A set of new nucleon coupling constants and the proto neutron star matter*

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Using a new set of nucleon coupling constants CZ11 the properties of a proto neutron star are examined within the framework of the relativistic mean-field theory for the baryon octet system. It is found that the relative number density of Λ, Ξ^-, and Ξ^0 for CZ11 are all smaller than those for GL97 and for both CZ11 and GL97, Σ^-, Σ^0, and Σ^+ do not appear. It is also found that the pressure and the maximum mass for CZ11 are all smaller than those for GL97. The maximum mass for CZ11 decreases by approximately 9 percent compared with that for GL97.

Keywords: nucleon coupling constants, relativistic mean-field theory, proto neutron star

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1. Introduction

The proto neutron star (PNS) is formed after the explosion of the supernova and its temperature is more than 10 MeV.[1] Its maximum mass, which will determine whether the proto neutron evolves from proto neutron star to black hole or to stable neutron star,[2] is of crucial importance.[3]

The relativistic mean-field (RMF) theory, which can successfully describe the nuclear system,[4] was first used to describe neutron star matter in 1985 by Glendenning,[5] and since then it has been extensively applied to the research of the neutron stars.[6] The theoretical study of neutron stars shows that neutron star matter is very sensitive to the nucleon coupling constants and the hyperon-to-nucleon coupling constants.[7,8]

The hyperon-to-nucleon vector couplings can be obtained by the constituent quark model [$SU(6)$ symmetry],[9] and the hyperon-to-nucleon scalar couplings can then be determined by fitting the Λ, Σ, and Ξ well depths in nucleon matter.[10,11] The experimental data of the depth $U_\Lambda^{(\mathrm{N})}$ is -30 MeV.[12] The depth $U_\Xi^{(\mathrm{N})}$ also is well determined to be about -16 MeV,[13] and -14 MeV[14] or -24 MeV~ -21 MeV.[15] In actual calculation, a more relativistic potential value of $U_\Xi^{(\mathrm{N})} = -28$ MeV can be chosen.[16] By the analysis of Σ nucleus optical potential constructed, Mars obtained the depth $U_\Sigma^{(\mathrm{N})}$ with a repulsive real part in the nuclear interior.[17] The inclusive (π^-, K^+) spectra on medium-to-heavy nuclear targets measured at KEK give a more direct evidence for a strongly repulsive Σ–nucleus potential of 90 MeV.[18,19] More sophisticated theoretical analyses of these KEK (π^-, K^+) spectra have also concluded that the Σ–nuclear potential is repulsive within the nuclear volume and they yield a weaker repulsion in the range of 10 MeV $\sim$ 40 MeV.[20−24]

The nucleon coupling constants are connected with four bulk properties of nuclear matter together with the choice of an effective mass at saturation.[5] By now, GL85 sets,[5] GL97 sets,[9] TM1 sets,[25] TM2 sets,[25] NL2 sets,[26] and NLSH sets,[27] and so on, have been obtained and applied to the calculations of neutron star matter. A recent analysis of the symmetry energy comes from isospin diffusion data for reactions involving ^{112}Sn and ^{124}Sn from the NSCL/MSU (the National Superconducting Cyclotron Laboratory at Michigan State University).[28,29] From the results of these works a new set of nucleon coupling constants (CZ11 set[30]) can be obtained.

In this paper, we use the CZ11 set to examine the properties of the PNS within the RMF approach

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considering the baryon octet.

2. The RMF theory of a PNS

The Lagrangian density of hadron matter reads as follows:[5]

$$\begin{aligned}\mathscr{L} =& \sum_B \bar{\Psi}_B \Big[\mathrm{i}\gamma_\mu \left(\partial^\mu + \mathrm{i} g_{\omega B}\gamma_\mu \omega^\mu + \frac{\mathrm{i}}{2} g_{\rho B}\tau \cdot \rho^\mu \right) \\ & - \left(m_B - g_{\sigma B}\sigma \right) \Big] \Psi_B \\ & + \frac{1}{2}\left(\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2 \right) - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} \\ & + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4}\rho_{\mu\nu}\cdot\rho^{\mu\nu} + \frac{1}{2} m_\rho^2 \rho_\mu \cdot \rho^\mu \\ & - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4 + \sum_{\lambda=\mathrm{e},\mu} \bar{\Psi}_\lambda \left(\mathrm{i}\gamma_\mu \partial^\mu - m_\lambda \right) \Psi_\lambda . \end{aligned} \tag{1}$$

We solve the field equations in the usual RMF approximation. The equations of baryon field are obtained as follows:

$$\begin{aligned}& \left(\gamma_\mu \kappa^\mu - m_B + g_{\sigma B}\sigma - g_{\omega B}\gamma_0 \omega_0 \right. \\ & \left. - g_{\rho B}\gamma_0 \tau_3 \rho_{03} \right) \Psi_B \left(\kappa, B \right) = 0, \end{aligned} \tag{2}$$

and their eigenvalues are

$$e_B\left(\kappa\right) = g_{\omega B}\omega_0 + g_{\rho B}\rho_{03} I_{3B} + \sqrt{\kappa^2 + m_B^{*2}}. \tag{3}$$

Here, I_{3B} is the isospin 3-component for baryon B and m^* is the effective mass of baryons

$$m_B^* = m_B - g_{\sigma B}\sigma. \tag{4}$$

The mesons field equations are given by

$$\begin{aligned} m_\sigma^2 \sigma =& -g_2\sigma^2 - g_3\sigma^3 + \sum_B \frac{2J_B+1}{2\pi^2} g_{\sigma B} \int_0^\infty \frac{m^*}{\sqrt{k^2+m^{*2}}} \\ & \times \left(\exp[(\varepsilon_B(k) - \mu_B)/T] + 1 \right)^{-1} k^2 \mathrm{d}k, \end{aligned} \tag{5}$$

$$m_\omega^2 \omega_0 = \sum_B \frac{2J_B+1}{2\pi^2} g_{\omega B} b_B \int_0^\infty \left(\exp[(\varepsilon_B(k) - \mu_B)/T] + 1 \right)^{-1} k^2 \mathrm{d}k, \tag{6}$$

$$m_\rho^2 \rho_{03} = \sum_B \frac{2J_B+1}{2\pi^2} g_{\rho B} I_{3B} b_B \int_0^\infty \left(\exp[(\varepsilon_B(k) - \mu_B)/T] + 1 \right)^{-1} k^2 \mathrm{d}k. \tag{7}$$

The energy density and pressure of a PNS are given by

$$\begin{aligned} \varepsilon =& \frac{1}{3} g_2 \sigma^3 + \frac{1}{4} g_3 \sigma^4 + \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{2} m_\rho^2 \rho_{03}^2 \\ & + \sum_B \frac{2J_B+1}{2\pi^2} \int_0^\infty \kappa^2 \mathrm{d}\kappa \sqrt{\kappa^2 + m^{*2}} \left(\exp[(\varepsilon_B(k) - \mu_B)/T] + 1 \right)^{-1} \\ & + \sum_{\lambda=\mathrm{e},\mu} \frac{2J_\lambda+1}{2\pi^2} \int_0^\infty \kappa^2 \mathrm{d}\kappa \sqrt{\kappa^2 + m_\lambda^2} \left(\exp[(\varepsilon_\lambda(k) - \mu_\lambda)/T] + 1 \right)^{-1}, \end{aligned} \tag{8}$$

$$\begin{aligned} p =& -\frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4 + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{2} m_\rho^2 \rho_{03}^2 \\ & + \frac{1}{3} \sum_B \frac{2J_B+1}{2\pi^2} \int_0^\infty \frac{\kappa^4}{\sqrt{\kappa^2 + m^{*2}}} \mathrm{d}\kappa \left(\exp[(\varepsilon_B(k) - \mu_B)/T] + 1 \right)^{-1} \\ & + \frac{1}{3} \sum_{\lambda=\mathrm{e},\mu} \frac{2J_\lambda+1}{2\pi^2} \int_0^\infty \frac{\kappa^4}{\sqrt{\kappa^2 + m_\lambda^2}} \mathrm{d}\kappa \left(\exp[(\varepsilon_\lambda(k) - \mu_\lambda)/T] + 1 \right)^{-1}. \end{aligned} \tag{9}$$

The conditions of the conservation of baryon number and the charge neutrality are respectively given by

$$\begin{aligned} \rho =& \sum_B \frac{2J_B+1}{2\pi^2} b_B \\ & \times \int_0^\infty \left(\exp\left[\left(\varepsilon_B\left(k\right) - \mu_B \right)/T \right] + 1 \right)^{-1} k^2 \mathrm{d}k, \end{aligned} \tag{10}$$

$$\begin{aligned} 0 =& \sum_B \frac{2J_B+1}{2\pi^2} b_B \\ & \times \int_0^\infty \left(\exp[(\varepsilon_B(k) - \mu_B)/T] + 1 \right)^{-1} k^2 \mathrm{d}k \\ & + \sum_{\lambda=\mathrm{e},\mu} \frac{2J_\lambda+1}{2\pi^2} q_\lambda \\ & \times \int_0^\infty \left(\exp[(\varepsilon_\lambda(k) - \mu_\lambda)/T] + 1 \right)^{-1} k^2 \mathrm{d}k. \end{aligned} \tag{11}$$

We use the Oppenheimer–Volkoff equation (O-V equation) to obtain the mass and the radius of a neu-

tron star

$$\frac{\mathrm{d}p}{\mathrm{d}r} = -\frac{(p+r)\left(M+4\pi r^3 p\right)}{r\left(r-2M\right)}, \tag{12}$$

$$M = 4\pi \int_0^r \varepsilon r^2 \,\mathrm{d}r. \tag{13}$$

3. Parameters

The CZ11 set,[30] which is obtained by the analysis of the NSCL/MSU experimental results,[28,29,31] is listed in Table 1.

Table 1. The CZ11 nucleon coupling parameter set, 1 fm = 10^{-15} m.

$(g_\sigma/m_\sigma)^2/\mathrm{fm}^2$	$(g_\omega/m_\omega)^2/\mathrm{fm}^2$	$(g_\rho/m_\rho)^2/\mathrm{fm}^2$	$b/10^2$	$c/10^2$
10.1937	4.8557	4.2090	0.0105	−0.0100

We define the ratios: $x_{\sigma\mathrm{h}} = g_{\sigma\mathrm{h}}/g_{\sigma\mathrm{N}}$, $x_{\omega\mathrm{h}} = g_{\omega\mathrm{h}}/g_{\omega\mathrm{N}}$, $x_{\rho\mathrm{h}} = g_{\rho\mathrm{h}}/g_{\rho\mathrm{N}}$, with h denoting hyperons. For the coupling constants of meson ω, we use ratios

$$g_{\omega\mathrm{N}}/3 = g_{\omega\Sigma}/2 = g_{\omega\Lambda}/2 = g_{\omega\Xi}, \tag{14}$$

which is given by the constituent quark model [$SU(6)$ symmetry]. The couplings constants of meson σ are then determined by fitting the Λ, Σ, and Ξ well depths in nuclear matter as follows:[7]

$$U_{\mathrm{h}}^{(\mathrm{N})} = m_B \left(\frac{m_n^*}{m_n} - 1\right) x_{\sigma\mathrm{h}} + \left(\frac{g_{\omega\mathrm{N}}}{m_\omega}\right)^2 \rho_0 x_{\omega\mathrm{h}}. \tag{15}$$

In our calculation, we choose $U_\Lambda^{(\mathrm{N})} = -30$ MeV, $U_\Sigma^{(\mathrm{N})} = +30$ MeV, $U_\Xi^{(\mathrm{N})} = -28$ MeV, and the temperature $T = 15$ MeV. Because GL97 set[9] can describe the neutron star matter better,[9] we also consider it for comparison.

4. Theoretical results and analyses

The field strengths of mesons σ, ω, and ρ are shown in Fig. 1. The temperature of the PNS is $T = 15$ MeV. From Fig. 1 we can see that the field strengths of mesons ω for CZ11 are larger than those for GL97 and those of mesons σ are by far larger than those for GL97. For the field strengths of mesons ρ, the values for CZ11 are almost equal to those for GL97.

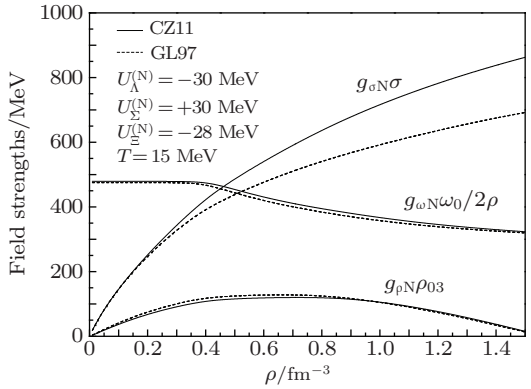


Fig. 1. The field strengths of σ, ω, and ρ as a function of baryon density.

Figure 2 gives the chemical potentials of neutrons and electrons. From Fig. 2 it can be seen that the chemical potentials of neutrons for CZ11 are smaller than those for GL97 while the chemical potentials of electrons for CZ11 are larger than those for GL97.

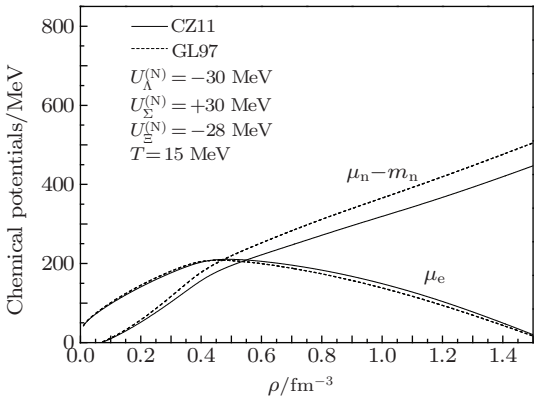


Fig. 2. The chemical potentials of neutrons and electrons as a function of baryon density.

The relative number density of particles is given in Fig. 3. From Fig. 3 we can see that the relative number density of neutrons for CZ11 is larger than that for GL97 while that of protons for CZ11 is smaller than that for GL97. As for the electrons and muons, the relative number densities for CZ11 are smaller than those for GL97 as $\rho < 0.4266$ fm^{-3} while as $\rho > 0.4266$ fm^{-3} the former are larger than the latter. The relative number densities of Λ, Ξ^-, and Ξ^0 for CZ11 are all smaller than those for GL97. The reason is the CZ11 sets are not of great advantage to the transfer from neutrons to Λ, Ξ^-, and Ξ^0. We can also see that for both CZ11 and GL97, Σ^-, Σ^0, and Σ^+ do not appear. This is because the positive well depth $U_\Sigma^{(\mathrm{N})} = +30$ MeV would restrain the occurrence of the Σ^-, Σ^0, and Σ^+.

The pressure as a function of energy density is charted in Fig. 4. We can see that the pressure for CZ11 is smaller than that for GL97.

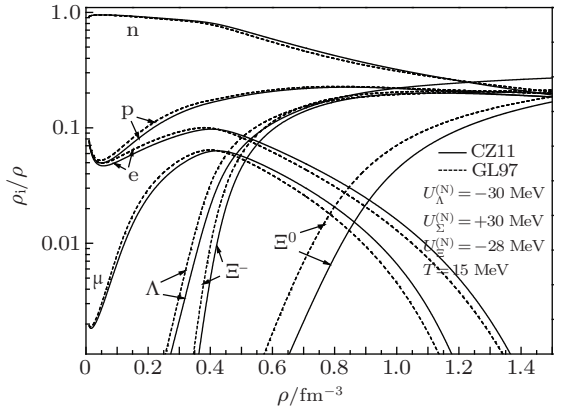


Fig. 3. The relative particle number density as a function of baryon density.

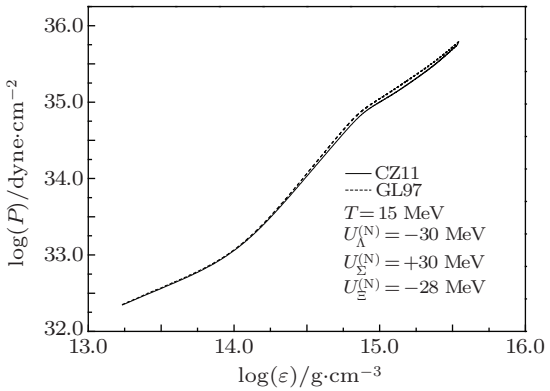


Fig. 4. The pressure as a function of energy density.

The mass as a function of central energy density is shown in Fig. 5 and the maximum mass and the corresponding radius are listed in Table 2. From Fig. 5 it can be seen that the mass for CZ11 is smaller than that for GL97. From Table 2 we can see that the maximum mass $M_{\max}$ for CZ11 is $M_{\max} = 1.4216\ M_\odot$ and that for GL97 is $M_{\max} = 1.5581\ M_\odot$. The maximum mass for CZ11 decreases by 8.76 percent compared with that for GL97.

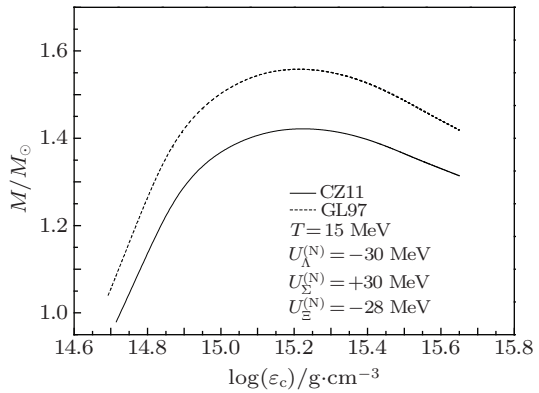


Fig. 5. The mass as a function of central energy density.

Table 2. The maximum mass and the corresponding radius.

	$\varepsilon_c/10^{15}$ g $\cdot$ cm^{-3}	$M_{\max}/M_\odot$	R/km
CZ11	1.6719	1.4216	13.4820
GL97	1.6344	1.5581	13.6870

5. Summary

In conclusion, in this paper using the new set of nucleon coupling constants CZ11, which was obtained in our previous work, the properties of a proto neutron star are examined within the framework of the relativistic mean-field theory for the baryon octet system. It is found that the relative number density of neutrons for CZ11 is larger than that for GL97 while that of protons for CZ11 is smaller than that for GL97. For the electrons and muons, the relative number densities for CZ11 are smaller than those for GL97 as $\rho < 0.4266$ fm^{-3} while as $\rho > 0.4266$ fm^{-3} the former are larger than the latter. The relative number densities of Λ, Ξ^-, and Ξ^0 for CZ11 are all smaller than those for GL97. We also can see that for both CZ11 and GL97, Σ^-, Σ^0, and Σ^+ do not appear. The pressure and the maximum mass for CZ11 are all smaller than those for GL97 and the maximum mass for CZ11 decreases by 8.76 percent compared with that for GL97.

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