Mean-field and high temperature series expansion calculations of some magnetic properties of Ising and XY antiferromagnetic thin-films

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In this work, the magnetic properties of Ising and XY antiferromagnetic thin-films are investigated each as a function of Néel temperature and thickness for layers (n = 2, 3, 4, 5, 6, and bulk (∞)) by means of a mean-field and high temperature series expansion (HTSE) combined with Padé approximant calculations. The scaling law of magnetic susceptibility and magnetization is used to determine the critical exponent γ, $\nu_{\rm eff}$ (mean), ratio of the critical exponents γ/ν, and magnetic properties of Ising and XY antiferromagnetic thin-films for different thickness layers n = 2, 3, 4, 5, 6, and bulk (∞).

Keywords: high-temperature series expansions, mean-field theory, antiferromagnetic thin film, Padé approximant, Néel temperature, critical exponent

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1. Introduction

Recently, the critical behaviours of magnetic films are of great interest.[1,2] In this regard, many approaches such as the effective field theory[3] and Monte Carlo (MC) simulation [4] have been developed. However, most of the studies relating to magnetic films have been devoted to the nearest-neighbour (NN) exchange interaction and few studies have dealt with the next-nearest neighbour (NNN). But for many materials with complex structures, it is necessary to take into account the NNN interactions.[5−7] Scott and Giebultowicz[8] have found that the Néel transition temperature varies linearly with NN interaction for the antiferromagnetic semiconductor EuTe, but they have not given a detailed description of the phase diagram. Lin *et al.*[9] have developed an analytical method based on the variational cumulant expansion to calculate the critical temperature of the Ising film as a function of the film thickness. The critical behaviour of the spin-1 Ising film in a random transverse field has been studied by the use of an effective field method (EFM) within the framework of a single cluster theory.[10] Mean-field theories (MFT) have been used,[11,12] which are particularly simple to apply for complicated systems. However, in those approaches both thermal and quantum fluctuations, which are crucial for low-dimensional systems, were completely neglected. Hu and Kawazoe[13] have treated the magnetic reorientation of ferromagnetic thin films within a continuum approach for the system and mean-field-like expressions for the magnetization density.

We consider $\infty \times \infty \times n$ body-centered-cubic antiferromagnetic films (see Fig. 1) composed of magnetic spins localized on the sites of a body-centered-cubic (bcc) lattice which are infinite in two dimensions but are of n finite layers in the third dimension (z direction).[14−16] We impose a free-surface boundary[14−16] condition, in which each surface spin lacks four nearest neighbouring spins on the body-centered-cubic lattice.

The Hamiltonian of the system is given as

$$H = \sum_{\langle i,j \rangle} J_{ij} \boldsymbol{S}_i . \boldsymbol{S}_j - h_{\rm s} \left[\sum_{i \in A} S_i^z - \sum_{j \in B} S_j^z \right], \quad (1)$$

where i and j refer to the sites of two distinct inter-

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penetrating sublattice, the pair interaction parameter J_{ij} is taken to be $J > 0$ when i and j are the nearest neighbours and zero otherwise, $h_{\rm s}$ in the Zeeman energy term is staggered on the magnetic field on two sublattices A and B for calculating the sublattice magnetization and staggered susceptibility. The series are analysed using the standard extrapolation techniques.[17] The dependence of the Néel temperature on the number of n-layers of the bcc lattice film is investigated from the sixth-order series.

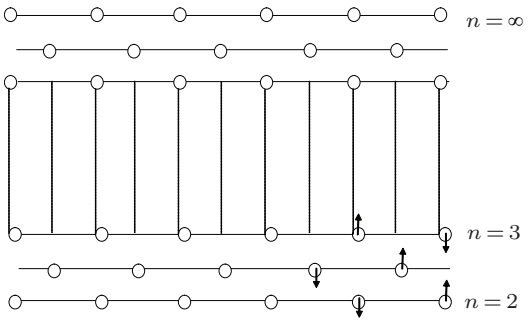


Fig. 1. Two-dimensional cross section of the thin film antiferromagnetic system on the body-centered cubic lattice with different (up or down) spin configurations.

In a recent paper,[18] the coefficients required for the calculation of the staggered susceptibility series of a magnetic thin film are given through order 6 in a $y = J/k_{\rm B}T$ study on the bcc lattice antiferromagnetic thin film. The rest of the present paper is organized as follows. In Section 2, we study the critical properties of a magnetic thin film through two models: Ising and XY using the high temperature series expansion (HTSE) method extrapolated with the Padé approximant method (PA).[19] The series expansion for the magnetic susceptibility is derived to order six in the reciprocal temperature in Ising and XY antiferromagnetic bcc layers. The results of the calculation and the analysis of series are presented in Section 3. Conclusions are given in Section 4.

2. Theoretical model (high temperature series expansion)

The Hamiltonian is divided into two parts, the mean-field Hamiltonian reads

$$H = H_0 + H_1, \tag{2}$$

where,

$$H_0 = -\sum_{i\in A}\left[8JM^+ + h_{\rm s}\right]S_i^z + \sum_{j\in B}\left[8JM^+ + h_{\rm s}\right]S_i^z + 4NJ\left(M^+\right)^2, \tag{3}$$

and the perturbation Hamiltonian

$$H_1 = J\sum_{\langle i,j\rangle}\left[S_i^z - M^+\right]\left[S_j^z + M^+\right] + \frac{J}{2}\sum_{\langle i,j\rangle}\left[S_i^+S_j^- + S_i^-S_j^+\right], \tag{4}$$

where $\sum_{\langle i,j\rangle}$ denotes the sum over the nearest neighbour spin pairs S_i and S_j, z is refers to the z direction of n-layer film, N is the total number of spins in a thin film, and M^+ is the sublattice magnetization which minimizes the free energy of the system.[20,21]

The high temperature series expansion of the susceptibility for a thin film of n spin layers is obtained from the free energy F by the relationship:

$$\chi_n^{\rm s}(\beta) = -\frac{1}{\beta}\frac{\partial^2(\beta F)}{\partial h_{\rm s}^2}, \tag{5}$$

$$\chi_l^{\rm s}(\beta) = N_2\frac{1}{\beta}\int_0^\beta {\rm d}\eta_k\int_0^\beta {\rm d}\eta_l \times\left\langle T_\eta\left\{\sum_{k\in A}\left(S_k^z\right)\sum_{l\in B}\left(-S_l^z\right)S\left(\beta\right)\right\}\right\rangle_\mu, \tag{6}$$

where

$$S(\beta) = \sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\int_0^\beta {\rm d}\eta_1\int_0^\beta {\rm d}\eta_2\cdots\int_0^\beta {\rm d}\eta_k T_\eta \times\left[H_1(\eta_1)H_2(\eta_2)\cdots H_k(\eta_k)\right] \tag{7}$$

in terms of the Dyson η-ordered cumulant product expansions,[20−23] the high temperature series expansions, with N_2 being the number of lattice sites in each spin layer. The subscript μ denotes that all connected diagrams need to be considered.

The zero-field ($h_{\rm s} = 0$) for the high temperature series expansion of magnetic susceptibility of the n-layer lattice with a free surface is obtained as a series in $y = \beta J$

$$\chi_n^{\rm s}(\beta) = \beta\sum_{m=0}^{\infty}a_m^n y^m, \tag{8}$$

where the coefficients a_m^n of an n-layer for a free surface for n = 2, 3, 4, 5, 6, and ∞ layers are given in Table 1 for the Ising model and in Table 2 for the XY model, by using the transformation of the coefficient[24] depending only on the dimension α of the spin (i.e., $\alpha = 1$ for Ising model and $\alpha = 2$ for XY model). The coefficients a_m^n are computed for the thicknesses (n = 2, 3 4, 5, 6, and ∞) and for the two models (α = 1 and 2) (see Tables 1 and 2).

Table 1. The series coefficients for the high-temperature staggered susceptibility series of body-centered-cubic (bcc) lattices with $n = 2$, 3, 4, 5, 6, and bulk layers and free-surface boundary condition for the Ising (I) model with a spin of $S = 3/2$.

n	a_0^n	a_1^n	a_2^n	a_3^n	a_4^n	a_5^n	a_6^n
2	1.25	18.75	206.253	2064.366	18520.65	154450.8	1.2342E6
3	1.25	24.9999	462.501	6877.494	111843.18	1.41674E6	2.10907E7
4	1.25	28.125	590.625	11393.433	205437.87	3.60556E6	6.04217E7
5	1.25	30	667.503	14102.991	286907.67	5.4884E6	1.05507E8
6	1.25	31.2501	718.749	15909.372	341220.6	7.06002E6	1.42762E8
bulk (∞)	1.25	37.5	974.997	24941.25	612785.25	1.49182E7	3.57515E8

Table 2. The series coefficients for the high-temperature staggered susceptibility series of the body-centered-cubic (bcc) lattices with $n = 2$, 3, 4, 5, 6, and bulk layers and free-surface boundary conditions for the XY model with a spin of $S = 3/2$.

n	a_0^n	a_1^n	a_2^n	a_3^n	a_4^n	a_5^n	a_6^n
2	1.25	9.375	51.56325	258.04575	1157.54063	4826.5875	19284.32813
3	1.25	12.49995	115.62525	859.68675	6990.19875	44273.08125	329542.17188
4	1.25	14.0625	147.65625	1424.17912	12839.86688	112673.78438	944089.17188
5	1.25	15	166.87575	1762.87388	17931.72937	171512.4375	1.64855×10^6
6	1.25	15.62505	179.68725	1988.6715	21326.2875	220625.775	2.23066×10^6
bulk (∞)	1.25	18.75	243.74925	3117.65625	38299.07813	466192.4625	5.58618×10^6

The high temperature series expansion of magnetic susceptibility obtained in the present calculation is directly evaluated from the two rooted diagrams. The high temperature series expansion of magnetic susceptibility could also be evaluated from the free energy to second order in h_s, and the derivative of the result is taken twice with respect to the field. Both ways must yield the same result. For the bulk ($n = \infty$) spin-1 antiferromagnetic Ising and XY models on both bcc, the results of the present calculation fit completely with previous results.[20,21,25,26] This is an independent check of the correctness of cumulants expansion, the corresponding weight and the value of the n-th-order η integral of the rooted diagrams.[20,21]

3. Results and discussion

3.1. Néel temperature

To determinate the Néel temperature $T_{\mathrm{N}}(n)$ for the n-layer film, we use the well-known ratio and Padé approximant techniques.[19] Since the ratio plot for a_m^n/a_{m-1}^n versus $1/m$ shows oscillatory behaviour, we utilize[27,28] the ratio plot for $\left(a_m^n/a_{m-1}^n\right)^{1/2}$ versus $(1/m\,(m-1))^{1/2}$. The plots of $\left(a_m^n/a_{m-1}^n\right)^{1/2}$ versus $(1/m\,(m-1))^{1/2}$ for n-layer films with a free surface are shown in Fig. 2 and in Fig. 3 for the Ising and XY models, respectively, with spin-3/2.

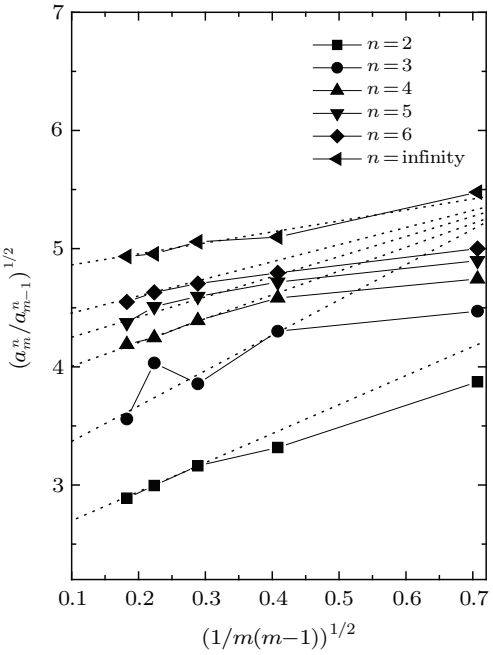


Fig. 2. The ratio diagrams of the high-temperature staggered susceptibility series $\left(a_m^n/a_{m-1}^n\right)^{1/2}$ versus $(1/m\,(m-1))^{1/2}$ n-layer bcc lattice film with the free surface for Ising (I) model. The dotted line represents the asymptotic line, which is extrapolated from the higher-order terms.

In Figs. 2 and 3, a straight line can be drawn through the points except those of the lower-order terms. The inverse Néel temperature is estimated

from extrapolating the straight line to the asymptotic limit (value of the intercept at $(1/m\,(m-1))^{1/2}=0$). The values of the critical temperature for Ising $I(1)$ and $XY(2)$ models in the bulk are estimated to be $T_{\rm N}(1)\approx 20$ K and $T_{\rm N}(2)\approx 10.736$ K. These values are close to the values obtained by Monte Carlo for $XY(2)$[29−35] and by the effective field theory and Monte Carlo for the Ising model ($T_{\rm N}=9.86$ K).[36,37]

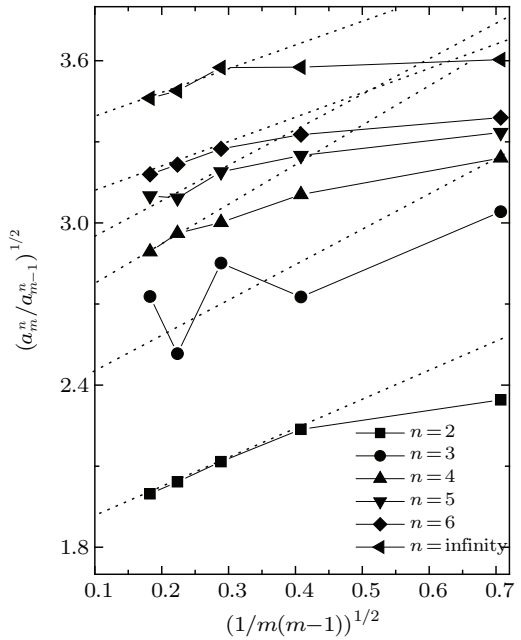


Fig. 3. Ratio plots of the high-temperature staggered susceptibility series $\left(a_m^n/a_{m-1}^n\right)^{1/2}$ versus $(1/m\,(m-1))^{1/2}$ n-layer bcc lattice film with the free surface for XY model. The dotted line represents the asymptotic line, which is extrapolated from the higher-order terms.

The plots for finite layers of $n=2$–6 exhibit appreciable curvatures and show the crossover phenomena appearing. These crossover phenomena represent a changeover from three-dimensional critical behaviour at low values of n (high temperatures) to two-dimensional critical behaviour at high values of n (low temperatures). For the thinner films, the present series is long enough to cover the crossover region and exhibits the two-dimensional critical behaviour of the ratio plots. The results of the Padé approximant analysis of the series are consistent with those obtained from the ratio method, except for the results of the $n=2$ and 3 layer films.

The ratio plot for $a_m^n(n)/a_{m-1}^n(n)$ versus $1/m$ shows oscillatory behaviour.

The results are found to comply with the analytic expression for $T_{\rm N}$ being made on the basis of a mean-field $T_{\rm N}$ examination[38] given as

$$\begin{aligned} &T_{\rm N}(n)\\ &=T_{\rm N}(\infty)\left(Z_0+2Z_1\cos(\pi/n+1)\right)/\left(Z_0+2Z_1\right). \end{aligned} \tag{9}$$

Note that $Z_0=0$ and $Z_1=4$ are the number of the nearest neighbours to a lattice point in the same layer and the number of nearest neighbours in one of the adjacent layers for cubic lattices, respectively. The obtained values of the critical temperature are given in Table 3 for the Ising model and in Table 4 for the XY model.

Table 3. The Néel temperature, exponents γ and ν for different layers for the Ising model with a body-centered-cubic and a spin of $S=3/2$.

n	2	3	4	5	6	∞
$T_{\rm N}$ (K) obtained by HTSE	8.612	12.685	13.153	12.819	16.945	20
$T_{\rm N}$ (K) obtained by MFT	10.009	14.147	16.184	17.323	18.021	20
$T_{\rm N}$ (K) obtained by MC and MFT[37]					9.86	
γ	2.952	2.9785	2.98	2.9795	2.988	2.991
ν (mean effective)					0.90	
ν obtained by MC and HTSE methods	$(\nu(1)=0.81)$[47,48], $(\nu(1)=0.79(1))$[49], $(\nu(1)\approx 0.8)$[31−35] and $(\nu(1)\approx 0.83)$					

Table 4. The Néel temperature, exponents γ and ν for different layers for the XY model with a body-centered-cubic and a spin of $S=3/2$.

n	2	3	4	5	6	∞
$T_{\rm N}$ (K) obtained by HTSE	4.306	6.041	6.573	7.883	8.477	10.736
$T_{\rm N}$ (K) obtained by MFT	5.372	7.594	8.687	9.299	9.673	10.736
$T_{\rm N}$ (K)[40]	–	–	–	–	–	10
γ	2.809	2.9065	2.9215	2.945	2.97	2.968
ν (mean effective)				0.65		

3.2. Critical exponents and magnetization

According to the finite-size scaling theory, the shift of the reduced Néel temperature $k_{\rm B}T_{\rm N}(n)/J$ from the three-dimensional bulk value $k_{\rm B}T_{\rm N}(\infty)/J$ for thicker layers of n can be described by a power law of the form as[30−43]

$$\lambda_{\rm eff} = -1/2\log(T_{\rm N}(\infty) - T_{\rm N}(n)/T_{\rm N}(\infty) - T_{\rm N}(n-1))/\log(n/n-1). \quad (10)$$

The obtained values of $\nu_{\rm mean} = 1/\lambda_{\rm eff}$ are $\nu(1)_{\rm mean} \approx 0.90$ for the Ising model and $\nu(2)_{\rm mean} \approx 0.65$ for the XY model according to the scaling prediction.[40,42,43] The obtained values of the critical exponent γ are given in Table 3 for the Ising model and in Table 4 for the XY model. These values are close to those obtained by a high temperature series expansion,[42−47,30−35] and using the Monte Carlo technique for the Ising model ($\nu(1) = 0.81$)[48] and ($\nu(1) = 0.79(1)$).[49] These results substantiate the general universality principle.[49] We also use the Padé approximant analysis of the series to estimate the Néel temperature. The Néel temperature corresponds to the simple pole of $[\chi_n^{\rm s}]^{1/2}$ with $n = 2$, 3, 4, 5, 6, and bulk (∞). The obtained central values are $\gamma(1) \approx 2.982$ for the Ising model and $\gamma(2) \approx 2.92$ for the XY model. These values are close to those obtained by the Monte Carlo simulations.[30−35] On the other hand,

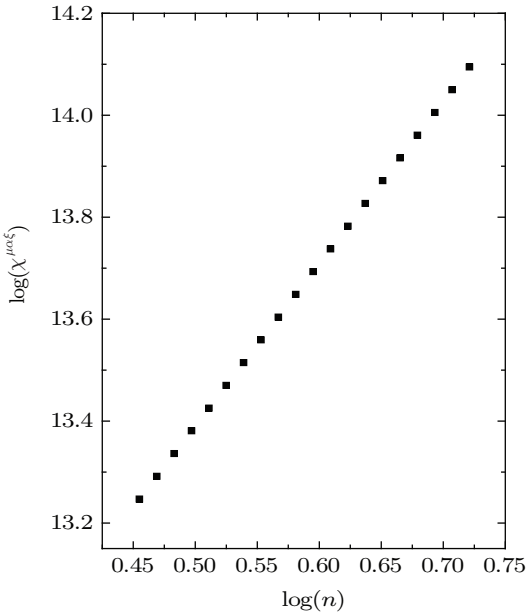


Fig. 4. The $\log(\chi^{\max})$ versus $\log(n)$ for the Ising model with a bcc lattice with a spin of $S = 3/2$.

the critical exponent γ related to the magnetic susceptibility by $\chi_n^{\rm s} \approx (T - T_{\rm N})^{-\gamma}$ is given for different values of the Padé approximant. The obtained values for the Ising and XY models are given in Tables 3 and 4, respectively.

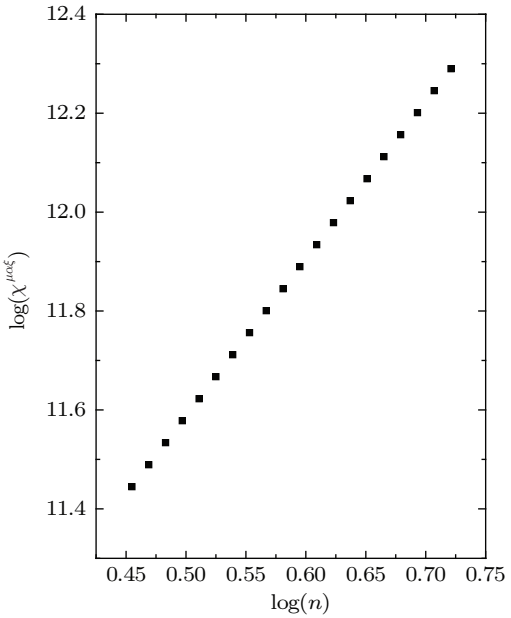


Fig. 5. Maximum sublattice susceptibility $\chi^{\max}$ versus $\log(n)$ in the log–log scale, for the XY model with a bcc lattice with a spin of $S = 3/2$.

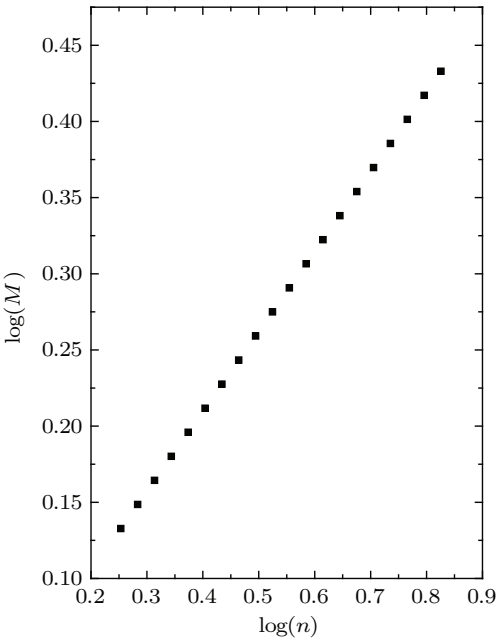


Fig. 6. Magnetization versus $\log(n)$ for the Ising model with $S = 3/2$.

The critical exponent ratio γ/ν is obtained by the slope of the straight line connecting the data points of each layer. By using the scaling law (maximal sublattice susceptibility) $\chi^{\max} \propto n^{\gamma/\nu}$ we plot $\ln(\chi^{\max})$ versus $\ln(n)$ in Figs. 4 and 5, respectively, for the Ising and the XY model for each layer. The obtained values

of γ/ν for the Ising and XY models are 3.183 ± 0.333 and 3.174 ± 0.323, respectively. These ratios are independent of the model used.

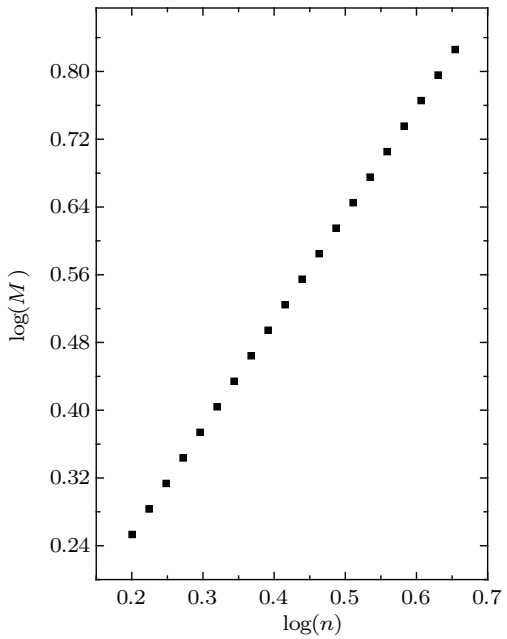


Fig. 7. Magnetization versus $\log(n)$ for the XY model with $S = 3/2$.

From the expression of magnetization $M \propto n^{1/\nu}$, we determinate the variation of M versus n-layer (see Figs. 6 and 7 for Ising and XY models, respectively).

4. Conclusions

The magnetic properties of spin-3/2 antiferromagnetic XY and Ising (I) models on an n-layers body-centred-cubic lattice films for free-surface boundary conditions are studied by using the high-temperature series expansion of magnetic susceptibility and mean-field theory. We give the dependences of the Néel temperature on the number of layers with $n = 2$, 3, 4, 5, 6, and bulk. The Néel temperatures $T_{\mathrm{N}}(n)$ depends on the studied model ($T_{\mathrm{N}}(1) > T_{\mathrm{N}}(2)$). The obtained values of the critical temperature by HTSEs are comparable to those obtained by the mean-field theory for two models with a spin of $S = 3/2$ (see Tables 3 and 4). Néel temperatures $T_{\mathrm{N}}(n)$ for the n-layer films are estimated from the divergence of the staggered susceptibility with exponents being $\gamma(1) \approx 2.982$ for the Ising model and $\gamma(2) \approx 2.92$ for the XY model, which is consistent with the basic assumptions of scaling laws. Our estimates for the shift exponent of the critical temperature for the $XY$and Ising (I) models are $\nu(1)_{\mathrm{mean}} \approx 0.90$ and $\nu(2)_{\mathrm{mean}} \approx 0.65$, respectively. These results do not belong to the same universality class. However, the uncertainties in our numerical estimates, allowing for the scatter of the results in the ratio and Padé approximant analysis, leave small discrepancy between our estimates and those from the Monte Carlo and high-temperature series. The lack of accuracy in the values of the critical temperatures and the limited number of layers due to the shortness of the series lead to uncertainties in the estimates of the shift exponent of the critical temperature $\lambda = 1/\nu$. We conclude that these exponents γ and ν are dependent on the studied model and increase when crossing the surface to bulk. The critical values of γ and ν are independent of each layer.

References

[1] Abid M, Ouahmane H, Lassri H, Khmouo A and Krishnan R 1999 *J. Magn. Magn. Mater.* **202** 335
[2] Custellanos D, Farach H A, Creswick R J and Poole C P Jr 1993 *Phys. Rev.* B **47** 5037
[3] Bengrine M, Benyoussef A, El Kenz A, Loulidi A and Mhirech F 1998 *Phys. Status Solidi* B **208** 207
[4] Huang Z, Feng Q, Chen Z, Chen S and Du Y 2003 *Microeletron. Eng.* **66** 128
[5] Seehra M S and Giebultowicz T M 1988 *Phys. Rev.* B **38** 11898
[6] Kachkachi H, Ezzir A, Nogues M and Tronc E 2000 *Eur. Phys. J.* B **14** 681
[7] Kodama R H and Berkowitz A E 1999 *Phys. Rev.* B **59** 6321
[8] Scott S M and Giebultowicz T M 2002 *J. Appl. Phys.* **91** 6842
[9] Lin D L, Che H and Xia Y 1992 *Phys. Rev.* A **46** 1805
[10] Moutie A, Kerouad M and Laaboudi B 2000 *Chin. J. Phys.* **38** 975
[11] Capehart T W and Fisher M E 1976 *Phys. Rev.* B **13** 5021
[12] Chui S T 1994 *Phys. Rev.* B **50** 12559
[13] Hu X and Kawazoe Y 1994 *Phys. Rev.* B **50** 12647
[14] Landau D P and Binder K 1990 *Phys. Rev.* B **41** 4633
[15] Allan G A T 1970 *Phys. Rev.* B **1** 352
[16] Capehart T W and Fisher M E 1976 *Phys. Rev.* B **13** 5021
[17] Gaunt D S and Guttman A J 1974 *Phase Transitions and Critical Phenomena*, ed. Wortis M (New York: Academic) p. 3
[18] Pan K K 2005 *Phys. Rev.* B **71** 134524
[19] Baker G A and Morris P G (eds.) 1981 *Padé Approximants* (London: Addison-Wesley)
[20] Pan K K 2000 *Phys. Lett.* A **271** 291
[21] Pan K K 1999 *Phys. Rev.* B **59** 1168
[22] Wentworth C and Wang Y L 1987 *Phys. Rev.* B **36** 8687
[23] Wang Y L, Wentworth C and Westwanski B 1985 *Phys. Rev.* B **32** 1805
[24] Stanley E 1974 "D-Vector Model or Universality Hamiltonian: Properties of Isotropically Interacting D-Dimensional Classical Spins" in *Phase Transition and Critical Phenomena*, Vol. 3 p. 520
[25] Belmamoun Y, Ez-Zahraouy H and Kerouad M 2009 *J. Magn. Magn. Mater.* **321** 635
[26] Masrour R, Hamedoun M, Hourmatallah A, Bouslykhane K and Benzakour N 2008 *Phys. Lett.* A **372** 5203

[27] Binder K and Hohenberg P C 1972 *Phys. Rev.* B **6** 3461
[28] Binder K and Hohenberg P C 1974 *Phys. Rev.* B **9** 2194
[29] Landau D P and Pandey R 1989 *Phys. Rev.* B **39** 16
[30] Kosterlitz J M and Thouless D J 1973 *J. Phys.* C **5** L124
[31] George A, Baker Jr, Nickel B G and Meiron D I 1978 *Phys. Rev.* B **17** 1365
[32] Carr C E H Y 1977 *Phys. Rev. Lett.* **39** 1558
[33] Ferrer M, Moore M A and Wortis M 1973 *Phys. Rev.* B **8** 5205
[34] Swendsen R H 1983 *Phys. Rev.* B **27** 391
[35] Wilson K G 1972 *Phys. Rev. Lett.* **28** 548
[36] Ritchie D S and Fisher M E 1973 *Phys. Rev.* B **7** 480
[37] Laosiritaworn Y, Poulter J and Staunten J B 2004 *Phys. Rev.* B **70** 104413
[38] Haubenreisser W, Brodkorb W, Corciovei A and Costache G 1972 *Phys. Status Solidi* B **53** 9
[39] Kaneyoshi T 1990 *J. Magn. Magn. Mater.* **89** L1
[40] Fisher M E 1971 *Critical Phenomena* (New York: Academic)
[41] Barber M N 1983 *Phase Transitions and Critical Phenomena* (New York: Academic) p. 8
[42] Ginzburg V L and Pitaevskii L P 1958 *Sov. Phys.* **7** 858
[43] Kaganov M I and Omelyanchuk A N 1972 *Sov. Phys.* **34** 895
[44] Hamedoun M, Bouslykhane K, Bakrima H, Hourmatallah A and Benzakour N 2005 *Physica* A **358** 102
[45] Masrour R, Hamedoun M and Benyoussef A 2012 *Appl. Surf. Sci.* **258** 1902
[46] Butera P and Comi M 1995 *Phys. Rev.* B **52** 6185
[47] Zuluaga J M and Restrepo J 2006 *Physica* B **384** 224
[48] Cossio P, Zuluaga J M and Restrepo J 2006 *Physica* B **384** 227
[49] Kadanoff L P 2000 *Statistical Physics: Statics, Dynamics and Renormalization* (Singapore: World Scientific)
[50] Acquarone M and Ventura CI 2009 *J. Magn. Magn. Mater.* **321** 2648