Semi-classical theory and experimental research for polarization flipping in a single frequency laser with feedback effect*

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In this paper, we propose a semi-classical theory to successfully explain the polarization flipping in a single frequency laser. An experimental setup is built to verify this theory. The observed experimental phenomena are consistent with the theoretical analysis. We perform phase retardation measurements of birefringent components using this experimental system. The results show that the measurement repeatability is 0.12° and the measurement accuracy is 0.22°.

Keywords: semi-classical theory, interference, birefringence

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1. Introduction

The phenomenon of laser feedback was first observed by King in 1963.[1] Since then, the research into the feedback effect has attracted considerable interest, and some major progresses have been made.[2−5] The polarization flipping is an important feedback phenomenon, in which the polarization state of a laser with an anisotropic feedback cavity flips into the vertical direction when the length of the feedback cavity changes. Some work has been done to study the polarization flipping.[6−8]

In this article, a semi-classical theory is proposed for the research of polarization flipping in a single frequency He–Ne laser with an anisotropic feedback external cavity. Based on this theoretical analysis, an experimental system is built. The phase retardation in birefringent components can be measured using the experimental system.

2. The semi-classical theory

A model of the equivalent cavity of the Fabry–Perot interferometer has been used to analyze the intensity transfer and the polarization flipping in a laser with the optical feedback from an external birefringent cavity.[9] The central idea is that the effects of the feedback cavity are converted into a reflectivity change of the laser mirror. The model can be expressed as[10]

$$E = R_1^{1/2} R_2^{1/2} E\, \mathrm{e}^{\mathrm{i}2kL}\, \mathrm{e}^{2gL} + (1-R_2) \times R_\mathrm{e}^{1/2} R_1^{1/2}\, \mathrm{e}^{\mathrm{i}2k(L+l)} E\, \mathrm{e}^{2gL}, \tag{1}$$

where E is the laser intensity in the internal laser cavity, R_1 and R_2 are the reflectivities of the laser mirrors, R_e is the reflectivity of the feedback mirror, L is the length of the laser cavity, l is the length of the feedback cavity, g is the gain coefficient of the laser, $k = 2\pi/\lambda$, and λ is the wavelength. From Eq. (1), the laser intensity is given by

$$I = I_\mathrm{o} - \frac{R_\mathrm{e}^{1/2}}{R_2^{1/2}}(1-R_2)\cos(2kl), \tag{2}$$

where I is the laser intensity with feedback, while I_o is the intensity without feedback. From Eq. (2), we find that the variation in laser intensity follows a cosine function. The theoretical analysis is consistent with the experimental results found by King.[1]

Equation (1) can be rewritten as

$$\begin{aligned} E &= R_1^{1/2}\, \mathrm{e}^{\mathrm{i}2kL}\, \mathrm{e}^{2gL} E \left[R_2^{1/2} + (1-R_2) R_\mathrm{e}^{1/2}\, \mathrm{e}^{\mathrm{i}2kl}\right] \\ &= R_1^{1/2} R_\mathrm{eff}^{1/2}\, \mathrm{e}^{\mathrm{i}2kL}\, \mathrm{e}^{2gL} E, \end{aligned} \tag{3}$$

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where $R_{\rm eff} = R_2 + 2(1-R_2)R_2^{1/2}R_{\rm e}^{1/2}\cos(2kl)$ is the equivalent reflectivity of the laser mirror R_2 and the feedback mirror $R_{\rm e}$. The effects of the feedback cavity can be affected by the reflectivity variation of the laser mirror R_2. The variation is expressed as $R_{\rm eff}$.

Based on the model of the equivalent cavity of the Fabry–Perot interferometer, the phenomenon of polarization flipping can be explained.[9] However, the work basis in Ref. [9] is the Floch's mechanism.[6] In this paper, we use a semi-classical theory to explain the polarization flipping.

The new theory integrates the model of the equivalent cavity of the Fabry–Perot interferometer and Lamb's semi-classical theory. In our experimental system, the feedback cavity is an anisotropic cavity. From formula (3), the equivalent reflectivities of ordinary (o) light and extraordinary (e) light are respectively expressed as

$$\begin{aligned} R_{\rm eff}^{\rm o} &= R_2 + 2R_2^{1/2}R_{\rm e}^{1/2}(1-R_2)\cos(2kl),\\ R_{\rm eff}^{\rm e} &= R_2 + 2R_2^{1/2}R_{\rm e}^{1/2}(1-R_2)\cos(2kl+\delta). \end{aligned} \quad (4)$$

Lamb's semi-classical theory is expressed as[11]

$$\begin{aligned} \dot{I}_{\rm o} &= 2I_{\rm o}(\alpha_{\rm o} - \beta_{\rm o}I_{\rm o} - \theta_{\rm oe}I_{\rm e}),\\ \dot{I}_{\rm e} &= 2I_{\rm e}(\alpha_{\rm e} - \beta_{\rm e}I_{\rm e} - \theta_{\rm eo}I_{\rm o}), \end{aligned} \quad (5)$$

where $I_{\rm o}$ and $I_{\rm e}$ are the intensities of the o light and the e light, respectively, $\alpha_{\rm o}$ and $\alpha_{\rm e}$ are the net gain coefficients, $\beta_{\rm o}$ and $\beta_{\rm e}$ are the self-saturation coefficients, and $\theta_{\rm oe}$ and $\theta_{\rm eo}$ are the mode competition coefficients. The quasi-steady-state solutions for $I_{\rm o}$ and $I_{\rm e}$ are given by

$$I_{\rm o} = \frac{\alpha_{\rm o}\beta_{\rm e} - \alpha_{\rm e}\theta_{\rm oe}}{\beta_{\rm o}\beta_{\rm e} - \theta_{\rm oe}\theta_{\rm eo}}, \quad I_{\rm e} = \frac{\alpha_{\rm e}\beta_{\rm o} - \alpha_{\rm o}\theta_{\rm eo}}{\beta_{\rm o}\beta_{\rm e} - \theta_{\rm oe}\theta_{\rm eo}}. \quad (6)$$

Now we introduce the effective gains and the coupling coefficient. The effective gains are defined as

$$\alpha'_{\rm o} = \alpha_{\rm o} - \alpha_{\rm e}\frac{\theta_{\rm oe}}{\beta_{\rm e}}, \quad \alpha'_{\rm e} = \alpha_{\rm e} - \alpha_{\rm o}\frac{\theta_{\rm eo}}{\beta_{\rm o}}, \quad (7)$$

where $\alpha'_{\rm o}$ and $\alpha'_{\rm e}$ are the net gains of the o light and the e light, respectively. The coupling coefficient C is given by

$$C = \frac{\theta_{\rm oe}\theta_{\rm eo}}{\beta_{\rm o}\beta_{\rm e}}. \quad (8)$$

Equation (5) can be rewritten as

$$I_{\rm o} = \frac{\alpha'_{\rm o}/\beta_{\rm o}}{1-C}, \quad I_{\rm e} = \frac{\alpha'_{\rm e}/\beta_{\rm e}}{1-C}. \quad (9)$$

The steady-state solution is shown in Table 1. From Table 1, we can see that the polarization flipping occurs under two sets of conditions: $\alpha_{\rm o} > 0$, $\alpha_{\rm e} > 0$, $\alpha'_{\rm o} > 0$, $\alpha'_{\rm e} < 0$ and $\alpha_{\rm o} > 0$, $\alpha_{\rm e} > 0$, $\alpha'_{\rm o} < 0$, $\alpha'_{\rm e} > 0$.

Table 1. Steady-state solutions under different conditions.

Conditions		$I_{\rm o}$	$I_{\rm e}$
$\alpha_{\rm o} < 0$, $\alpha_{\rm e} < 0$		0	0
$\alpha_{\rm o} < 0$, $\alpha_{\rm e} > 0$		0	$\alpha_{\rm e}/\beta_{\rm e}$
$\alpha_{\rm o} > 0$, $\alpha_{\rm e} < 0$		$\alpha_{\rm o}/\beta_{\rm o}$	0
$\alpha_{\rm o} > 0$, $\alpha_{\rm e} > 0$	$\alpha'_{\rm o} > 0$, $\alpha'_{\rm e} > 0$	$\alpha'_{\rm o}/[\beta_{\rm o}(1-C)]$	$\alpha'_{\rm e}/[\beta_{\rm e}(1-C)]$
	$\alpha'_{\rm o} > 0$, $\alpha'_{\rm e} < 0$	$\alpha_{\rm o}/\beta_{\rm o}$	0
	$\alpha'_{\rm o} < 0$, $\alpha'_{\rm e} > 0$	0	$\alpha_{\rm e}/\beta_{\rm e}$
	$\alpha'_{\rm o} < 0$, $\alpha'_{\rm e} < 0$	determined by the initial conditions	

The net gain coefficients are given by

$$\alpha_{\rm o} = F_{\rm o}\left[\frac{Z_{\rm i}(\xi_{\rm o})}{Z_{\rm i}(0)} - \frac{1}{\eta}\right], \quad \alpha_{\rm e} = F_{\rm e}\left[\frac{Z_{\rm i}(\xi_{\rm e})}{Z_{\rm i}(0)} - \frac{1}{\eta}\right], \quad (10)$$

where $Z_{\rm i}$ is the plasma dispersion function, η is the relative excitation degree, which is approximately 1.33, and ξ is a frequency parameter. The $Z_{\rm i}$, ξ, and F are given by

$$\begin{aligned} &Z_{\rm i}(\xi) = \sqrt{\pi}\exp(-\xi^2),\\ &F_{\rm o} = \frac{\eta}{2\pi}\Delta[0.003 + (0.006 + (1-R_{\rm eff}^{\rm o}))0.5 + 0.007],\\ &\xi_{\rm o} = \frac{\nu_{\rm o}-\nu_0}{10^9}, \quad \xi_{\rm e} = \frac{\nu_{\rm e}-\nu_0}{10^9},\\ &F_{\rm e} = \frac{\eta}{2\pi}\Delta[0.003 + (0.006\\ &\quad + (1-R_{\rm eff}^{\rm e}))0.5 + 0.007], \end{aligned} \quad (11)$$

where $v_{\rm o}$ and $v_{\rm e}$ are the frequencies of the o light and the e light, respectively, v_0 is the center frequency of the laser, and Δ is the longitudinal mode spacing and is approximately 833 MHz.

The self-saturation coefficients are given by[12]

$$\beta_{\rm o} = F_{\rm o}\frac{\sqrt{\pi}\gamma_a\gamma_{\rm b}H_{\rm o}^{\rm r}}{8Z_{\rm i}(0)\left[\sum\limits_m|\wp_{mm}|^2\right]^2}(B+C+D),$$
$$\beta_{\rm e} = F_{\rm e}\frac{\sqrt{\pi}\gamma_a\gamma_{\rm b}H_{\rm e}^{\rm r}}{8Z_{\rm i}(0)\left[\sum\limits_m|\wp_{mm}|^2\right]^2}(B+C+D), \quad (12)$$

where $\gamma_{\rm a} = 24$ MHz, $\gamma_{\rm b} = 83$ MHz, and $H^{\rm r}$ is the real part of the complex function H. The other parameters are expressed as

$$\begin{aligned}
\wp_{m,m} &= \wp\left[(J_{\rm a}-m+1)(J_{\rm a}+m+1)\right]^{1/2},\\
\wp_{m,m+1} &= -\frac{1}{2}{\rm i}\wp\left[(J_{\rm a}-m+1)(J_{\rm a}+m+1)\right]^{1/2},\\
\wp_{m,m-1} &= -\frac{1}{2}{\rm i}\wp\left[(J_{\rm a}-m+1)(J_{\rm a}-m-2)\right]^{1/2},\\
B &= \sum_{-J_{\rm a}}^{J_{\rm a}}|\wp_{m,m}|^2\left(|\wp_{m+1,m}|^2+|\wp_{m-1,m}|^2\right),\\
C &= \sum_{-J_{\rm a}}^{J_{\rm a}}|\wp_{m,m}|^2\left(|\wp_{m,m-1}|^2+|\wp_{m,m+1}|^2\right),\\
D &= \sum_{-J_{\rm a}}^{J_{\rm a}}\wp_{m,m}(\wp_{m,m-1}\wp_{m-1,m-1}\wp_{m-1,m}\\
&\quad+\wp_{m,m+1}\wp_{m+1,m+1}\wp_{m+1,m}),\\
H_{\rm o} &= \left[D(0)+D(\xi_{\rm o})\right]\left[D_{\rm a}(0)+D_{\rm b}(0)\right]\bar{N},\\
H_{\rm e} &= \left[D(0)+D(\xi_{\rm e})\right]\left[D_{\rm a}(0)+D_{\rm b}(0)\right]\bar{N},\\
D(\nu) &= \frac{1}{\gamma+{\rm i}\nu 10^9}, \quad D_{\rm a}(\nu) = \frac{1}{\gamma_{\rm a}+{\rm i}\nu 10^9},\\
D_{\rm b}(\nu) &= \frac{1}{\gamma_{\rm b}+{\rm i}\nu 10^9}, \quad (13)
\end{aligned}$$

with $\gamma = 210$ MHz. The mode competition coefficients are expressed as

$$\begin{aligned}
\xi_{\rm oe} &= \frac{\xi_{\rm o}+\xi_{\rm e}}{2},\\
\theta_{\rm oe} &= F_{\rm e}\frac{\sqrt{\pi}\gamma_{\rm a}\gamma_{\rm b}}{8Z_{\rm i}(0)\left(\sum\limits_m|\wp_{mm}|^2\right)^2}\left[BM_{\rm oe}^{\rm r}+CM'^{\rm r}_{\rm oe}\right]\theta_{\rm eo}\\
&= F_{\rm o}\frac{\sqrt{\pi}\gamma_{\rm a}\gamma_{\rm b}}{8Z_{\rm i}(0)\left(\sum\limits_m|\wp_{mm}|^2\right)^2}\left(BM_{\rm eo}^{\rm r}+CM'^{\rm r}_{\rm eo}\right),\\
M_{\rm oe} &= D_{\rm b}(0)\left[D\left(\frac{\xi_{\rm e}-\xi_{\rm o}}{2}\right)+D(\xi_{\rm oe})\right]\\
&\quad+D_{\rm a}(\xi_{\rm e}-\xi_{\rm o})\left[\bar{N}D\left(\frac{\xi_{\rm e}-\xi_{\rm o}}{2}\right)+N_2D(\xi_{\rm o})\right],\\
M'_{\rm oe} &= D_{\rm a}(0)\left[D\left(\frac{\xi_{\rm e}-\xi_{\rm o}}{2}\right)+D(\xi_{\rm oe})\right]\\
&\quad+D_{\rm b}(\xi_{\rm e}-\xi_{\rm o})\left[\bar{N}D\left(\frac{\xi_{\rm e}-\xi_{\rm o}}{2}\right)+N_2D(\xi_{\rm o})\right],\\
M_{\rm eo} &= D_{\rm b}(0)\left[D\left(\frac{\xi_{\rm o}-\xi_{\rm e}}{2}\right)+D(\xi_{\rm oe})\right]\\
&\quad+D_{\rm a}(\xi_{\rm o}-\xi_{\rm e})\left[\bar{N}D\left(\frac{\xi_{\rm o}-\xi_{\rm e}}{2}\right)+N_2D(\xi_{\rm e})\right],\\
M'_{\rm eo} &= D_{\rm a}(0)\left[D\left(\frac{\xi_{\rm o}-\xi_{\rm e}}{2}\right)+D(\xi_{\rm oe})\right]\\
&\quad+D_{\rm b}(\xi_{\rm o}-\xi_{\rm e})\left[\bar{N}D\left(\frac{\xi_{\rm o}-\xi_{\rm e}}{2}\right)\right.\\
&\quad\left.+N_2D(\xi_{\rm e})\right]. \quad (14)
\end{aligned}$$

The $F_{\rm o}$ and $F_{\rm e}$ build the connection between the model of the equivalent cavity of the Fabry–Perot interferometer and Lamb's semi-classical theory, and they are modulated by the external cavity length. From the equations given above, the effective gain can be calculated, and some new conclusions can be derived. One is that the magnitude of the effective gain is modulated by the external cavity length. The other is that the magnitude of the effective gain increases against the increasing frequency difference between the o light and the e light. The relationship between the effective gain and the external cavity length is shown in Fig. 1.

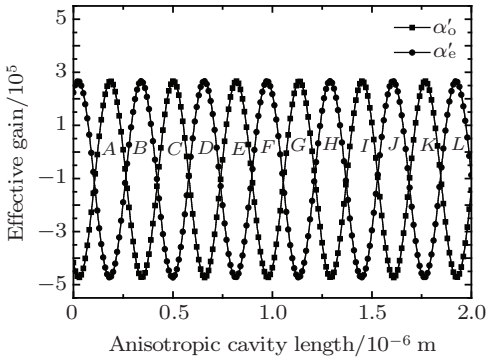


Fig. 1. The effective gain versus the length variation of feedback cavity.

In Fig. 1, the effective gain is modulated by the external cavity length. The variation of the effective gain approximately follows a cosine function. The period of variation is about 316 nm. According to the conclusions given above, the polarization states will flip when the effective gain is bigger than 0. The polarization flipping can be observed at points A, B, C, D, E, F, G, H, I, J, K, and L.

3. Experiment

Experiments are carried out with a linearly polarized He–Ne laser. The experimental setup is shown in

Fig. 2. The wavelength is 632.8 nm.

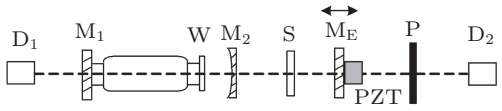


Fig. 2. Experimental setup, where W is the antireflection window, the other components are explained in the text.

The laser cavity is made up of mirrors M_1, M_2 with the reflectivities of 99.8%, 98.8% and a gain medium. The cavity length is 150 mm. The external cavity is made up of sample S and feedback mirror M_E. The length is 100 mm. Mirror M_E with the reflectivity of 15% is used to reflect laser beams back into the laser cavity. A piezoelectric transducer (PZT) is used to push and pull mirror M_E. Polarizer P is used to separate different polarization states from the laser. When the polarization state of the laser is in the same direction of the polarizer, the laser can reach detector D_2, and the voltage of D_2 increases. When the polarization state of the laser is in the orthogonal direction of the polarizer, the voltage of D_2 maintains the minimum. So, the voltage flipping of detector D_2 indicates the polarization flipping of the laser. Detector D_1 is used to detect the laser intensity.

The sample material with anisotropic is placed in the feedback external cavity, so, the feedback cavity is anisotropic. When the length of the feedback cavity is modulated, the losses of the o light and the e light in the laser cavity are changed, as shown in Fig. 1. Sometimes, the loss of the o light is bigger than that of the e light, and the e light resonates in the laser cavity. Other times, the o light resonates in the laser cavity. The laser intensity and polarization will change, as shown in Fig. 3.

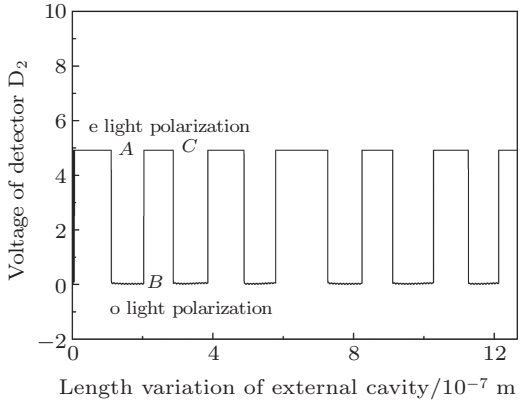


Fig. 3. The polarization flipping.

In the experiment, we find that the position of the polarization flipping is modulated by the phase retardation of the birefringent component inserted into the external cavity. We define the duty ratio of polarization flipping as

$$D = \frac{l_{BC}}{l_{AC}}, \tag{15}$$

where D is the duty ratio, and l_{AC} and l_{BC} are the length changes of the external cavity between C, A and C, B. The relationship between the phase retardation and the duty ratio is given by

$$\delta = D \times 90, \tag{16}$$

where δ is the phase retardation. In our experimental setup, the stress birefringence of the optical component and the phase retardation of the birefringent component can be measured.

4. Measurement of phase retardation

In optical systems for precision measurement, the accuracy of many phase retardation measurement instruments has not met the needs of industry. In this paper, the experimental system can be used to measure the phase retardation of a birefringent component. The measurement repeatability is 0.12°, and the measurement accuracy is 0.22°. The measurement results and errors are given in Table 2.

Table 2. Measurement results of phase retardation at 22 °C.

Actual value/(°)	Measured value/(°)	Error/(°)
18.38	18.34	0.04
29.85	29.91	0.06
46.83	46.69	0.14
71.44	71.61	0.17
80.91	80.93	0.02
85.77	85.55	0.22
99.15	99.26	0.11
108.8	108.8	–
130.9	131	0.1
140.2	140.2	–
159.1	159.3	0.2

5. Conclusion

A theoretical research has been conducted to explain the polarization flipping in a single frequency laser with phase anisotropic cavity. The theoretical analysis is consistent with the experimentally observed

phenomena. The experimental setup can also be used to measure the phase retardation of a birefringent component.

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