Response of thermal source in a transversely isotropic thermoelastic half-space with mass diffusion by using a finite element method

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The two-dimensional problem of generalized thermoelastic diffusion material with thermal and diffusion relaxation times is investigated in the context of the Lord–Shulman theory. As an application of the problem, a particular type of thermal source is considered and the problem is solved numerically by using a finite element method. The components of displacement, stress, temperature distribution, chemical potential, and mass concentration are obtained. The resulting quantities are depicted graphically for a special model. An appreciable effect of relaxation times is observed on various resulting quantities.

Keywords: thermoelastic diffusion, thermal source, finite element method, relaxation time

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1. Introduction

The generalized theory of thermoelasticity is one of the modified versions of the classical uncoupled and coupled theory of thermoelasticity and has been developed in order to remove the paradox of the physically impossible phenomena of infinite velocity of thermal signals in the classical coupled thermoelasticity.

Lord and Shulman[1] formulated a generalized theory of thermoelasticity with one thermal relaxation time, who obtained a wave-type equation by postulating a new law of heat conduction instead of classical Fourier's law. Green and Lindsay[2] developed a temperature rate-dependent thermoelasticity that includes two thermal relaxation times and does not violate the classical Fourier's law of heat conduction, when the body under consideration has a center of symmetry. One can refer to Hetnarski and Ignaczak[3] for a review and presentation of generalized theories of thermoelasticity.

Diffusion can be defined as the movement of particles from an area of high concentration to an area of lower concentration until equilibrium is reached. It occurs as a result of the second law of thermodynamic, which states that the entropy or disorder of any system must always increase with time. Diffusion is important in many life processes. Nowadays, there is a great deal of interest in the study of this phenomenon, due to its many applications in geophysics and industrial applications. In integrated circuit fabrication, diffusion is used to introduce "dopants" in controlled amounts into semiconductor substrate. In particular, diffusion is used to form the base and emitter in bipolar transistors from, intregrated resistors from the source/drain regions in metal–oxide semiconductor (MOS) transistors. In most applications, the concentration is calculated using Fick's law. This is a simple law which does not take into consideration the mutual interaction between introducing the substance and the medium into which it is introduced. Study of this phenomenon diffusion is used to improve the conditions of oil extractions. These days, oil companies are interested in the thermoelastic diffusion process for more efficient extraction of oil from oil deposits.

Until recently, thermodiffusion in solids, especially in metals, was considered as a quantity that is independent of body deformation. Practice however indicates that the process of thermodiffusion could have a very considerable influence on the deformation

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of the body. Thermodiffusion in an elastic solid is due to the coupling of temperature, mass diffusion, and strain in addition to the exchange of heat and mass with the environment.

Nowacki[4−7] developed the theory of thermoelastic diffusion by using a coupled thermoelastic model. This implies an infinite speed of propagation of thermoelastic waves. Dudziak and Kowalski,[8] and Olesiak and Pyryev,[9] investigated the theory of thermoelastic diffusion and coupled quasi-stationary problems of thermal diffusion for an elastic layer. They studied the influence of cross effects arising from the coupling of the fields of temperature, mass diffusion, and strain due to which the thermal excitation results in an additional mass concentration and generates additional fields of temperature.

Sherief *et al.*[10] developed the generalized theory of thermoelastic diffusion with one relaxation time which allows finite speeds of propagation of waves. Sherief and Saleh[11] investigated the problem of a thermoelastic half space in the context of the theory of generalized thermoelastic diffusion with one relaxation time. Singh[12] discussed the reflection phenomena of waves from free surface of an thermoelastic diffusion with one relaxation time and with two relaxation times in Ref. [13]. Aouadi[14−16] investigated different problems in thermoelastic diffusion. Sharma[17,18] discussed the plane harmonic generalized thermoelastic diffusive waves and elasto-thermodiffusive surface waves in heat conducting solids. Kumar and Kansal[19] derived the basic equations for generalized thermoelastic diffusion (GL model) and discussed the Lamb waves. Recently Kumar and Chawla[20] discussed the plane-wave propagation in an anisotropic three-phase-lag and two-phase-lag model. Kumar and Chawla[21] discussed the fundamental solution in an orthotropic thermoelastic diffusion medium.

The exact solution of the governing equations of the generalized thermodiffusion elastic theory for a coupled and nonlinear/linear system exists only for very special and simple initial and boundary problem. A numerical solution technique is used to calculate the solution of general problems. For this reason the finite element method is chosen.

The finite element method is a powerful technique originally developed for the numerical solution of complex problems in structural mechanics, and it remains the method of choice for complex systems. A further benefit of this method is that it allows physical effects to be visualized and quantified regardless of experimental limitations. Abbas *et al.*[22−25] and Othman *et el.*[26] successfully tge applied finite element method to generalized thermoelastic material.

Keeping in mind these applications, a general finite element model is proposed to study the thermal interaction in transversely isotropic thermoelastic diffusion media in the context of Lord and Shulman's theory. Numerical results for components of displacement, stress, temperature distribution, chemical potential, and mass diffusion are given and illustrated graphically.

2. Basic equations

Following Sherief *et al.*,[11] the basic governing equations for an anisotropic homogeneous generalized thermoelastic diffusion (Lord–Shulman theory) solid in the absence of body forces, heat, and mass diffusion sources are as follows.

Constitutive relations

$$\sigma_{ij} = c_{ijkm}e_{km} + a_{ij}T + b_{ij}C, \tag{1}$$

$$P = b_{km}e_{km} + bC - aT. \tag{2}$$

Equations of motion

$$c_{ijkm}e_{km,j} + a_{ij}T_{,j} + b_{ij}C_{,j} = \rho\ddot{u}_i. \tag{3}$$

Equation of heat conduction

$$\left(1 + \tau_0\frac{\partial}{\partial t}\right)(\rho C_E\dot{T} + aT_0\dot{C} - a_{ij}T_0\dot{e}_{ij}) = K_{ij}T_{,ij}. \tag{4}$$

Equation of mass diffusion

$$-\alpha^*_{ij}b_{km}e_{km,ij} - \alpha^*_{ij}bC_{,ij} + \alpha^*_{ij}aT_{,ij} = -\left(1 + \tau^0\frac{\partial}{\partial t}\right)\dot{C}. \tag{5}$$

Here, $c_{ijkm}(= c_{kmij} = c_{jikm} = c_{ijmk})$ are elastic parameter; $a_{ij}(= a_{ji})$, $b_{ij}(= b_{ji})$ are, respectively, the tensor of thermal and diffusion moduli. ρ is the density and C_E is the specific heat at constant strain, a, b are, respectively, the coefficients describing the measure of thermoelastic diffusion effects and of diffusion effects, T_0 is the reference temperature assumed to be such that $|T/T_0| \ll 1$. $K_{ij}(= K_{ji})$, $\sigma_{ij}(= \sigma_{ji})$, and $\varepsilon_{ij} = (u_{i,j} + u_{j,i})/2$ denote the components of thermal conductivity, stress, and strain tensor, respectively. $T(x, y, z, t)$ is the temperature change from the reference temperature T_0 and C is the mass concentration. u_i are components of displacement vector. $\alpha^*_{ij}(= \alpha^*_{ji})$ are diffusion parameters.

In the above equations symbol ("$\prime$") followed by a suffix denotes differentiation with respect to spatial coordinate and a superposed dot (".") denotes the derivative with respect to time.

3. Formulation of the problem

We consider a homogeneous, transversely isotropic generalized thermoelastic diffusion half-space ($x \geqslant 0$) initially at uniform temperature T_0. All the considered function will depend on time t and coordinates x and z. Thus we assume the displacement vector, temperature distribution, and mass concentration are, respectively, of the form

$$\boldsymbol{u} = (u, 0, w), T(x, z, t), C(x, z, t). \tag{6}$$

We define the dimensionless quantities

$$x' = \frac{\omega_1^* x}{v_1}, \quad z' = \frac{\omega_1^* z}{v_1}, \quad t' = \omega_1^* t, \quad u' = \frac{\omega_1^* u}{v_1},$$
$$w' = \frac{\omega_1^* w}{v_1}, \quad T' = \frac{a_1 T}{\rho v_1^2}, \quad C' = \frac{b_1 C}{\rho v_1^2},$$
$$\tau_0' = \omega_1^* \tau_0, \quad \tau^{0\prime} = \omega_1^* \tau^0, \quad P' = \frac{P}{b_1}, \quad \sigma_{ij}' = \frac{\sigma_{ij}}{a_1 \mathrm{T}_0},$$

where

$$v_1^2 = \frac{c_{11}}{\rho}, \quad \omega_1^* = \frac{\rho C_E v_1^2}{K_1}. \tag{7}$$

Applying the transformation

$$x' = x\cos\phi + y\sin\phi, \quad y' = -x\sin\phi + y\cos\phi,$$
$$z' = z, \tag{8}$$

where ϕ is the angle of rotation of x–y plane, in Eqs. (1)–(5), applying the dimensionless quantities defined by Eq. (7) and with the aid of Eq. (6), the basic equations for a transversely isotropic generalized thermoelastic diffusion elastic solid are

$$\sigma_{xx} = \bar{c}_{11}\left[\frac{\partial u}{\partial x} + (\delta_2 - \delta_1)\frac{\partial w}{\partial z} - T - C\right], \tag{9a}$$
$$\sigma_{zz} = \bar{c}_{11}\left[(\delta_2 - \delta_1)\frac{\partial u}{\partial x} + \delta_3\frac{\partial w}{\partial z} - \chi_1 T - \chi_2 C\right], \tag{9b}$$
$$\sigma_{xz} = \delta_1 \bar{c}_{11}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right), \tag{9c}$$
$$P = -(u_{1,1} + \chi_4 u_{3,3}) + \chi_5 C - \chi_6 T, \tag{9d}$$

$$\frac{\partial^2 u}{\partial x^2} + \delta_1\frac{\partial^2 u}{\partial z^2} + \delta_2\frac{\partial^2 w}{\partial x z} - \frac{\partial T}{\partial x} - \frac{\partial C}{\partial x} = \frac{\partial^2 u}{\partial t^2}, \tag{10}$$
$$\delta_2\frac{\partial^2 u}{\partial x z} + \delta_1\frac{\partial^2 w}{\partial x^2} + \delta_3\frac{\partial^2 w}{\partial z^2} - \chi_1\frac{\partial T}{\partial z} - \chi_2\frac{\partial C}{\partial z} = \frac{\partial^2 w}{\partial t^2}, \tag{11}$$
$$\left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)\left[T + \varsigma_1 C + \varsigma_2\left(\frac{\partial u}{\partial x} + \chi_1\frac{\partial w}{\partial z}\right)\right] = \frac{\partial^2 T}{\partial x^2} + \chi_3\frac{\partial^2 T}{\partial z^2}, \tag{12}$$
$$q_1^*\frac{\partial^3 u}{\partial x^3} + q_2^*\frac{\partial^3 w}{\partial z \partial x^2} + q_7^*\frac{\partial^3 u}{\partial x \partial z^2} + q_8^*\frac{\partial^3 w}{\partial z^3} - q_5^*\frac{\partial^2 C}{\partial x^2} - q_6^*\frac{\partial^2 C}{\partial z^2} + q_3^*\frac{\partial^2 T}{\partial x^2} + q_4^*\frac{\partial^2 T}{\partial z^2} = -\left(\frac{\partial}{\partial t} + \tau^0\frac{\partial^2}{\partial t^2}\right)C, \tag{13}$$

where $a_{ij} = -a_i\delta_{ij}$, $a_{ji} = -b_i\delta_{ij}$, $\alpha_{1j}^* = \alpha_i^*\delta_{ij}$, $K_{ij} = K_i\delta_{ij}$, (i is not summed)

$$a_1 = (c_{11} + c_{12})\alpha_1 + c_{13}\alpha_3, \quad a_3 = 2c_{13}\alpha_1 + c_{33}\alpha_3,$$
$$b_1 = (c_{11} + c_{12})\alpha_{1c} + c_{13}\alpha_{3c}, \quad b_3 = 2c_{13}\alpha_{1c} + c_{33}\alpha_{3c},$$

and

$$(\delta_1, \delta_2, \delta_3) = \frac{1}{c_{11}}(c_{44}, c_{13} + c_{44}, c_{33}),$$
$$\chi_1 = \frac{a_3}{a_1}, \quad \chi_2 = \frac{b_3}{b_1}, \quad \chi_3 = \frac{K_3}{K_1},$$
$$(\chi_4,\ \chi_5,\ \chi_6) = \frac{1}{b_1}\left(b_3, \frac{bc_{11}}{b_1}, \frac{ac_{11}}{a_1}\right), \quad \bar{c}_{11} = \frac{c_{11}}{a_1 T_0},$$
$$(q_1^*, q_2^*, q_7^*, q_8^*) = \frac{1}{\rho v_1^4}(\alpha_1^* b_1^2\omega_1^*, \alpha_3^* b_1 b_3\omega_1^*, \alpha_3^* b_1^2\omega_1^*, b_1 b_3\omega_1^*\alpha_1^*),$$
$$(q_3^*, q_4^*) = \frac{1}{a_1 v_1^2}(\alpha_1^* b_1 a\omega_1^*, \alpha_3^* b_1 a\omega_1^*),$$
$$(q_5^*, q_6^*) = \frac{1}{v_1^2}(\alpha_1^* b\omega_1^*, \alpha_3^* b\omega_1^*),$$
$$(\varsigma_1, \varsigma_2) = \frac{a_1 T_0}{K_1\omega_1^*}\left(\frac{av_1^2}{b_1}, \frac{a_1}{\rho}\right).$$

4. Initial and boundary condition

The above equations are solved subjected to initial conditions

$$u = w = T = C = 0, \quad \dot{u} = \dot{w} = \dot{T} = \dot{C} = 0, \quad t = 0.$$

The boundary conditions for the problem may be taken as

$$T(0, z, t) = T_0 H(t) H(2l - |z|),$$
$$\sigma_{xx}(0, z, t) = 0, \sigma_{xz}(0, z, t) = 0, \quad P(0, z, t) = 0,$$

where H is the Heaviside unit step and T_1 is the constant temperature applied on the boundary.

5. Finite element formulation

In this section, the governing equations of generalized thermoelastic diffusion with relaxation times (Lord and Shulman's theory) are summarized, followed by the corresponding finite element equations. In the finite element method, the displacement components u, w, temperature T, and mass concentration C are related to the corresponding nodal values by

$$u=\sum_{i=1}^{m}N_iu_i(t),\quad w=\sum_{i=1}^{m}N_iw_i(t),$$
$$T=\sum_{i=1}^{m}N_iT_i(t),\quad C=\sum_{i=1}^{m}N_iC_i(t),\qquad(14)$$

where m denotes the number of nodes per element, and N_i are the shape functions. The eight-node isoparametric, quadrilateral element is used for displacement components, temperature, and mass concentration calculations. The weighting functions and the shape functions coincide. Thus,

$$\delta u=\sum_{i=1}^{m}N_i\delta u_i,\quad \delta w=\sum_{i=1}^{m}N_i\delta w_i,$$
$$\delta T=\sum_{i=1}^{m}N_i\delta T_i,\quad \delta C=\sum_{i=1}^{m}N_i\delta C_i.\qquad(15)$$

It should be noted that appropriate boundary conditions associated with the governing equations (10)–(13) must be adopted in order to properly formulate a problem. Boundary conditions are either essential (or geometric) or natural (or traction) types. Essential conditions are prescribed displacements u, w, temperature T, and mass concentration C while, the natural boundary conditions are prescribed tractions, heat flux, and mass flux which are expressed as

$$\sigma_{xx}n_x+\sigma_{xz}n_z=\bar{\tau}_x,\quad \sigma_{xz}n_x+\sigma_{zz}n_z=\bar{\tau}_z,$$
$$q_xn_x+q_zn_z=\bar{q},\quad \eta_xn_x+\eta_zn_z=\bar{\eta},\qquad(16)$$

where n_x and n_z are direction cosines of the outward unit normal vector at the boundary, $\bar{\tau}_x$, $\bar{\tau}_z$ are the given traction values, $\bar{q}$ is the given surface heat flux and $\bar{\eta}$ is the given surface mass flux. In the absence of body force, the governing equations are multiplied by weighting functions and then are integrated over the spatial domain Ω with the boundary Γ. Applying integration by parts and making use of the divergence theorem reduce the order of the spatial derivatives and allows for the application of the boundary conditions. Thus, the finite element equations corresponding to Eqs. (10)–(13) can be obtained as

$$\sum_{e=1}^{m}\left(\begin{bmatrix}M_{11}^e&0&0&0\\0&M_{22}^e&0&0\\M_{31}^e&M_{32}^e&M_{33}^e&M_{34}^e\\0&0&0&M_{44}^e\end{bmatrix}\begin{Bmatrix}\ddot{u}^e\\\ddot{w}^e\\\ddot{T}^e\\\ddot{C}^e\end{Bmatrix}+\begin{bmatrix}0&0&0&0\\0&0&0&0\\R_{31}^e&R_{32}^e&R_{33}^e&R_{34}^e\\0&0&0&R_{44}^e\end{bmatrix}\begin{Bmatrix}\dot{u}^e\\\dot{w}^e\\\dot{T}^e\\\dot{C}^e\end{Bmatrix}\right.$$
$$\left.+\begin{bmatrix}K_{11}^e&K_{12}^e&K_{13}^e&K_{14}^e\\K_{21}^e&K_{22}^e&K_{23}^e&K_{24}^e\\0&0&K_{33}^e&0\\K_{41}^e&K_{42}^e&K_{43}^e&K_{44}^e\end{bmatrix}\begin{Bmatrix}u^e\\w^e\\T^e\\C^e\end{Bmatrix}=\begin{Bmatrix}F_1^e\\F_2^e\\F_3^e\\F_4^e\end{Bmatrix},\right)\qquad(17)$$

where the coefficients in Eq. (17) are given below

$$M_{11}^e=\int_\Omega[N]^{\mathrm{T}}[N]\,\mathrm{d}\Omega,\quad M_{22}^e=\int_\Omega[N]^{\mathrm{T}}[N]\,\mathrm{d}\Omega,$$
$$M_{31}^e=\int_\Omega\zeta_2\tau_0[N]^{\mathrm{T}}\left[\frac{\partial N}{\partial x}\right]\mathrm{d}\Omega,$$
$$M_{32}^e=\int_\Omega\zeta_2\chi_1\tau_0[N]^{\mathrm{T}}\left[\frac{\partial N}{\partial z}\right]\mathrm{d}\Omega,$$
$$M_{33}^e=\int_\Omega\tau_0[N]^{\mathrm{T}}[N]\,\mathrm{d}\Omega,$$
$$M_{34}^e=\int_\Omega\zeta_1\tau_0[N]^{\mathrm{T}}[N]\,\mathrm{d}\Omega,$$
$$M_{44}^e=\int_\Omega\tau^0[N]^{\mathrm{T}}[N]\,\mathrm{d}\Omega,$$
$$R_{31}^e=\int_\Omega\zeta_2[N]^{\mathrm{T}}\left[\frac{\partial N}{\partial x}\right]\mathrm{d}\Omega,$$
$$R_{32}^e=\int_\Omega\zeta_2\chi_1[N]^{\mathrm{T}}\left[\frac{\partial N}{\partial z}\right]\mathrm{d}\Omega,$$
$$R_{33}^e=\int_\Omega[N]^{\mathrm{T}}[N]\,\mathrm{d}\Omega,\quad R_{34}^e=\int_\Omega\zeta_1[N]^{\mathrm{T}}[N]\,\mathrm{d}\Omega,$$
$$R_{44}^e=\int_\Omega[N]^{\mathrm{T}}[N]\,\mathrm{d}\Omega,$$
$$K_{11}^e=\int_\Omega\left(\left[\frac{\partial N}{\partial x}\right]^{\mathrm{T}}\left[\frac{\partial N}{\partial x}\right]\right.$$

$$+\delta_1\left[\frac{\partial N}{\partial z}\right]^{\mathrm{T}}\left[\frac{\partial N}{\partial z}\right]\Big)\,\mathrm{d}\Omega,$$

$$K^e_{12}=\int_\Omega\Big((\delta_2-\delta_1)\left[\frac{\partial N}{\partial x}\right]^{\mathrm{T}}\left[\frac{\partial N}{\partial z}\right]+\delta_1\left[\frac{\partial N}{\partial z}\right]^{\mathrm{T}}\left[\frac{\partial N}{\partial x}\right]\Big)\,\mathrm{d}\Omega,$$

$$K^e_{13}=-\int_\Omega\left[\frac{\partial N}{\partial x}\right]^{\mathrm{T}}[N]\,\mathrm{d}\Omega,$$

$$K^e_{14}=-\int_\Omega\left[\frac{\partial N}{\partial x}\right]^{\mathrm{T}}[N]\,\mathrm{d}\Omega,$$

$$K^e_{21}=\int_\Omega\Big(\delta_1\left[\frac{\partial N}{\partial x}\right]^{\mathrm{T}}\left[\frac{\partial N}{\partial z}\right]+(\delta_2-\delta_1)\left[\frac{\partial N}{\partial z}\right]^{\mathrm{T}}\left[\frac{\partial N}{\partial x}\right]\Big)\,\mathrm{d}\Omega,$$

$$K^e_{22}=\int_\Omega\Big(\delta_1\left[\frac{\partial N}{\partial x}\right]^{\mathrm{T}}\left[\frac{\partial N}{\partial x}\right]+\delta_3\left[\frac{\partial N}{\partial z}\right]^{\mathrm{T}}\left[\frac{\partial N}{\partial z}\right]\Big)\,\mathrm{d}\Omega,$$

$$K^e_{23}=-\int_\Omega\chi_1\left[\frac{\partial N}{\partial z}\right]^{\mathrm{T}}[N]\,\mathrm{d}\Omega,$$

$$K^e_{24}=-\int_\Omega\chi_2\left[\frac{\partial N}{\partial z}\right]^{\mathrm{T}}[N]\,\mathrm{d}\Omega,$$

$$K^e_{33}=\int_\Omega\left(\left[\frac{\partial N}{\partial x}\right]^{\mathrm{T}}\left[\frac{\partial N}{\partial x}\right]+\chi_3\left[\frac{\partial N}{\partial z}\right]^{\mathrm{T}}\left[\frac{\partial N}{\partial z}\right]\right)\mathrm{d}\Omega,$$

$$K^e_{41}=\int_\Omega\Big(q_1^*\left[\frac{\partial N}{\partial x}\right]^{\mathrm{T}}\left[\frac{\partial^2 N}{\partial x^2}\right]+q_7^*\left[\frac{\partial N}{\partial z}\right]^{\mathrm{T}}\left[\frac{\partial^2 N}{\partial x\partial z}\right]\Big)\,\mathrm{d}\Omega,$$

$$K^e_{42}=\int_\Omega\Big(q_2^*\left[\frac{\partial N}{\partial x}\right]^{\mathrm{T}}\left[\frac{\partial^2 N}{\partial x\partial z}\right]+q_8^*\left[\frac{\partial N}{\partial z}\right]^{\mathrm{T}}\left[\frac{\partial^2 N}{\partial z^2}\right]\Big)\,\mathrm{d}\Omega,$$

$$K^e_{43}=\int_\Omega\Big(q_3^*\left[\frac{\partial N}{\partial x}\right]^{\mathrm{T}}\left[\frac{\partial N}{\partial x}\right]+q_4^*\left[\frac{\partial N}{\partial z}\right]^{\mathrm{T}}\left[\frac{\partial N}{\partial z}\right]\Big)\,\mathrm{d}\Omega,$$

$$K^e_{44}=-\int_\Omega\Big(q_5^*\left[\frac{\partial N}{\partial x}\right]^{\mathrm{T}}\left[\frac{\partial N}{\partial x}\right]+q_6^*\left[\frac{\partial N}{\partial z}\right]^{\mathrm{T}}\left[\frac{\partial N}{\partial z}\right]\Big)\,\mathrm{d}\Omega,$$

$$F^e_1=\int_\Gamma[N]^{\mathrm{T}}\bar{\tau}_x\,\mathrm{d}\Gamma,\quad F^e_2=\int_\Gamma[N]^{\mathrm{T}}\bar{\tau}_y\,\mathrm{d}\Gamma,$$

$$F^e_3=\int_\Gamma[N]^{\mathrm{T}}\bar{q}\,\mathrm{d}\Gamma,\quad F^e_4=\int_\Gamma[N]^{\mathrm{T}}\bar{\eta}\,\mathrm{d}\Gamma.$$

Symbolically, the discretized equations of Eq. (17) can be written as

$$M\ddot{d}+R\dot{d}+Kd=F^{\mathrm{ext}},\tag{18}$$

where M, R, K, and F^{ext} represent the mass, damping, stiffness matrices, and external force vectors, respectively; $d=[u\ w\ TC]^{\mathrm{T}}$. On the other hand, the time derivatives of the unknown variables have to be determined by the Newmark time integration method (see Ref. [27]).

6. Numerical results and discussion

For the numerical computation, we take the following values of relevant parameters as $c_{11}=18.78\times10^{10}$ kg·m^{-1}·s^{-2}, $c_{12}=8.76\times10^{10}$ kg·m^{-1}·s^{-2}, $c_{13}=8.0\times10^{10}$ kg·m^{-1}·s^{-2}, $c_{33}=18.2\times10^{10}$ kg·m^{-1}·s^{-2}, $c_{44}=5.06\times10^{10}$ kg·m^{-1}·s^{-2}, $T_0=0.293\times10^3$ K, $\alpha_1=2.98\times10^{-5}$ K^{-1}, $\alpha_3=2.4\times10^{-5}$ K^{-1}, $\alpha_{1c}=2.1\times10^{-4}$ m^3, $\alpha_{3c}=2.5\times10^{-4}$ m^3·kg^{-1}, $a=2.4\times10^4$ m^2·s^{-2}·K^{-1}, $b=13.2\times10^5$ m^5·s^{-2}·kg^{-1}, $\alpha_1^*=0.95\times10^{-8}$ kg·s·m^{-3}, $\alpha_3^*=1.25\times10^{-8}$ kg·s·m^{-3}, $\rho=8.954\times10^3$ kg·m^{-3}, $K_1=0.433\times10^3$ W·m^{-1}·K^{-1}, $K_3=0.450\times10^3$ W·m^{-1}·K^{-1}, $t=0.3$, $l=0.25$, $T_1=1$, $\tau_0=0.1$, $\tau^0=0.2$, $z=0.2$.

Figures 1–8 exhibit the variation of displacement, temperature distribution, mass concentration, stress, and chemical potential. The solid line corresponds to the coupled thermoelastic (CT) theory and the dotted line corresponds to the Lord–Shulman (LS) theory.

Figure 1 depicts the variations of horizontal displacement (u) with x. The values of u increase for smaller values of x, but for higher values of x, it becomes constant (for both LS and CT theories). It is noticed that the values of u for the LS theory remain higher initially, but for higher values of x the reverse behavior occurs, whereas the values of u for both theories approximately equal at the boundary surface. Figure 2 shows that the variation of normal displacement (w) with x and it indicates that the values of w decrease initially, but for higher values of x, it becomes constant (for both LS and CT theories). It is evident that the values of vertical displacement for CT theory remain more (in comparison with the LS theory).

Figure 3 shows that the variation of temperature distribution (T) with x and it exhibits that the values of T has maximum value at the boundary and then decrease to zero for both theories. It is evident that the values of T in the case of the LS theory remain higher in comparison with the CT theory initially, but

as x increases the reverse behavior occurs and finally the values of T become approximately equal at the boundary surface. Figure 4 depicts the variation of mass concentration with x and it is noticed that the behavior of variation of mass concentration is similar as temperature distribution, although the magnitude values of mass concentration are different.

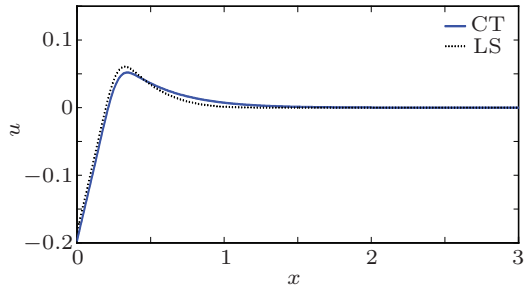


Fig. 1. (colour online) Tangential displacement distribution.

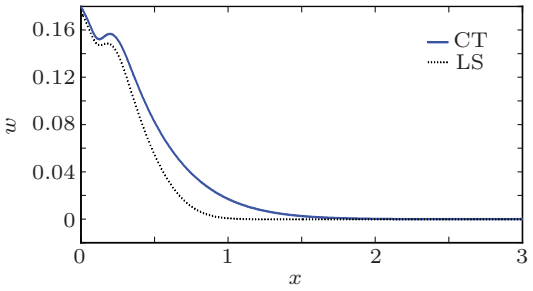


Fig. 2. (colour online) Normal displacement distribution.

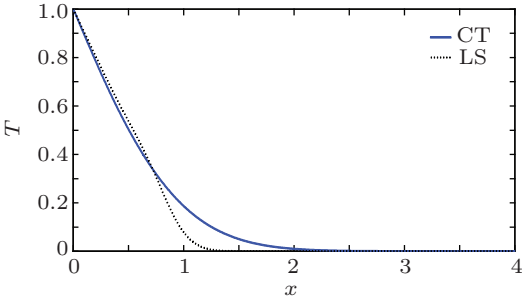


Fig. 3. (colour online) Temperature distribution.

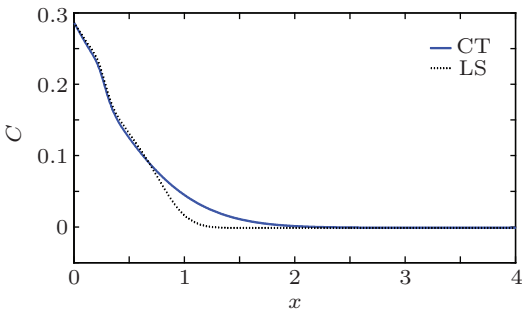


Fig. 4. (colour online) Concentration distribution.

Figure 5 depicts the variation of normal stress components (σ_{xx}) with x. The variation of σ_{xx} remains oscillatory, for smaller values of x, but for higher values of x it becomes stable for both theories. It is evident that for smaller values of x, the values of σ_{xx} in the case of CT theory remain higher, whereas for higher values of x reverse behavior occurs. Figures 6 and 7 depict the variation of tangential stress component (σ_{zx}) and normal stress component (σ_{zz}), respectively. It is evident that the behavior and variation of σ_{zx}, σ_{zz} are similar as σ_{xx} with difference in their magnitude values.

Figure 8 exhibits the variation of chemical potential with x and it indicates that the values of chemical potential decrease for initial values of x and increase for higher values of x whereas it becomes constant at the boundary surface. It is evident that the values of chemical potential in the case of LS theory remain higher, in comparison with GL theory, for higher values of x.

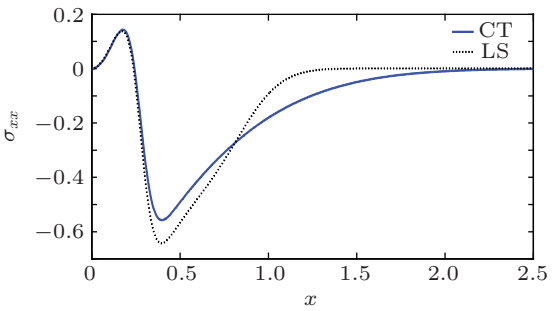


Fig. 5. (colour online) Normal stress σ_{xx} distribution.

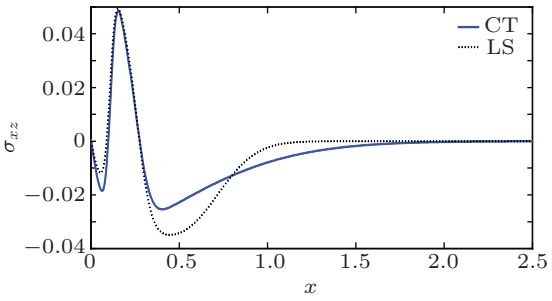


Fig. 6. (colour online) Tangential stress σ_{xz} distribution.

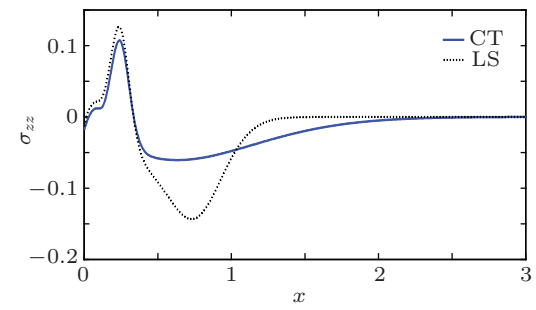


Fig. 7. (colour online) Normal stress σ_{zz} distribution.

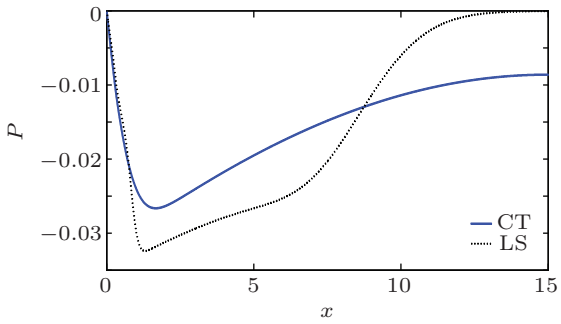


Fig. 8. (colour online) Chemical potential distribution.

7. Conclusion

In this paper we have studied the two-dimensional problem of a generalized thermoelastic diffusion elastic material with thermal and diffusion relaxation times in the context of the Lord–Shulman theory. The problem has been solved numerically by using the finite element method. From the numerically results, we observed that the relaxation time has very small effect on the horizontal displacement and mass concentration while, it has a significant effect on the other field quantities as expected.

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