Distillability sudden death in a two qutrit systems under a thermal reservoir*

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The dynamics of distillability entanglement between qutrit–qutrit systems interacting with a thermal reservoir is investigated in this paper. We discovered an interesting phenomenon that under a thermal reservoir certain initially prepared free-entangled states become bound-entangled states in a finite time, which is called distillability sudden death (DSD). We use a realignment criterion to measure the nine-dimensional density matrix of the entanglement. Moreover, we analyze some other parameters to investigate the effects to the systems. Explanations are then given.

Keywords: distillability entanglement, distillability sudden death, realignment criterion

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1. Introduction

As we know that quantum entanglement, as a key physical resource, plays an essential role in various kinds of quantum information processing, including quantum state teleportation,[1,2] cryptographic key distribution,[3] classical communication over quantum channels,[4−6] quantum error correction,[7] quantum computational speedups,[8] and distributed computation.[9,10] In the development of both theory and experiment, much attention has been attracted to the abrupt disappearance of entanglement, called entanglement sudden death (ESD),[11−16] and it has been tested through experiments.[17] This phenomenon stirred up much attention, and many works have been done. For example, Lopez[18] found that when there is ESD in the main system, there is entanglement sudden birth (ESB) in the environment most of time.

Recently, entanglement of qubit–qubit systems characterized by concurrence[19] and qubit–qutrit systems has been measured by the Peres–Horodecki criterion.[20] Both of the upper systems are lower dimensional bipartite systems. The higher dimensional bipartite systems such as qutrit–qutrit systems are hard to define a criterion for. We divide the bipartite entangled state into a free-entangled state and a bound-entangled state.[21,22] Under local operations and classical communication, the free-entangled state can be distilled but the bound-entangled state cannot. We use the change of distillability to measure the qutrit–qutrit systems, although this is not a perfect method.[23,24] References [23] and [24] showed that the free-entangled states can be converted to bound-entangled states under some quantum processes, this behavior is called DSD. This discovery raises great interest and DSD has been found in many systems. In this paper, we consider the dynamics of a two-qutrit atomic system of V-type. The distillability entanglement phenomenon is discussed and the effect of the other parameters is analyzed, too. Lastly, we give our explanations.

This paper is organized as follows. In Section 2, we build our model and give the differential equation from the density matrix. In Section 3, we discuss DSD and its effect, and give a brief reasonable explanation. Some figures are also in this section. Finally, we draw some conclusions in Section 4.

2. Two-qutrit dynamics

We start with the sketch of the derivation of the dynamical equation in a three-level atom with the V configuration interacted with a common thermal reservoir. We call the two qutrit atoms A and B. Consider the two nondegenerate excited states $|e\rangle$ and $|u\rangle$ with transition frequencies to the ground state $|g\rangle$ repre-

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sented by ω_1 and ω_2, respectively. The electric dipole moments are $\boldsymbol{\mu}_1$ and $\boldsymbol{\mu}_2$. For simplicity, we assume that the excited levels can decay to the ground state by spontaneous emission, whereas a direct transition between excited levels is forbidden. As we know, spontaneous decay constants of the excited states $|i\rangle$, $i = e, u$, are given as

$$\gamma_i = \frac{2|\boldsymbol{\mu}_i|^2}{3\hbar}\left(\frac{\omega_i}{c}\right)^3. \tag{1}$$

If the dipole moments of these two transitions are parallel, then indirect coupling between $|e\rangle$ and $|u\rangle$ can appear due to the interaction with vacuum.[25]

The master equation driven by the time evolution of the system is given as[26]

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\mathrm{i}[H,\rho] + L\rho, \tag{2}$$

where the damping term is

$$\begin{aligned} L\rho = \sum_{i=A,B}\Bigg[&\frac{N+1}{2}\gamma_e(2\sigma_{ge}^i\rho\sigma_{eg}^i - \sigma_{eg}^i\sigma_{ge}^i\rho - \rho\sigma_{eg}^i\sigma_{ge}^i) \\ &+ \frac{N}{2}\gamma_e(2\sigma_{eg}^i\rho\sigma_{ge}^i - \sigma_{ge}^i\sigma_{eg}^i\rho - \rho\sigma_{ge}^i\sigma_{eg}^i) \\ &+ \frac{N+1}{2}\gamma_u(2\sigma_{gu}^i\rho\sigma_{ug}^i - \sigma_{ug}^i\sigma_{gu}^i\rho - \rho\sigma_{ug}^i\sigma_{gu}^i) \\ &+ \frac{N}{2}\gamma_u(2\sigma_{gu}^i\rho\sigma_{ug}^i - \sigma_{ug}^i\sigma_{gu}^i\rho - \rho\sigma_{ug}^i\sigma_{gu}^i)\Bigg], \end{aligned}$$

here

$$\sigma_{ij}^A = \sigma_{ij}^A \otimes I_3^B, \sigma_{ij}^B = I_3^A \otimes \sigma_{ij}^B, \quad i,j = e,u,g, \tag{3}$$

and

$$N = \langle n\rangle = \frac{\mathrm{e}^{-\hbar\omega/kT}}{1-\mathrm{e}^{-\hbar\omega/kT}} \tag{4}$$

is the mean number of photons. k is the Boltzman constant and T is the temperature of the reservoir. The spontaneous emission of atoms A and B from their excited states $|e\rangle$ to their ground states $|g\rangle$ is described by the spontaneous decay rate γ_e, whereas γ_u is the spontaneous decay rate of the excited states $|u\rangle$ to the ground states $|g\rangle$. The atomic transition operator $\sigma_{ij} = |i\rangle\langle j|$ takes an atom from its state $|j\rangle$ to the state $|i\rangle$. We choose the basis $\{|gg\rangle, |ge\rangle, |gu\rangle, |eg\rangle, |ee\rangle, |eu\rangle, |ug\rangle, |ue\rangle, |uu\rangle\}$. The solution of the master equation is tedious and complex, we just give the diagonal matrix element which we will use in this paper

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\rho_{gg,gg} =& \gamma_e[(N+1)\rho_{eg,eg} + (N+1)\rho_{ge,ge} \\ &- 2N\rho_{gg,gg}] + \gamma_u[(N+1)\rho_{gu,gu} \\ &+ (N+1)\rho_{ug,ug} - 2N\rho_{gg,gg}], \end{aligned} \tag{5}$$

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\rho_{ge,ge} =& \gamma_e[(N+1)\rho_{ee,ee} - N\rho_{ge,ge} + N\rho_{gg,gg}] \\ &+ \gamma_u[(N+1)\rho_{ue,ue} - 2N\rho_{ge,ge}], \end{aligned} \tag{6}$$

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\rho_{gu,gu} =& \gamma_e[(N+1)\rho_{eu,eu} - N\rho_{gu,gu}] \\ &+ \gamma_u[(N+1)\rho_{uu,uu} + N\rho_{gg,gg} \\ &- 2N\rho_{gu,gu} - \rho_{gu,gu}], \end{aligned} \tag{7}$$

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\rho_{eg,eg} =& \gamma_e[(N+1)\rho_{ee,ee} + N\rho_{gg,gg} \\ &- 2N\rho_{eg,eg} - \rho_{eg,eg}] \\ &+ \gamma_u[(N+1)\rho_{eu,eu} - N\rho_{eg,eg}], \end{aligned} \tag{8}$$

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\rho_{ee,ee} =& \gamma_e[N\rho_{gg,gg} + N\rho_{eg,eg} \\ &- 2(N+1)\rho_{ee,ee}], \end{aligned} \tag{9}$$

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\rho_{eu,eu} =& \gamma_e[N\rho_{gu,gu} - (N+1)\rho_{eu,eu}] \\ &+ \gamma_u[N\rho_{eg,eg} - (N+1)\rho_{eu,eu}], \end{aligned} \tag{10}$$

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\rho_{ug,ug} =& \gamma_e[(N+1)\rho_{ue,ue} - N\rho_{ug,ug}] \\ &+ \gamma_u[(N+1)\rho_{uu,uu} + N\rho_{gg,gg} \\ &- 2N\rho_{ug,ug} - \rho_{ug,ug}], \end{aligned} \tag{11}$$

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\rho_{ue,ue} =& \gamma_e[N\rho_{ug,ug} - (N+1)\rho_{ue,ue}] \\ &+ \gamma_u[N\rho_{ge,ge} - (N+1)\rho_{ue,ue}], \end{aligned} \tag{12}$$

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\rho_{uu,uu} =& \gamma_u[N\rho_{gu,gu} + N\rho_{ug,ug} \\ &- 2(N+1)\rho_{uu,uu}]. \end{aligned} \tag{13}$$

3. Distillability entanglement

Now we analyze a pair of three-level systems in the V configuration with the characterization of bound-entangled states and distillability entanglement behavior. Reference [22] showed that bound-entangled states have PPT and there is non-distillability under LOCC. As is known, there is not a perfect way to detect all the bound-entangled states in qutrit–qutrit systems. The most common method is the realignment criterion,[23,24] which can detect certain bound-entangled states. For a given density matrix, the realignment is operated as $(\rho^R)_{ij,kl} = \rho_{ik,jl}$. If ρ is a separable state, we can obtain $\|\rho^R\| \leq 1$ from the realignment criterion. Here we define $\Gamma = \|\rho^R\| - 1$. So, for a PPT state, the positive value of Γ can prove the bound-entangled state.

Here we choose familiar quantum states

$$\rho_\alpha(0) = \begin{pmatrix} \alpha/21 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2/21 & 0 & 2/21 & 0 & 0 & 0 & 0 & 2/21 \\ 0 & 0 & (5-\alpha)/21 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2/21 & 0 & 2/21 & 0 & 0 & 0 & 0 & 2/21 \\ 0 & 0 & 0 & 0 & (5-\alpha)/21 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha/21 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & (5-\alpha)/21 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha/21 & 0 \\ 0 & 2/21 & 0 & 2/21 & 0 & 0 & 0 & 0 & 2/21 \end{pmatrix}, \quad (14)$$

where $2 \leq \alpha \leq 5$. From Ref. [27], we know that ρ_α is separable for $2 \leq \alpha \leq 3$, bound-entangled for $3 < \alpha \leq 4$ and free-entangled for $4 < \alpha \leq 5$. We take $4 < \alpha \leq 5$.

Let us analyze that the time evolution of the density matrix $\rho_\alpha(t)$ under the interaction of the thermal reservoir. This could be most effectively demonstrated by checking the entanglement of $\rho_\alpha(t)$ after it becomes PPT.

In Fig. 1, we take $N = 1$, $\beta = \gamma_u/\gamma_e = 0.5$, $\alpha = 4.2$, expecting the phenomenon to appear. At the time $\gamma_e t \approx 0.0938$ the initial free-entangled state becomes a bound-entangled state. However, after time $\gamma_e t \approx 0.0938$, we cannot detect the possible entanglement any more and the evolution situation is not known. We want to explore its characters and effect factors more clearly, so we change the parameter values next. We change the value α in Fig. 2. From the figure we can see that the smaller the α is, the longer the duration time of DSD is. We can use this feature to control the value α to obtain the change time we needed. Next we investigate the ratio β. From Fig. 3, we can find that when β is bigger, the concave curve is higher. There is a change that we must keep in mind, when β is big enough, the concave curve is above the $\gamma_e t$ axis, and Γ is always larger than zero which is clearly shown in Fig. 4. It means that the free-entangled state always keeps its initial states, the realignment criterion is no longer suitable. As it is known that β is the ratio of γ_u and γ_e. When β is bigger, the value of γ_u is closer to γ_e than $\gamma_u \approx \gamma_e$. This implies that the two excited states are degenerate, which obey our presupposition, so it is no longer suitable for the model. This is the deeper reason for the phenomenon.

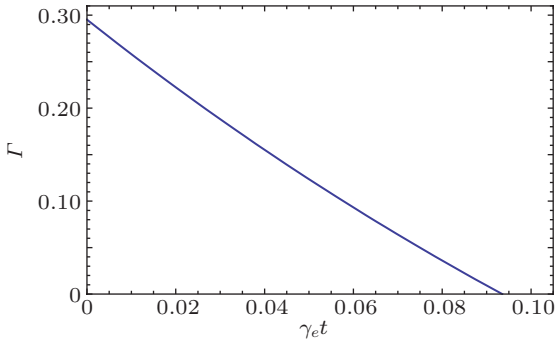


Fig. 1. (colour online) Γ is plotted against the decay parameter $\gamma_e t$ with $\alpha = 4.2$, $N = 1$, and $\beta = \gamma_u/\gamma_e = 0.5$ according to the realignment criterion.

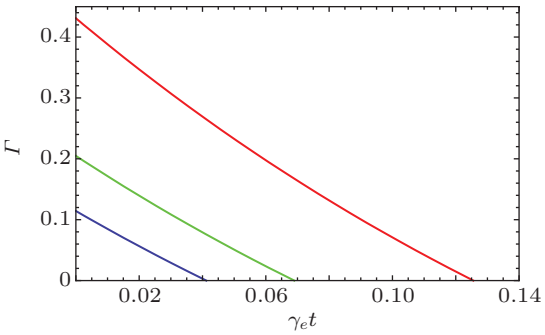


Fig. 2. (colour online) Γ is plotted against the decay parameter $\gamma_e t$ with $N = 1$ and $\beta = \gamma_u/\gamma_e = 0.5$ according to the realignment criterion. The red line is for $\alpha = 4.1$, the green line is for $\alpha = 4.4$, and the blue line is for $\alpha = 4.6$.

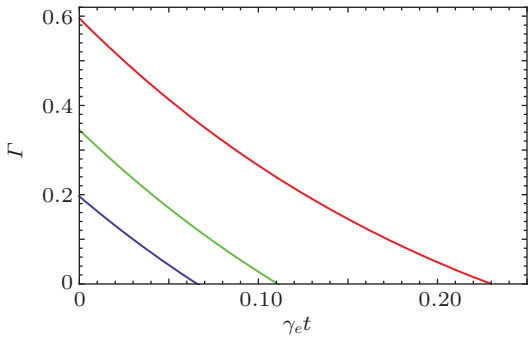


Fig. 3. (colour online) Γ is plotted against the decay parameter $\gamma_e t$ with $N = 1$ and $\alpha = 4.2$ according to the realignment criterion. The red line is for $\beta = 0.8$, the green line is for $\beta = 0.5$, and the blue line is for $\beta = 0.1$.

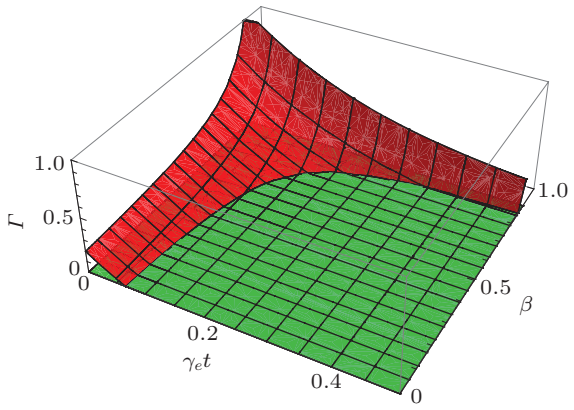


Fig. 4. (colour online) Γ is plotted against the decay parameter $\gamma_e t$ and β with $N = 1$ and $\alpha = 4.2$ according to the realignment criterion.

Now we consider the average photon number N. It corresponds to optical intensity. We can easily obtain rich information from Figs. 5 and 6. When $N = 0$, the reservoir is in vacuum, in this situation, the systems evolute only by atoms and the effect of the reservoir can be omitted. With the increasing of N, the system energy becomes higher, its effect is obvious, it can last for the distillability entanglement time, but when the N is bigger than 1.41 the curve never reaches zero, because our model is in the presupposition of a low quantum state when N is larger than a certain value, the system becomes disordered. That is outside of our discussion here.

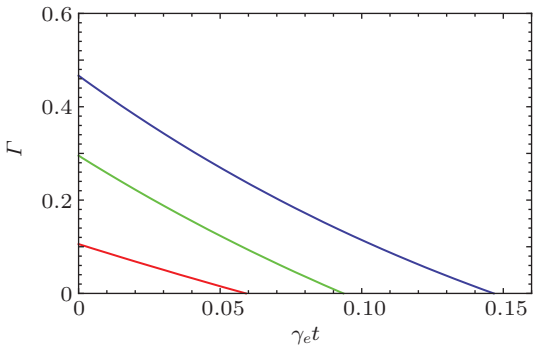


Fig. 5. (colour online) Γ is plotted against the decay parameter $\gamma_e t$ with $\alpha = 4.2$ and $\beta = 0.5$ according to the realignment criterion. The red line is for $N = 1.8$, the green line is for $N = 1.0$ and the blue line is for $N = 0.1$.

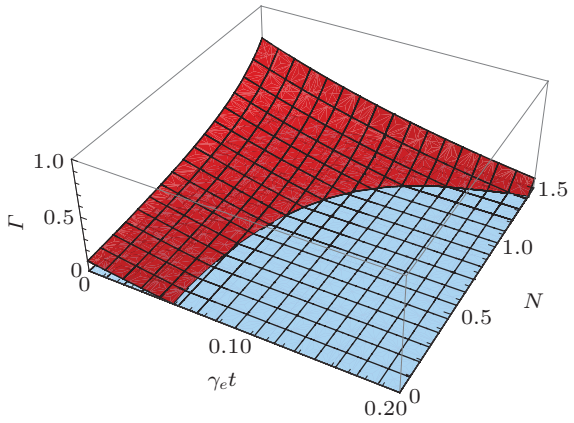


Fig. 6. (colour online) Γ is plotted against the decay parameter $\gamma_e t$ and N with $\alpha = 4.2$ and $\beta = 0.5$ according to the realignment criterion.

4. Conclusion

In summary, we have investigated the entanglement dynamics of two V-typed atomic systems in a thermal reservoir and found the distillability sudden death and discussed its effect factors. Our conclusions are summarized below.

(i) When α is bigger, the distillability sudden death time is longer and the maximum corresponding value Γ is bigger.

(ii) Parameter β is the ratio of γ_u and γ_e. We can obtain some limiting value because when β is close to 1, the distillability sudden death phenomenon disappears.

(iii) N is satisfied only in the range of $0 \leq N < 2$. The explanations are given in the text briefly.

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