(|2k\rangle\langle 2k| - |2k+1\rangle\langle 2k+1|)D^+(\alpha), \quad (5)\end{aligned}$$

where $D(\alpha) = \exp[\alpha\hat{a}^+ - \alpha^*\hat{a}]$ is the displacement operator for Bosonic operators $\hat{a}^+$ and $\hat{a}$. Such a measurement should be able to discriminate between even and odd numbers of photons. It should be pointed out that in order to maximize the violation of Bell's inequality for two-mode SECS, the Bell operator in the Banaszek–Wódkiewicz formalism needs to be written as

$$\begin{aligned}\hat{B}_{\mathrm{CHSH}} &= \hat{\Pi}_1(\alpha)\otimes\hat{\Pi}_2(\beta) + \hat{\Pi}_1(\alpha')\otimes\hat{\Pi}_2(\beta)\\ &\quad + \hat{\Pi}_1(\alpha)\otimes\hat{\Pi}_2(\beta') - \hat{\Pi}_1(\alpha')\otimes\hat{\Pi}_2(\beta'). \quad (6)\end{aligned}$$

Furthermore, in terms of the relationship between Wigner function and the quantum expectation value of the displaced parity operator, the two-mode Wigner function $W(\alpha,\beta)$ can be written explicitly as

$$\begin{aligned}W(\alpha,\beta) &= \frac{4}{\pi^2}\langle\hat{\Pi}_1(\alpha)\otimes\hat{\Pi}_2(\beta)\rangle\\ &= \frac{4}{\pi^2}\Pi_1(\alpha)\otimes\Pi_2(\beta), \quad (7)\end{aligned}$$

where $\Pi_1(\alpha)$ and $\Pi_2(\beta)$ are the the quantum expectation values of the operators $\hat{\Pi}_1(\alpha)$ and $\hat{\Pi}_2(\beta)$, respectively. Thus, the Wigner representation of Bell–CHSH inequality can be expanded in terms of the Wigner function as

$$\begin{aligned}B_{\mathrm{CHSH}} &= |\langle\hat{B}_{\mathrm{CHSH}}\rangle|\\ &= \frac{4}{\pi^2}|W(\alpha,\beta) + W(\alpha',\beta)\\ &\quad + W(\alpha,\beta') - W(\alpha',\beta')| \le 2. \quad (8)\end{aligned}$$

If inequality (8) is violated, then the corresponding field state exhibits the quantum nonlocality which conflicts with the local realistic theory, and the bound values for the Bell–CHSH inequality ($\pm 2\sqrt{2}$) are well known as Cirel'son's bound.[25] Using Eqs. (1) and (7), the Wigner functions for the SECS become

$$\begin{aligned}W_{\Phi_\varphi^{\mathrm{S}}}(\alpha,\beta) &= N_\varphi^2[W_\gamma(\tilde{\alpha})W_\gamma(\tilde{\beta}) + W_{-\gamma}(\tilde{\alpha})W_{-\gamma}(\tilde{\beta})\\ &\quad + W_{\gamma,\varphi}(\tilde{\alpha},\tilde{\beta}) + W_{-\gamma,-\varphi}(\tilde{\alpha},\tilde{\beta})], \quad (9)\end{aligned}$$

where

$$\begin{aligned}&\tilde{\alpha} = \alpha\cosh r + \beta^*\sinh r, \quad \tilde{\beta} = \beta\cosh r + \alpha^*\sinh r,\\ &W_{\pm\gamma}(\tilde{\alpha}) = 2\pi^{-1}\exp[-2|\tilde{\alpha}\mp\gamma|^2],\\ &W_{\pm\gamma}(\tilde{\beta}) = 2\pi^{-1}\exp[-2|\tilde{\beta}\mp\gamma|^2],\\ &W_{\pm\gamma,\pm\varphi}(\tilde{\alpha},\tilde{\beta}) = 4\pi^{-2}\exp[-2(\tilde{\alpha}\pm\gamma)(\tilde{\alpha}\mp\gamma)^*\\ &\qquad - (\tilde{\beta}\pm\gamma)(\tilde{\beta}\mp\gamma)^* \pm \mathrm{i}\varphi - 4|\gamma|^2].\end{aligned}$$

The Wigner functions for the photon-added SECS can be calculated in the same manner as

$$\begin{aligned}W_{\Phi_\varphi^{\mathrm{S}}}^a(\alpha,\beta) &= (N_\varphi^a)^2[W_\gamma(\tilde{\alpha})W_\gamma(\tilde{\beta})X_\gamma(\alpha,\beta)\\ &\quad + W_{-\gamma}(\tilde{\alpha})W_{-\gamma}(\tilde{\beta})X_{-\gamma}(\alpha,\beta)\\ &\quad + W_{-\gamma,-\varphi}(\tilde{\alpha},\tilde{\beta})Y_{-\gamma}(\alpha,\beta)\\ &\quad + W_{\gamma,\varphi}(\tilde{\alpha},\tilde{\beta})Y_\gamma(\alpha,\beta)], \quad (10)\end{aligned}$$

where

$$\begin{aligned}X_{\pm\gamma}(\alpha,\beta) &= \sinh^4 r[(|\tilde{\alpha}_{\pm\pm}|^2 - 1)(|\tilde{\beta}_{\pm\pm}|^2 - 1)\\ &\quad - \tanh r(\tilde{\alpha}_{\pm\pm}\tilde{\beta}_{\pm\pm} + \tilde{\alpha}_{\pm\pm}^*\tilde{\beta}_{\pm\pm}^*)\\ &\quad + \tanh^2 r], \quad (11)\\ Y_{\pm\gamma}(\alpha,\beta) &= \cosh^4 r[(\tilde{\alpha}_{\pm\mp}\tilde{\alpha}_{\mp\pm}^* - 1)(\tilde{\beta}_{\pm\mp}\tilde{\beta}_{\mp\pm}^* - 1)\\ &\quad - \tanh r(\tilde{\alpha}_{\pm\mp}\tilde{\beta}_{\pm\mp} + \alpha_{\pm\mp}^*\tilde{\beta}_{\pm\mp}^*)\\ &\quad + \tanh^2 r], \quad (12)\end{aligned}$$

$$\begin{aligned}&\tilde{\alpha}_{\pm\pm} = \cosh r\left(2\alpha \pm \frac{\gamma}{\cosh r}\right) + \sinh r\left(2\beta^* \pm \frac{\gamma^*}{\cosh r}\right),\\ &\tilde{\alpha}_{\pm\mp} = \cosh r\left(2\alpha \pm \frac{\gamma}{\cosh r}\right) + \sinh r\left(2\beta^* \mp \frac{\gamma^*}{\cosh r}\right),\\ &\tilde{\beta}_{\pm\pm} = \cosh r\left(2\beta \pm \frac{\gamma}{\cosh r}\right) + \sinh r\left(2\alpha^* \pm \frac{\gamma^*}{\cosh r}\right),\\ &\tilde{\beta}_{\pm\mp} = \cosh r\left(2\beta \pm \frac{\gamma}{\cosh r}\right) + \sinh r\left(2\alpha^* \mp \frac{\gamma^*}{\cosh r}\right), \quad (13)\end{aligned}$$

Next, we study the violation of Bell'S inequality in the phase space utilizing the Wigner function based upon photon number parity measurements and the displacement operation. In Fig. 1, we plot the Bell violation as a function of j and γ for $\varphi = 3\pi/4$, $\alpha' = \beta' = j\delta$, and r is assumed to be real. In the case of $0.01 \le r \le 0.05$, it is shown that the Bell violation can be enhanced by applying the photon creation operator to the SECS for appropriate amplitudes, but the difference between them is not very obvious.

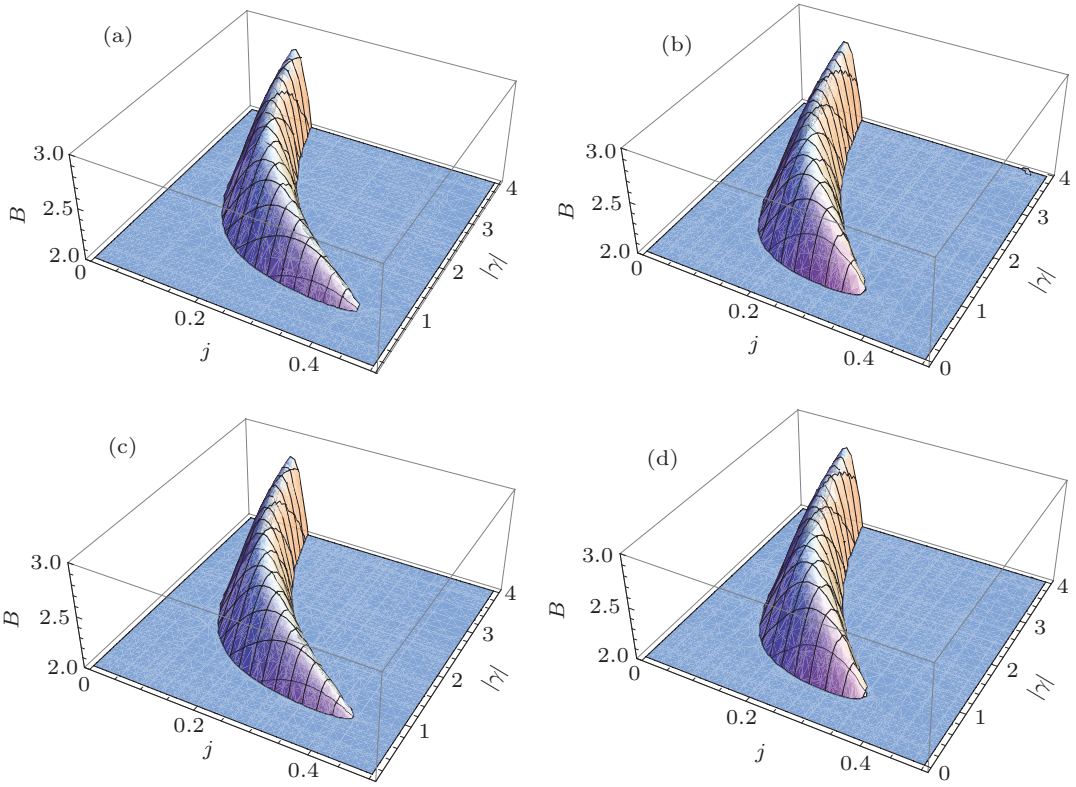


Fig. 1. (colour online) The Bell–CHSH function B as a function of j and γ for $|\Phi_{\varphi}^{\mathrm{S}}\rangle$ with (a) $r = 0.01$, and (c) $r = 0.05$. (b), (d) The Bell-CHSH function B lines of $|\Phi_{\varphi}^{\mathrm{S}}\rangle^{a}$, corresponding to (a) and (c), respectively.

In order to see obviously the change of Bell violation after adding photon, we plot the maximal value of an absolute Bell function $|B|_{max}$ as a function of the parameter $|\gamma|$ with $\varphi = 3\pi/4$ (see Fig. 2). It is well known that the Bell violation for an ECS approaches Cirel'son's bound[25] when the amplitude γ becomes large.[26] Actually, if we choose appropriate squeezing parameters, we can draw from Fig. 2 similar conclusions.

However, more importantly, figure 2 shows that some characteristic properties in common is applicative to the photon-added states considered. The photon addition increases significantly the degree of the Bell violation for small r, but has a tendency of degrading it for larger $|\gamma|$ in some specific directions. In other words, in photon-parity measurements, the photon addition operation can always enhance obviously Bell violations of tested states for small squeezing parameter r, and its contribution gets smaller for larger amplitudes of states. The maximum violations of Bell's inequalities of $|\Phi_{\varphi}^{\mathrm{S}}\rangle$ and $|\Phi_{\varphi}^{\mathrm{S}}\rangle^{a}$ are almost the same when the amplitude is large enough. At the same time, we find that the SECSs violate the CHSH inequality only using the photon-parity measurement. In addition, when $\varphi = 3\pi/4$ and $r = 0.01$, the lower bound of coherent amplitude to violate the CHSH inequality is found to be 0.7638 only using photon-parity measurements. However, under the same conditions, the lower bound of $|\gamma|$ is found to be 0.5226 after the photon addition operation, which indicates that the lower bound of the coherent amplitudes can be significantly reduced by adding photon. However, it is worth mentioning that the effect of photon addition on the degree of Bell violation is reduced first and then enhanced for larger r (i.e., $r = 0.5$). We think that this is a relatively interesting and novel physical phenomenon.

In Fig. 4, we plot the maximal value of an absolute Bell function $|B|_{\max}$ as a function of φ with $|\gamma| = 1$, $r = 0.01$, for $|\Phi_{\varphi}^{\mathrm{S}}\rangle$ (solid) and $|\Phi_{\varphi}^{\mathrm{S}}\rangle^{a}$ (dashed), respectively. It can be found that the maximal violation is increased upon adding photon. The numerical calculation shows that when the maximal value of absolute Bell function $|B|_{\max}$ increases from 2.4093 to 2.5751, the violation degree of Bell's inequality increases from 0.4093 to 0.5751. That is to say, it is improved by 40.5%.

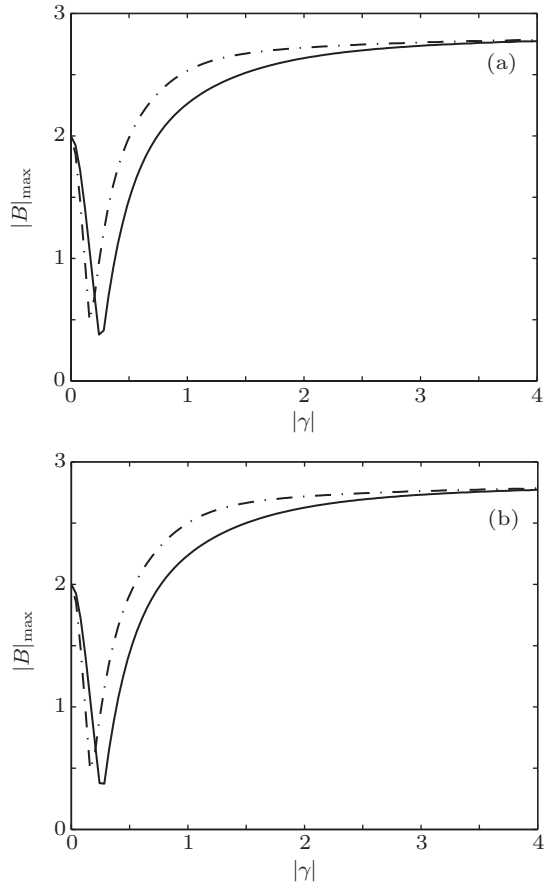


Fig. 2. The maximal value of absolute Bell function $|B|_{\max}$ as a function of $|\gamma|$ with (a) $r = 0.01$ and (b) $r = 0.05$ for $|\Phi_{\varphi}^{\mathrm{S}}\rangle$ (solid) and $|\Phi_{\varphi}^{\mathrm{S}}\rangle^a$ (dashed), respectively.

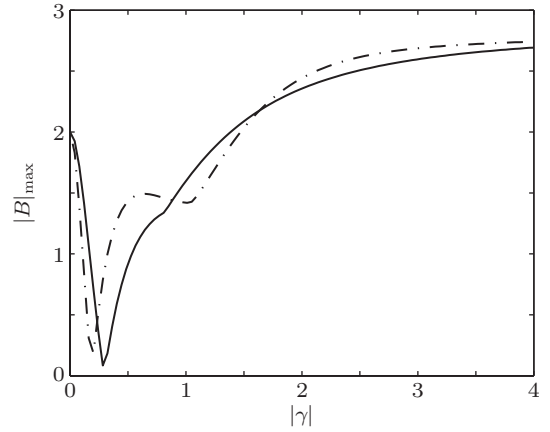


Fig. 3. The maximal value of absolute Bell function $|B|_{\max}$ as a function of $|\gamma|$ with $r = 0.5$ for $|\Phi_{\varphi}^{\mathrm{S}}\rangle$ (solid) and $|\Phi_{\varphi}^{\mathrm{S}}\rangle^a$ (dashed), respectively.

The conclusions above can be explained in a simple way. As a matter of fact, for the small $|\gamma|$ and r, the photon addition operation makes the interference fringes more sharp due to the increase in the Bell violations. However, when the squeezing parameter is relatively larger, the interference part $W_{\gamma,\varphi}(\tilde{\alpha},\tilde{\beta})Y_{\gamma}(\alpha,\beta) + W_{-\gamma,-\varphi}(\tilde{\alpha},\tilde{\beta})Y_{-\gamma}(\alpha,\beta)$ in the Wigner function fades out, which may play a crucial role in degrading the Bell violations. Meanwhile, the mean photon number of optical field gradually increases with increasing amplitude. Obviously, the effects of single-photon addition on Bell violations of tested states gradually disappears, so its contribution gets slighter as the amplitudes of the states become larger. Therefore, in photon-parity measurements, the photon addition can enhance the Bell violations of tested states for well-chosen values of amplitudes and squeezing.

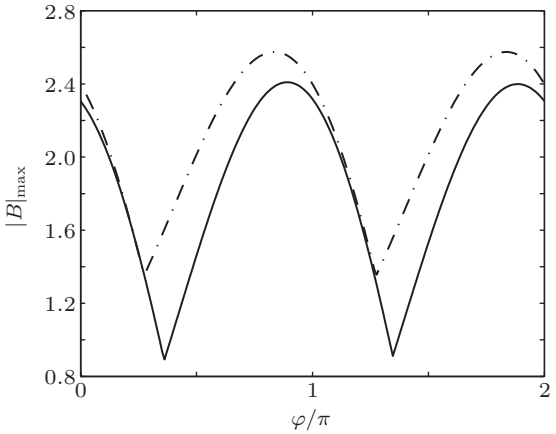


Fig. 4. The maximal value of absolute Bell function $|B|_{\max}$ as a function of φ with $|\gamma| = 1$ and $r = 0.01$ for $|\Phi_{\varphi}^{\mathrm{S}}\rangle$ (solid) and $|\Phi_{\varphi}^{\mathrm{S}}\rangle^a$ (dashed), respectively.

In some sense, the nonclassical properties of the quantum states are directly associated with their nonlocality. The photon addition can enhance nonclassical properties of SECS, which is the reason why the photon addition can enhance the nonlocality of SECS. Furthermore, the nonclassical properties of optical fields have been extensively studied both theoretically and experimentally in recent decades. The basic property of the Wigner function is that it may take negative values for a specific quantum state. This is an obvious nonclassical feature of the quantum state. Therefore, the nonclassical properties of the quantum state are discussed by using the negativity of the Wigner function.

Basically, if the Wigner function of a quantum state is negative, then such a quantum state should be nonclassical and thus certain nonclassical effects can be revealed. Therefore, calculation of the Wigner function of a quantum state may determine if such a quantum state is classical or nonclassical. In this section, we will investigate the effects of photon addition on the nonclassical properties of SECS by means of the Wigner function.

In Fig. 5, we plot the Wigner function as a function of φ with $|\gamma| = 0.2$ for $|\Phi_{\varphi}^{\rm S}\rangle$ (thick), and $|\Phi_{\varphi}^{\rm S}\rangle^a$ (dotted), and with $|\gamma| = 0.8$ for $|\Phi_{\varphi}^{\rm S}\rangle$ (thin), and $|\Phi_{\varphi}^{\rm S}\rangle^a$ (dashed), where $r = 2$, $|\alpha| = |\beta| = 0$, and γ is assumed to be real. It can be clearly seen from Fig. 5 the dependence of the Wigner function on the parameters φ and γ. However, under the photon addition operation, it is worth noting that the value of Wigner function will decrease in any direction and range for small γ. At the same time, the function image becomes wider, the negative peak of the Wigner function becomes gentler and the positive peak becomes sharper. That is to say, the negative range of the Wigner function becomes wider. In other words, the photon addition can always increase the nonclassical degree under given conditions, the photon-added SECS may become more nonclassical than SECS. However, in the case of $\varphi = k\pi + \pi$ $(k = 0, 1, 2\ldots)$, the photon addition has no effect on the Wigner function. That is to say, we can observe that the photon addition can decrease the value of the Wigner function in any direction and range for small γ except for $\varphi = k\pi + \pi$ $(k = 0, 1, 2\ldots)$.

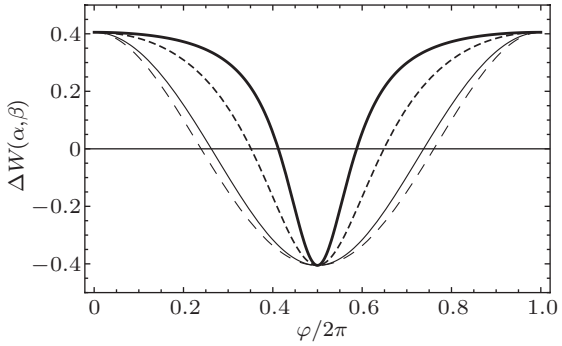


Fig. 5. The Wigner function as a function of φ with $|\gamma| = 0.2, |\alpha| = |\beta| = 0$, for $|\Phi_{\varphi}^{\rm S}\rangle$ (thick), and $|\Phi_{\varphi}^{\rm S}\rangle^a$ (dotted), respectively, and with $|\gamma| = 0.8, |\alpha| = |\beta| = 0$ for $|\Phi_{\varphi}^{\rm S}\rangle$ (thin), and $|\Phi_{\varphi}^{\rm S}\rangle^a$ (dashed).

Furthermore, in order to facilitate the comparison of the difference between the two functions for large amplitudes, we plot the function

$$\Delta W(\alpha,\beta) = W_{\Phi_{\varphi}^{\rm S}}\ (\alpha,\beta) - W^a_{\Phi_{\varphi}^{\rm S}}(\alpha,\beta), \qquad (14)$$

as a function of φ with $|\gamma| = 1.0$ (solid) and $|\gamma| = 1.5$ (dashed), where $r = 0.2$, and $|\alpha| = |\beta| = 0$. It is easy to see that the difference of the Wigner function values is not obvious. When the amplitude is large enough, the two function values are almost the same. Through numerical calculation, it is very easy to know that the maximal function value is 7×10^{-3} for $|\gamma| = 1.0$. However, when $|\gamma| = 1.5$, the maximal function value is only 5×10^{-5}. In view of the small difference of function values, we can say that the two function values are almost the same when the amplitude is large enough. In other words, the photon addition makes the Wigner function decrease more slowly with increasing γ. For this reason, we can also understand why photon addition can enhance Bell violations, though its contribution gets slighter for larger amplitudes of the states.

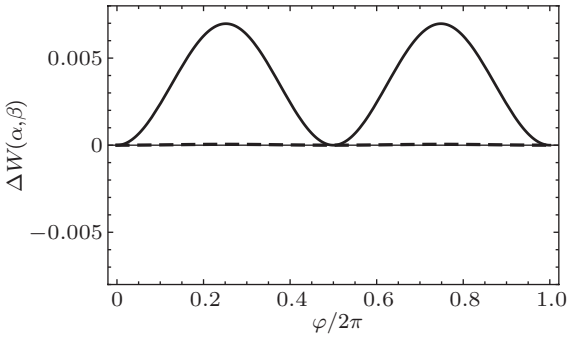


Fig. 6. The Wigner function $\Delta W(\alpha,\beta)$ as a function of φ with $|\gamma| = 1.0$ (solid) and $|\gamma| = 1.5$ (dashed), where r=0.2, and $|\alpha| = |\beta| = 0$.

4. A possible scheme for generating photon-added squeezed entangled coherent state

We have found that photon addition operation can increase the degree of Bell violations when the values of amplitude and squeezing parameters are properly chosen in photon-parity measurements. In the following, we will propose a possible scheme to generate photon-added SECS experimentally.

The single-photon-added SECS

$$|\psi_{\pm}\rangle = N_{\pm}a_1^+ a_2^+ S(r)(|\gamma\rangle|\gamma\rangle \pm |-\gamma\rangle|-\gamma\rangle)$$

with $\varphi = \pm\pi$, may be realized in experiment (see Fig. 7). It is worth noting that superpositions of coherent states $|{\rm SCS}_{\pm}(\sqrt{2}\gamma)\rangle = N_{\pm}(\sqrt{2}\gamma)(|\sqrt{2}\gamma\rangle \pm |-\sqrt{2}\gamma\rangle)$ have been generated in experiment, where $N_{\pm}(\sqrt{2}\gamma)$ is the normalization factor. For this reason, the SECS can be obtained by feeding $|{\rm SCS}_{\pm}(\sqrt{2}\gamma)\rangle$ into a 50/50 beam splitter (BS) and squeezing it into two-modes.[27] Figure 7 shows the schematic of the experimental apparatus,[28] where D1 and D2 are the single-photon detectors. If D1 and D2 can detect one photon simultaneously, then we may obtain the single-

photon-added squeezed entangled coherent states.

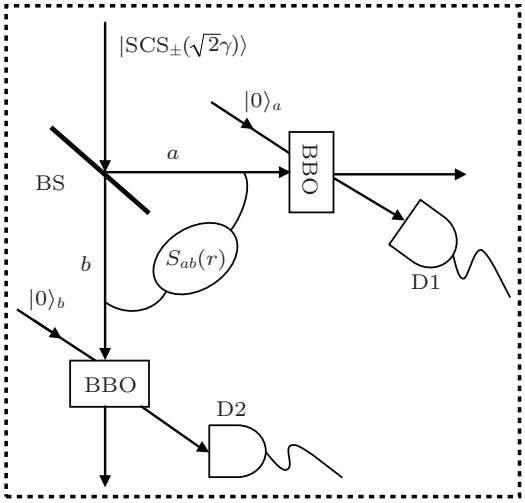


Fig. 7. Experimental scheme of generating the single-photon-added squeezed entangled coherent states. BS is the 50/50 beam splitter. $S_{\mathrm{ab}}(r)$ is two-mode squeezing operator with r being a real squeezing parameter, and the a (b) represents a (b) mode of the SECS. The $|0\rangle_{\mathrm{a}}$ and $|0\rangle_{\mathrm{b}}$ are all the vacuum states. BBO is beta-barium borate crystal.

5. Conclusions

We have studied the effects of photon addition operation on the quantum nonlocality of SECS for the test of Bell's inequality. It has been found that photon addition can enhance the Bell violation when the values of amplitude and squeezing parameters are properly chosen in the photon-parity measurements. Therefore, it should be noted that for photon-parity measurement schemes, the photon addition is not always helpful in enhancing Bell violations of squeezed entangled states. In addition, we also propose a feasible scheme to generate the single-photon-added SECS.

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