Second-harmonic generation in asymmetric quantum dots in the presence of a static magnetic field*

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The second-harmonic generation (SHG) coefficient in an asymmetric quantum dot (QD) with a static magnetic field is theoretically investigated. Within the framework of the effective-mass approximation, we obtain the confined wave functions and energies of electrons in the QD. We also obtain the SHG coefficient by the compact-density-matrix approach and the iterative method. The numerical results for the typical GaAs/AlGaAs QD show that the SHG coefficient depends strongly on the magnitude of magnetic field, parameters of the asymmetric potential and the radius of the QD. The resonant peak shifts with the magnetic field or the radius of the QD changing.

Keywords: quantum dot, nonlinear optical rectification, magnetic field

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1. Introduction

With the development of the fabrication technique of nanostructures, many low-dimensional structures such as quantum wells, quantum well wires, and quantum dots (QDs),[1−8] have been fabricated by using molecular beam epitaxy and etching technique.[9] Due to the discrete and isolated energy levels and the particular optical characters, the nonlinear effects in these structures are much stronger than in the bulk materials.[10−18] These particular properties lead to interesting applications, for instance, the QDs that confine electrons in all three spatial dimensions are extensively applied in optical memories and infrared photodetectors.[19] Li *et al.*[14] investigated the SHG in parabolic QDs in the presence of electric and magnetic fields and the results showed that the the radius of the QD and the magnitude of electric and magnetic fields have a great influence on the SHG coefficient. Atoyan *et al.*[20] investigated direct interband light absorption in a cylindrical QD in a quantizing magnetic field and they found the selection rules corresponding to different transitions between quantum levels. Khordad[21] investigated the effects of magnetic field and geometrical size on the interband light absorption in a quantum pseudodot system and found that the magnetic field and quantum pseudodot size play important roles in the absorption threshold frequency. The SHG in an asymmetric QD in the presence of a static magnetic field is a valuable issue but it has not been studied so far. We investigate it in this paper. Firstly, we derive analytical results for the single-electron system. Secondly, using the obtained energy levels and wave functions, we calculate the SHG in an asymmetric QD in the presence of magnetic fields. From our results, we can find the radius of the QD, the asymmetric potential, and the magnitude of the magnetic field have great influence on the SHG.

The paper is organized as follows: in Section 2, the eigenfunctions and eigenenergies of electron states are attained by using the effective-mass approximation, and the analytical expression for the SHG is derived by means of the compact-density-matrix approach and the iterative method. In Section 3, the numerical results and discussions are presented in typical GaAs/AlGaAs asymmetric QDs. The results show that the SHG depends strongly on the size of the QDs, the static magnetic fields, and the asymmetric confined potential. A brief summary is given in Section 4.

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2. Model and analysis

The motion of an electron in the QD confined by the radial potential of the form $m^*\omega_0^2\rho^2/2$ and an asymmetrical potential of the form $U_0(L/z - z/L)^2$ along z direction in the external static magnetic field can be written by the following equation[22,23]

$$\frac{1}{2m^*}\left(\boldsymbol{P}-\frac{e}{c}\boldsymbol{A}\right)^2\Psi + \left[\frac{1}{2}m^*\omega_0^2\varrho^2 + U_0\left(\frac{L}{z}-\frac{z}{L}\right)^2\right]\Psi = E\Psi, \tag{1}$$

where $\boldsymbol{A}$ is the vector potential of the magnetic field $\boldsymbol{B}$, and m^* is the effective mass of the electron in the conduction band. The Schrödinger equation in cylindrical coordinates has a form

$$\begin{aligned} &-\frac{\hbar^2}{2m^*}\left[\frac{1}{\rho}\frac{\partial}{\partial\varrho}\left(\rho\frac{\partial}{\partial\rho}\right)+\frac{\partial^2}{\partial z^2}+\frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2}\right]\Psi \\ &+\frac{1}{2}\omega_c\hat{l}_z\Psi \\ &+\frac{m^*\omega_c^2\rho^2}{8}\Psi + U_0\left(\frac{L}{z}-\frac{z}{L}\right)^2\Psi \\ &+\frac{1}{2}m^*\omega_0^2\rho^2\Psi = E\Psi, \end{aligned} \tag{2}$$

where $\hat{l}_z$ is the projection of the angular momentum onto the magnetic field direction, $\omega_c = eB/m^*c$ is the cyclotron frequency, $\omega_0 = \hbar/m^*R^2$ is the frequency of electron[24] and $\Omega = \sqrt{\omega_c^2 + 4\omega_0^2}$.

The corresponding eigenfunctions of this system can be written as

$$\Psi = f(\rho,\varphi)\chi(z), \tag{3}$$

$$f(\rho,\varphi) = \frac{1}{a^{1+|m|}}\sqrt{\frac{(|m|+n)!}{2\pi 2^{|m|}n!}}\frac{1}{m!}\exp\left(-\frac{\rho^2}{4a^2}\right) \times \exp(\mathrm{i}m\varphi)\rho^{|m|}\mathrm{F}\left(-n,|m|+1,\frac{\rho^2}{2a^2}\right), \tag{4}$$

$$\chi(z) = C_{n_z}z^{\nu}\exp\left(-\sqrt{\frac{m^*U_0}{2\hbar^2L^2}}\cdot z^2\right) \times \mathrm{F}\left(-n_z,\nu+\frac{1}{2},\sqrt{\frac{m^*U_0}{2\hbar^2L^2}}\cdot z^2\right), \tag{5}$$

where $a = \sqrt{\hbar/m^*\Omega}$ is the effective length scale, $\Omega = \sqrt{\omega_c^2+4\omega_0^2}$, $\mathrm{F}(a,b,x)$ is the confluent hypergeometric function, n is the radial quantum number, m is the magnetic quantum number, C_{n_z} is the normalization constant,

$$\nu = \frac{1}{2}\sqrt{\frac{8m^*U_0L^2}{\hbar^2}+1}+1,$$

and n_z is the quantum number.

The electronic eigenenergies E are given as follows:

$$\begin{aligned} E =\ & \hbar\Omega\left(n+\frac{1+|m|}{2}\right)+\frac{m\hbar\omega_c}{2}+\frac{2\hbar}{L}\sqrt{\frac{2U_0}{m^*}} \\ &\times\left\{n_z+\frac{1}{2}+\frac{1}{4}\left[\sqrt{\frac{8m^*U_0L^2}{\hbar^2}+1}\right.\right. \\ &\left.\left.-\sqrt{\frac{8m^*U_0L^2}{\hbar^2}}\right]\right\}. \end{aligned} \tag{6}$$

Next, the formula of the SHG in the QD can be obtained by the compact-density-matrix method and the iterative procedure.[9,25] Consider the system excited by an incidence light $E(t) = \widetilde{E}\,\mathrm{e}^{-\mathrm{i}\omega t} + \widetilde{E}^*\,\mathrm{e}^{\mathrm{i}\omega t}$. The evolution of the density matrix is given by the time-dependent Schrödinger equation

$$\frac{\partial\rho_{ij}}{\partial t} = \frac{1}{\mathrm{i}\hbar}\left[H_0 - qzE(t),\rho\right]_{ij} - \Gamma_{ij}\left(\rho-\rho^{(0)}\right)_{ij}, \tag{7}$$

where H_0 is the Hamiltonian for this system without the light field $E(t)$, $\rho^{(0)}$ is the unperturbed density matrix, and Γ_{ij} is the relaxation rate. Equation (7) can be calculated by the following iterative method

$$\rho(t) = \sum_n \rho^n(t) \tag{8}$$

with

$$\begin{aligned} \frac{\partial\rho_{ij}^{(n+1)}}{\partial t} =\ & \frac{1}{\mathrm{i}\hbar}\left\{\left[H_0,\rho^{(n+1)}\right]_{ij} - \mathrm{i}\hbar\Gamma_{ij}\rho_{ij}^{(n+1)}\right\} \\ &-\frac{1}{\mathrm{i}\hbar}\left[qz,\rho^{(n)}\right]_{ij}E(t). \end{aligned} \tag{9}$$

The electric polarization of the QD due to $E(t)$ can be expressed as

$$\begin{aligned} P(t) =\ & (\varepsilon_0\chi^{(1)}\widetilde{E}\,\mathrm{e}^{-\mathrm{i}\omega t} + \varepsilon_0\chi_0^{(2)}|\widetilde{E}|^2 \\ &+\varepsilon_0\chi_{2\omega}^{(2)}\widetilde{E}^2\,\mathrm{e}^{-\mathrm{i}2\omega t} \\ &+\varepsilon_0\chi_{3\omega}^{(3)}\widetilde{E}^3\,\mathrm{e}^{-\mathrm{i}3\omega t}) + \mathrm{c.c.}, \end{aligned} \tag{10}$$

where $\chi^{(1)}$, $\chi_0^{(2)}$, $\chi_{2\omega}^{(2)}$, and $\chi_{3\omega}^{(3)}$ are the linear susceptibility, optical rectification, second-harmonic generation coefficient, and third-harmonic generation susceptibilities, respectively. ε_0 is the vacuum dielectric constant. The electronic polarization of the n-th order is given as

$$P^{(n)}(t) = \frac{1}{V}\,\mathrm{Tr}\left(\rho^{(n)}ez\right), \tag{11}$$

where V is the volume of the interaction.

Then, the SHG susceptibility per unit volume is given as[24]

$$\chi_{2\omega}^{(2)} = \frac{\mathrm{e}^3\sigma_\nu}{\varepsilon_0\hbar^2}\frac{M_{01}M_{12}M_{20}}{(\omega-\omega_{10}+\mathrm{i}\Gamma_{10})(2\omega-\omega_{20}+\mathrm{i}\Gamma_{20})}, \tag{12}$$

where σ_ν is the density of electrons in the QD, $\omega_{ij} = (E_i - E_j)/\hbar$ is the transition frequency, and $M_{ij} = |\langle\Psi_j|z|\Psi_i\rangle|$ is the off-diagonal matrix element.

The SHG susceptibility has a resonant peak in the energy of two-photon resonance, i.e., $\hbar\omega = \hbar\omega_{10} = \hbar\omega_{21}$, obtained by

$$\chi^{(2)}_{2\omega,\max} = \frac{e^3\sigma_\nu}{\varepsilon_0}\frac{M_{01}M_{12}M_{20}}{(\hbar\Gamma)^2}, \tag{13}$$

where the off-diagonal relaxation rate $\Gamma = \Gamma_{10} = \Gamma_{20}$.

3. Results and discussions

In this section, we will discuss the SHG coefficient $\chi^{(2)}_{2\omega}$ in the typical GaAs/AlGaAs QDs in the presence of a static magnetic field. In calculation we use the following parameters: $m^* = 0.067m_0$ (m_0 is the mass of a free electron), $\Gamma = 5\times10^{12}$ Hz, $\sigma_\nu = 5\times10^{24}$ m^{-3}.[27]

We present the SHG susceptibility $|\chi^{(2)}_{2\omega}|$ as a function of the incident photon energy $\hbar\omega$ for three different values of the static magnetic field $B = 0$, 5, and 10 T, with $L = 2.0$ nm, $R = 8.0$ nm, and $U_0 = 0.1$ eV as illustrated in Fig. 1. From Fig. 1, it can be seen that the magnetic field has a great influence on the SHG susceptibility $|\chi^{(2)}_{2\omega}|$, and three resonant peaks occur at $\hbar\omega = 0.0356$ eV, 0.0366 eV, and 0.0395 eV, respectively. The resonant peaks will move to the higher energy regions, and the smaller the magnitude of peak will be, as magnetic field B is increasing. The results clearly show that the magnetic field can not only influence the magnitude of the SHG susceptibility $|\chi^{(2)}_{2\omega}|$ but also shift the resonant position.

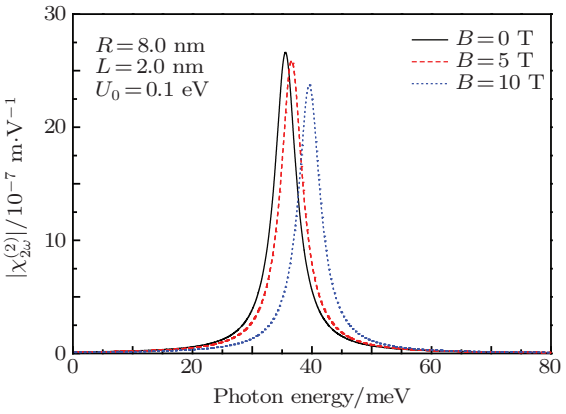


Fig. 1. (colour online) The SHG susceptibility $|\chi^{(2)}_{2\omega}|$ as a function of incident photon energy $\hbar\omega$ for three different values of B, $B = 0$ T, $B = 5$ T, $B = 10$ T, with $L = 2.0$ nm, $R = 8.0$ nm, $U_0 = 0.1$ eV.

In Fig. 2, we show the SHG susceptibility $|\chi^{(2)}_{2\omega}|$ as a function of the incident photon energy $\hbar\omega$ with $R = 8.0$ nm, $B = 10$ T, $U_0 = 0.1$ eV for three different values of parameter L, $L = 1.0$ nm, $L = 1.5$ nm, and $L = 2.0$ nm, respectively. We can see that the peak value will visibly enhance with increasing L. Therefore, it is necessary to increase L if one wants to obtain a large SHG susceptibility.

As shown in Fig. 3, we demonstrate the SHG susceptibility $|\chi^{(2)}_{2\omega}|$ as a function of the incident photon energy $\hbar\omega$, with $R = 8.0$ nm, $L = 2.0$ nm, $B = 10$ T for three different values of potential parameters U_0, $U_0 = 0.1$ eV, $U_0 = 0.2$ eV, and $U_0 = 0.3$ eV. We can clearly see that the magnitude of $|\chi^{(2)}_{2\omega}|$ is sensitive to the values of U_0. The value of the resonant peak will reduce as U_0 is decreasing.

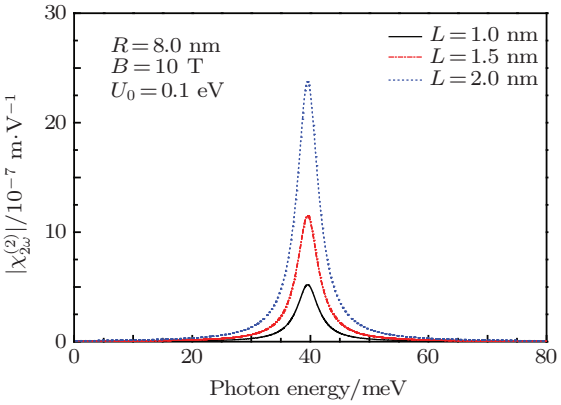


Fig. 2. (colour online) The SHG susceptibility $|\chi^{(2)}_{2\omega}|$ as a function of incident photon energy $\hbar\omega$ for three different values of L, $L = 1.0$ nm, $L = 1.5$ nm, $L = 2.0$ nm, with $R = 8.0$ nm, $B = 10$ T, $U_0 = 0.1$ eV.

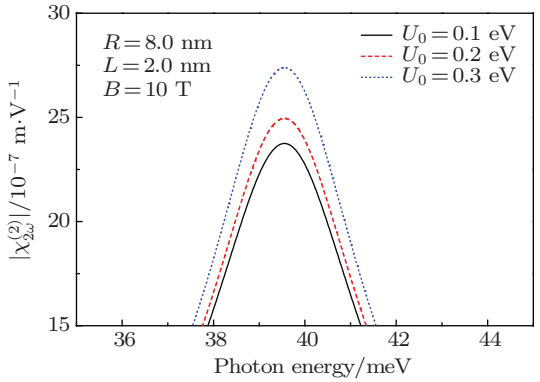


Fig. 3. (colour online) The SHG susceptibility $|\chi^{(2)}_{2\omega}|$ as a function of incident photon energy $\hbar\omega$ for three different values of U_0, $U_0 = 0.1$ eV, $U_0 = 0.2$ eV, $U_0 = 0.3$ eV, with $R = 8.0$ nm, $L = 2.0$ nm, $B = 10$ T.

In Fig. 4, the SHG susceptibility $|\chi^{(2)}_{2\omega}|$ is plotted as a function of the incident photon energy $\hbar\omega$

for three different values of radius, $R = 8.0$ nm, $R = 9.0$ nm, and $R = 10.0$ nm, with $L = 2.0$ nm, $B = 10.0$ T, and $U_0 = 0.1$ eV. It can be easily seen from Fig. 4 that R of the QD has an important influence on the SHG susceptibility $|\chi^{(2)}_{2\omega}|$, and three resonant peaks occur at $\hbar\omega = 0.0397$ eV, 0.0331 eV, 0.0285 eV, respectively. As R increases, the resonant peak will move to the lower-energy regions, and the smaller the magnitudes of $|\chi^{(2)}_{2\omega}|$ will be. Because of the fact that the quantum confinement effects will result in the separation of energy levels, the stronger the confinement effects are, the broader the separation will be. The resonant position shifts to the higher energy with decreasing R, which indicates a strong confinement-induced blueshift in the QD. The SHG susceptibilities $|\chi^{(2)}_{2\omega}|$ we have obtained are larger than those in the previous work.[14]

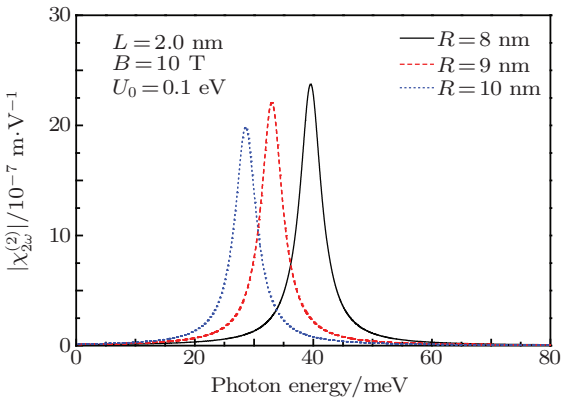


Fig. 4. (colour online) The dependence of the SHG susceptibility $|\chi^{(2)}_{2\omega}|$ on the incident photon energy $\hbar\omega$ for $R = 8$ nm, 9 nm, and 10 nm with $L = 2.0$ nm, $B = 10.0$ T, $U_0 = 0.1$ eV.

4. Summary

We present a study of the SHG susceptibility $|\chi^{(2)}_{2\omega}|$ in detail for a GaAs/AlGaAs asymmetric QD. The results show that the SHG susceptibility is not only affected by the size of the QD, but also related to the parameters of the asymmetric potential and the magnitude of magnetic field. To obtain the larger values of the SHG susceptibility, we should select reasonable values of the QDs' sizes, the parameters of the asymmetric potential, and the magnitude of magnetic field. Moreover, the redshifts occur when the sizes of the QDs increase. Interestingly, the blueshifts of resonant position can be found when the magnetic field increases. We hope that our theoretical results may have some influences on experimental study for optical devices.

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