Propagation properties of stochastic electromagnetic double-vortex beams in a turbulent atmosphere*

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Based on the extended Huygens–Fresnel principle, we study the propagation properties of stochastic electromagnetic double-vortex beams in a turbulent atmosphere. The result shows that the spreading of partially coherent double-vortex beams can be smaller than that of fully coherent ones. The degree of polarization of this kind of beam will experience change, which is dependent on the degree of polarization of the source plane, the atmospheric turbulence, topological charge, and the spatial coherence. The results may have applications in space optical communication.

Keywords: double-vortex beams, coherent, polarization, turbulent atmosphere

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1. Introduction

It is well known that stochastic electromagnetic beams are less sensitive to the effects of turbulence in contrast to fully coherent ones.[1,2] In recent years, a good deal of attention has been paid to the propagation of stochastic electromagnetic beams in a turbulent atmosphere.[3−9] Moreover, the study of the polarization properties of optical beams is very important in practical applications of beams, such as optical communications and remote sensing.[10] Since Wolf[11] presented a unified theory of coherence and polarization, there has been much research on the polarization of random electromagnetic beams on propagation.[12−15] It has been demonstrated that the degree of polarization of a stochastic electromagnetic beam may change as it propagates in turbulent atmosphere.[16−19] In the present paper, we study the propagation of stochastic electromagnetic double-vortex beams in a turbulent atmosphere. The spreading of this kind of beams is studied, and more attention will be focused on the polarization changes of the stochastic electromagnetic double-vortex beams propagating in turbulent atmosphere.

2. Theoretical analysis

Here we take linearly polarized Laguerre–Gaussian beams as the double-vortex model. The field of the completely coherent beam can be expressed as

$$E_j(r,0,\omega) = A_{j0}\left(\frac{r}{w_{j0}}\right)^{n_j}\exp(\mathrm{i}n_j\varphi)\exp\left(-\frac{r^2}{w_{j0}^2}\right), \quad j=x,y, \tag{1}$$

where A_{j0} and w_{j0} are the characteristic amplitude and beam size in the source plane, ω is the angular frequency , n_j denotes the topological charge, and $\exp(\mathrm{i}n_j\varphi)$ is the vortex phase factor. Furthermore, we assume the correlation function to be

$$C_{ij}^{(0)}(\boldsymbol{r}_1,\boldsymbol{r}_2) = B_{ij}\exp\left[-\frac{(\boldsymbol{r}_1-\boldsymbol{r}_2)^2}{\delta_{ij}^2}\right], \tag{2}$$

where $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ are the position vectors located in the transverse plane $z=0$, δ_{ij} is the transverse coherence length, and the factor B_{ij} has the following properties:

$$B_{ij} \equiv 1, \quad \text{when} \quad i=j, \tag{3a}$$

$$|B_{ij}| \le 1, \quad \text{when} \quad i\neq j, \tag{3b}$$

$$B_{ji} = B_{ij}^*. \tag{3c}$$

We consider the case that the double-vortex beams are generated by coherent superimposition of

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two coaxial linearly polarized beams with different topological charges n_1 and n_2 ($n_1 < n_2$), respectively. Let the polarization directions of the two beams be orthogonal to each other, and the field of the double-vortex beams at $z = 0$ plane can be denoted as

$$E(r, 0, \omega) = E_x(r, 0, \omega) \cdot \boldsymbol{e}_x + E_y(r, 0, \omega) \cdot \boldsymbol{e}_y, \quad (4)$$

where $\boldsymbol{e}_x$ and $\boldsymbol{e}_y$ are unit vectors in the x and y direction. respectively.

The second-order coherence and polarization properties of a random, statistically stationary electromagnetic beam in the $z = 0$ plane may be characterized by a 2×2 electric cross-spectral density matrix[11]

$$W^{(0)}(\boldsymbol{r}_1, \boldsymbol{r}_2, 0, \omega) = \begin{bmatrix} W_{xx}^{(0)}(\boldsymbol{r}_1, \boldsymbol{r}_2, 0, \omega) & W_{xy}^{(0)}(\boldsymbol{r}_1, \boldsymbol{r}_2, 0, \omega) \\ W_{yx}^{(0)}(\boldsymbol{r}_1, \boldsymbol{r}_2, 0, \omega) & W_{yy}^{(0)}(\boldsymbol{r}_1, \boldsymbol{r}_2, 0, \omega) \end{bmatrix}, \quad (5)$$

where the element of the matrix is given by

$$W_{ij}^{(0)}(\boldsymbol{r}_1, \boldsymbol{r}_2, 0, \omega) = \left\langle E_i(\boldsymbol{r}_1, 0, \omega) E_j^*(\boldsymbol{r}_2, 0, \omega) \right\rangle, \quad i = x, y, \quad j = x, y, \quad (6)$$

where $E(\boldsymbol{r}, 0, \omega)$ is the electric field of the beams. The asterisk denotes the complex conjugate. The angular brackets indicate an ensemble average.

For brevity, the off-diagonal elements of the electric cross-spectral density matrix of the beams in the source plane are assumed to be zero (i.e. $W_{xy}^{(0)}(\boldsymbol{r}_1, \boldsymbol{r}_2, 0, \omega) = W_{yx}^{(0)}(\boldsymbol{r}_1, \boldsymbol{r}_2, 0, \omega) = 0$). Substituting Eqs. (1)–(3) into Eq. (6), we obtain the cross-spectral density of a partially coherent vortex beam in the $z = 0$ plane as

$$\begin{aligned} & W_{ii}^{(0)}(\boldsymbol{r}_1, \boldsymbol{r}_2, 0, \omega) \\ = {} & A_{i0}^2 \left(\frac{\boldsymbol{r}_1}{w_{i0}}\right)^{n_i} \left(\frac{\boldsymbol{r}_2}{w_{i0}}\right)^{n_i} \\ & \times \exp(-\mathrm{i} n_i(\varphi_1 - \varphi_2) \cdot \exp\left[-\frac{\boldsymbol{r}_1^2 + \boldsymbol{r}_2^2}{w_{i0}^2}\right] \\ & \times \exp\left[-\frac{\boldsymbol{r}_1^2 + \boldsymbol{r}_2^2 - 2\boldsymbol{r}_1\boldsymbol{r}_2\cos(\varphi_1 - \varphi_2)}{\delta_{ii}^2}\right], \\ & i = x, y. \end{aligned} \quad (7)$$

According to the extended Huygens–Fresnel diffraction integral formula, the elements of the cross-spectral density matrix for the beams propagating in a turbulent atmosphere can be expressed as follows:[1,6,18]

$$\begin{aligned} & W_{ii}(\rho_1, \rho_2, z, \omega) \\ = {} & \frac{k^2}{4\pi^2 z^2} \iiiint W_{ii}^{(0)}(\boldsymbol{r}_1, \boldsymbol{r}_2, 0, \omega) \\ & \times \exp\left[-\frac{\mathrm{i}k}{2z}(\boldsymbol{r}_1 - \boldsymbol{\rho}_1)^2 + \frac{\mathrm{i}k}{2z}(\boldsymbol{r}_2 - \boldsymbol{\rho}_2)^2\right] \\ & \times \langle \exp[\psi(\boldsymbol{r}_1, \boldsymbol{\rho}_1, z, \omega) + \psi^*(\boldsymbol{r}_2, \boldsymbol{\rho}_2, z, \omega)]\rangle \\ & \times \mathrm{d}^2 r_1 \mathrm{d}^2 r_2, \end{aligned} \quad (8)$$

where $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$ are the position vectors in the z plane and k is the wave number. The first exponential term expresses the diffraction effect that the propagating beams are subjected to, which is also common in free space propagation. The second exponential term describes the effects of turbulence on propagation, which can be expressed as[20]

$$\begin{aligned} & \langle \exp[\psi(\boldsymbol{r}_1, \boldsymbol{\rho}_1, z, \omega) + \psi^*(\boldsymbol{r}_2, \boldsymbol{\rho}_2, z, \omega)]\rangle \\ = {} & \exp\left[-\frac{1}{\rho_0^2}(\boldsymbol{r}_1 - \boldsymbol{r}_2)^2\right], \quad (\text{if } \boldsymbol{\rho}_1 = \boldsymbol{\rho}_2), \end{aligned} \quad (9)$$

where $\rho_0 = (0.545 C_n^2 k^2 z)^{-3/5}$ is the coherent length of a spherical wave propagating in the turbulent medium, and C_n^2 is the constant of refraction index structure and describes the turbulence level. Substituting Eqs. (7) and (9) into Eq. (8), we obtain

$$\begin{aligned} & W_{ii}(\rho_1, \rho_2, \theta_1, \theta_2, z) \\ = {} & \frac{A_{i0}^2 k^2}{4\pi^2 z^2} \exp\left[-\frac{\mathrm{i}k}{2z}\left(\rho_1^2 - \rho_2^2\right)\right] \\ & \times \iiiint \left(\frac{r_1}{w_{i0}}\right)^{n_i} \left(\frac{r_2}{w_{i0}}\right)^{n_i} \\ & \times \exp\left[-\left(\frac{1}{\rho_0^2} + \frac{1}{\delta_{ii}^2} + \frac{1}{w_0^2}\right)\left(r_1^2 + r_2^2\right)\right] \\ & \times \exp\left[-\frac{\mathrm{i}k}{2z}\left(r_1^2 - r_2^2\right)\right] \\ & \times \exp\left[\frac{\mathrm{i}k\rho_1 r_1}{z} \cdot \cos\left(\theta_1 - \varphi_1\right)\right] \\ & \times \exp\left[-\frac{\mathrm{i}k\rho_2 r_2}{z} \cdot \cos\left(\theta_2 - \varphi_2\right)\right] \\ & \times \exp\left[\left(\frac{1}{\rho_0^2} + \frac{1}{\delta_{ii}^2}\right) \cdot 2 r_1 r_2 \cos\left(\varphi_1 - \varphi_2\right)\right] \\ & \times \exp\left[-\mathrm{i} n_i \left(\varphi_1 - \varphi_2\right)\right] \\ & \times r_1 r_2 \mathrm{d}r_1 \mathrm{d}r_2 \mathrm{d}\varphi_1 \mathrm{d}\varphi_2. \end{aligned} \quad (10)$$

By making use of the following formulae:[21]

$$\begin{aligned} & \exp\left[\frac{\mathrm{i}kr\rho}{z}\cos(\theta - \varphi)\right] \\ = {} & \sum_{l=-\infty}^{+\infty} i^l J_l\left(\frac{kr\rho}{z}\right) \exp[\mathrm{i}l(\theta - \varphi)], \end{aligned} \quad (11)$$

$$\begin{aligned} & \int_0^{2\pi} \exp\left[-\mathrm{i}m\varphi_1 + \frac{2r_1 r_2}{\rho_0^2}\cos(\varphi_1 - \varphi_2)\right] \mathrm{d}\varphi_1 \\ = {} & 2\pi \exp(-\mathrm{i}m\varphi_2) I_m\left(\frac{2r_1 r_2}{\rho_0^2}\right), \end{aligned} \quad (12)$$

$$\int_0^{2\pi} \exp(\mathrm{i}m\varphi)\mathrm{d}\varphi = \begin{cases} 2\pi, & m = 0, \\ 0, & m \neq 0, \end{cases} \tag{13}$$

and by setting $\theta_1 = \theta_2 = 0$, $\rho_1 = \rho_2 = \rho$, equation (10) can be simplified as

$$\begin{aligned} & W_{ii}(\rho, \rho, z) \\ &= \frac{A_{i0}^2 k^2}{4\pi^2 z^2} \sum_{l=-\infty}^{+\infty} \iint \left(\frac{r_1}{w_{i0}}\right)^{n_i} \left(\frac{r_2}{w_{i0}}\right)^{n_i} \\ &\times I_{(n_i+l)}\left[2r_1 r_2 \left(\frac{1}{\rho_0^2} + \frac{1}{\delta_{ii}^2}\right)\right] \\ &\times J_l\left(\frac{kr_1\rho}{z}\right) J_l\left(\frac{kr_2\rho}{z}\right) \times \exp\left[-\frac{\mathrm{i}k}{2z}\left(r_1^2 - r_2^2\right)\right] \\ &\times \exp\left[-\left(\frac{1}{\rho_0^2} + \frac{1}{\delta_{ii}^2} + \frac{1}{w_0^2}\right)\left(r_1^2 + r_2^2\right)\right] \\ &\times r_1 r_2 \mathrm{d}r_1 \mathrm{d}r_2. \end{aligned} \tag{14}$$

The expression for the intensity distribution and the degree of polarization of the beams in the propagation field ($z > 0$) are given by[11]

$$\begin{aligned} I(\rho, z) &= \mathrm{Tr}\, \overleftrightarrow{W}(\rho, \rho, z) \\ &= W_{xx}(\rho, \rho, z) + W_{yy}(\rho, \rho, z), \end{aligned} \tag{15}$$

and

$$P(\rho, z) = \sqrt{1 - \frac{4\det \overleftrightarrow{W}(\rho, \rho, z)}{[\mathrm{Tr}\, \overleftrightarrow{W}(\rho, \rho, z)]^2}}$$

$$= \frac{|W_{xx}(\rho, \rho, z) - W_{yy}(\rho, \rho, z)|}{W_{xx}(\rho, \rho, z) + W_{yy}(\rho, \rho, z)}, \tag{16}$$

where Tr denotes the trace and det denotes the determinant. Let $I_{i0} = W_{ii}^{(0)}(r_1, r_2, 0, \omega)$, the degree of polarization in the source plane which is denoted as $P^{(0)}$ can be expressed as

$$P^{(0)} = \frac{|I_{x0} - I_{y0}|}{I_{x0} + I_{y0}}. \tag{17}$$

In light of the above derived expressions, we can perform some numerical calculations. In the following section, we will give some numerical calculation results to show the propagation properties of stochastic electromagnetic double-vortex beams in a turbulent atmosphere.

3. Results and discussion

First of all, we study the intensity distributions of the partially coherent double-vortex beams, in which the topological charge of the inner vortex beam is fixed, and topological charge of the outer vortex beam is varying. Figure 1 shows the intensity distributions of the partially coherent double-vortex beams with

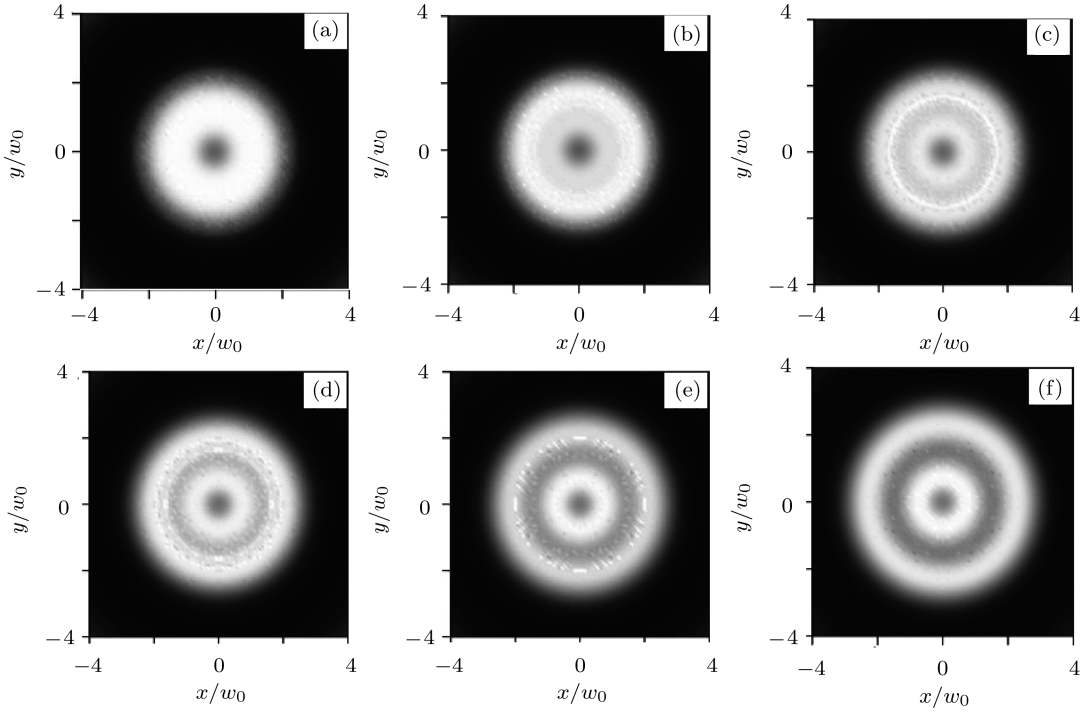


Fig. 1. The intensity distributions of the partially coherent double-vortex beams with different topological charges of the outer ring at $z = 50$ m: $n_2 = 5$ (a), 6 (b), 7 (c), 8 (d), 9 (e), and 10 (f). The other parameters are chosen to be $n_1 = 1$, $w_0 = 5$ mm, $\lambda = 632.8$ nm, $\delta_{xx} = \delta_{yy} = w$, and $C_n^2 = 5 \times 10^{-15}$ m$^{2/3}$.

different topological charges of the outer ring in a turbulent atmosphere at propagation distance $z = 50$ m. It is clearly indicated that the intensity distributions have a double-ring structure and the gap between the inner and outer ring increases with the increase of the topological charges of the outer ring. This is due to the fact that the outer ring of the larger topological charge has a larger divergence angle, so that the two vortex rings separate after propagation.

Now we focus on the spreadings of the stochastic electromagnetic double-vortex beams propagating in turbulent atmosphere and in free space. Because the output beam is not pure Gaussian, we use the definition of 86.5% power content to calculate the beam size. The calculation results are shown in Fig. 2. It can be seen that, in free space, the spreading of the partially coherent double-vortex beams is larger than that of fully coherent ones. However in a turbulent atmosphere, we can choose a suitable transverse coherence length to reduce the beam-spreading. As $\delta_{xx} = \delta_{yy} = w$, the beam size of the partially coherent double-vortex beams is smaller than that of fully coherent ones.

Then we study the changes in the degree of polarization of this kind of beam for different source parameters and different atmospheric turbulences. In Fig. 3, the changes in the degree of polarization at the axial point are plotted. The changes in the degree of polarization for different source polarizations are shown in Fig. 3(a). We find that for a long distance propagation, the degree of polarization increases with the propagation distance increasing, owing to the fact that after propagation in turbulent atmosphere, two vortex rings with different topological charges have different divergence angles, so that the two vortex rings with different polarization directions separate. Figure 3(b) shows the influence of the turbulence on the degree of polarization. It can be seen that as the propagation distance increases, the degree of polarization first decreases and then increases. For a long propagation distance, the degree of polarization increases with strength of turbulence increasing. Figure 3(c) shows the different topological charges of the outer ring of the double-vortex beams. After a sufficiently long propagation distance ($z > 5 \times 10^4$ m), the degree of polarization will tend to a certain value, which is equal to the degree of polarization in the source plane (i.e. $P^{(0)} = 0.5$). In Fig. 3(d), the influence of transverse coherence length on the degree of polarization is investigated. It is clear that the degree of polarization will have a great change when it propagates in turbulent atmosphere, and at large z value ($z > 5 \times 10^4$ m), the degree of polarization will tend to a certain value.

The changes in the degree of polarization of this kind of beam in the cross section of $z = 1000$ m are shown in Fig. 4. It can be found that the degree of polarization in the cross section is influenced by the source polarization, the strength of turbulence, and the topological charges as well. It is clear from Fig. 4 that with the increase of transverse distance (i.e., ρ), all the degrees of polarization first decrease and reach a minimal value, and then increase. However, we can find from Fig. 4(b) that all the curves tend to be the same as ρ increases. This indicates that the degree of polarization in the cross section is less influenced by turbulence.

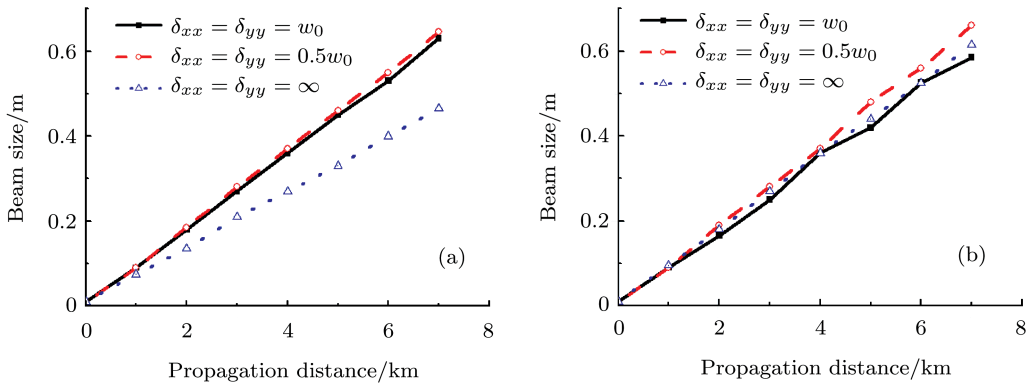


Fig. 2. (colour online) Beam widths of the double-vortex beams propagating (a) in free space ($C_n^2 = 0$), (b) in turbulent atmosphere ($C_n^2 = 10^{-14}$ m$^{2/3}$), where solid curve denotes $\delta_{xx} = \delta_{yy} = w_0$, dashed curve $\delta_{xx} = \delta_{yy} = 0.5w_0$, and dot curve $\delta_{xx} = \delta_{yy} = \infty$ (fully coherent). The other parameters are chosen to be $n_1 = 1$, $n_2 = 7$, $w_0 = 5$ mm, and $\lambda = 632.8$ nm.

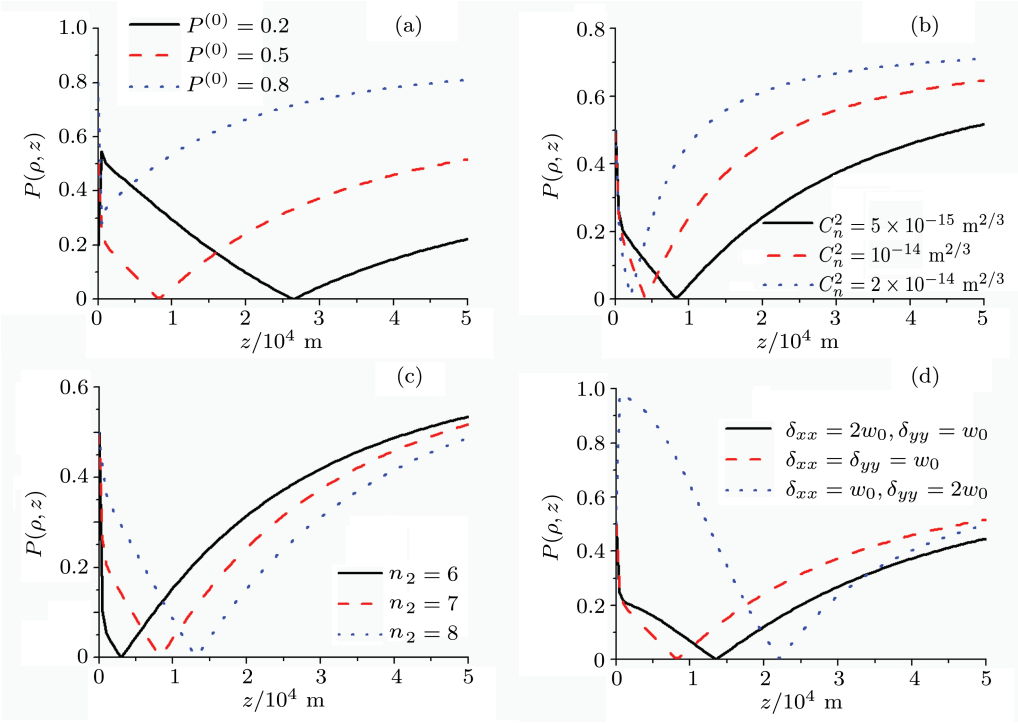


Fig. 3. (colour online) Degrees of polarization in the axis. Panel (a) shows different source polarizations, where solid curve denotes $P^{(0)} = 0.2$, dashed curve $P^{(0)} = 0.5$, dot curve $P^{(0)} = 0.8$. Panel (b) displays different turbulences, where solid curve denotes $C_n^2 = 5 \times 10^{-15}$ m$^{2/3}$, dashed curve $C_n^2 = 10^{-14}$ m$^{2/3}$, and dot curve $C_n^2 = 2 \times 10^{-14}$ m$^{2/3}$. Panel (c) exhibits different topological charges of the outer ring, where solid curve refers to $n_2 = 6$; dashed curve $n_2 = 7$, and dot curve $n_2 = 8$. Panel (d) indicates different transverse coherence lengths, where solid curve denotes $\delta_{xx} = 2w_0$ and $\delta_{yy} = w_0$, dashed curve $\delta_{xx} = \delta_{yy} = w_0$, dot curve $\delta_{xx} = w_0$ and $\delta_{yy} = 2w_0$. The default parameters are chosen to be $n_1 = 1$, $n_2 = 7$, $w_0 = 5$ mm, $\lambda = 632.8$ nm, $\delta_{xx} = \delta_{yy} = w_0$, $P^{(0)} = 0.5$, and $C_n^2 = 5 \times 10^{-15}$ m$^{2/3}$.

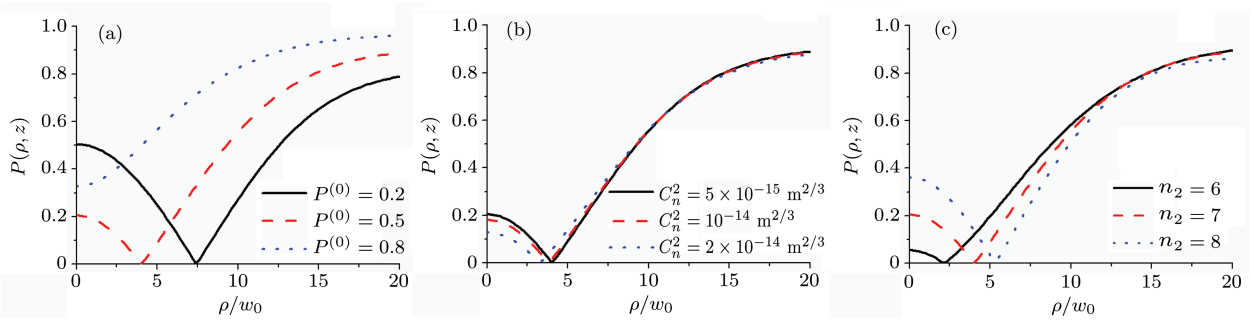


Fig. 4. (colour online) Degrees of polarization in the transverse plane $z = 1000$ m. Panel (a) is for different source polarizations, where solid curve denotes $P^{(0)} = 0.2$, dashed curve $P^{(0)} = 0.5$, dot curve $P^{(0)} = 0.8$. Panel (b) is for different turbulences, where solid curve denotes $C_n^2 = 5 \times 10^{-15}$ m$^{2/3}$, dashed curve $C_n^2 = 10^{-14}$ m$^{2/3}$, dot curve $C_n^2 = 2 \times 10^{-14}$ m$^{2/3}$. Panel (c) is for different topological charges of the outer ring, where solid curve denotes $n_2 = 6$, dashed curve $n_2 = 7$, and dot curve $n_2 = 8$. The default parameters are chosen to be $n_1 = 1$, $n_2 = 7$, $w_0 = 5$ mm, $\lambda = 632.8$ nm, $\delta_{xx} = \delta_{yy} = w_0$, $P^{(0)} = 0.5$, and $C_n^2 = 5 \times 10^{-15}$ m$^{2/3}$.

4. Conclusion

In this paper, we have studied the propagation properties of stochastic electromagnetic double-vortex beams propagating in turbulent atmosphere. The result shows that the spreading of the partially coherent double-vortex beams can be smaller than that of the fully coherent ones. In addition, the degree of polarization of the double-vortex beams will experience change, which is dependent on the degree of polarization of the source plane, the strength of the atmospheric turbulence, the topological charges of the outer ring of the double-vortex beams, and the spatial coherence. The results may have applications in space

optical communication.

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