Ultrashort optical solitons in the dispersion-decreasing fibers*

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We derive analytical bright and dark solitons of the modified nonlinear Schrödinger equations with variable coefficients. Under constraint conditions between system parameters, the optical soliton transmission in the dispersion-decreasing fibers can be exactly controlled by proper dispersion management. The analytical description of the interactions between the bright and dark solitons are first obtained.

Keywords: nonlinear Schrödinger equation, soliton interaction and control, dispersion decreasing fibers

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1. Introduction

In modern soliton theory, a vast variety of methods have been established to obtain exact soliton solutions to a given nonlinear partial differential equation, such as the Jacobian elliptic function method,[1] the improved projective approach,[2−4] (G'/G)-expansion method,[5] the undetermined coefficient method,[6] and so on. Optical solitons, as good information carriers for long distance communication,[7] have flourished into a research area of great importance and interest due to their important application. They form with the balance between the group velocity dispersion (GVD) and the self-phase modulation (SPM). The theoretical description of the optical solitons is usually based on the constant-coefficient nonlinear Schrödinger equation (NLSE).[8] However, in a real fiber, the core medium is always nonuniform due to the variation in the lattice parameters of the fibre medium and (or) the variation of the fiber geometry (diameter fluctuations, etc.).[9] These nonuniformities influence various effects such as gain/loss, dispersion, phase modulation, and so on. Considering the inhomogeneities in the fiber, the dynamics of the pulse propagation are governed by variable coefficient (VC) NLSE.[10−13] The soliton control[10,11] and interactions[12,13] have been studied extensively. However, relatively less research work for the control and interaction between one bright and one dark soliton was carried out, which we shall pay attention to in this paper.

The dispersion-decreasing fiber (DDF), first introduced in Ref. [14], is used to avoid soliton broadening resulting from inherent optical loss of the fiber during the propagation of the pulse. Recent numerical and experimental results have shown the possibility to generate parabolic pulses via a single DDF with normal dispersion.[15] However, we notice that there has not been much discussion about the soliton transmission control in the DDF based on the special analytic solutions. Motivated by this, we will perform an analytic study on the transmission control using the DDF for the following modified VCNLSE[16,17]

$$\begin{aligned} &\mathrm{i}u_z + D(z)u_{tt} + \mathrm{i}F(z)(|u|^2u)_t \\ &\quad + R(z)|u|^2u + \mathrm{i}G(z)u = 0, \end{aligned} \tag{1}$$

where $u(z,t)$ is the complex envelope of the electrical field, z and t respectively, represent the propagation distance and retarded time. They are normalized by the dispersion length $L_{\mathrm{D}} = T_0^2/\beta_2(0)$ and the half-width T_0 (at $1/e$ intensity point) of the input pulse. In practice, it is customary to use the full width at half maximum (FWHM) in place of T_0. For a hyperbolic-secant pulse, the two variables are related as $T_{\mathrm{FWHM}} = 2\ln(1+2^{1/2})T_0 \simeq 1.763T_0$. $D(z) = |\beta_2(z)/\beta_2(0)|$ is the dispersion management function, $F(z)$ accounts for the self-steepening, $R(z)$ represents the nonlinear Kerr coefficient of the fiber

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with the localized inhomogeneity, and $G(z)$ is the inhomogeneous linear gain/loss coefficient. Of physical and optical interest, specific significance of this model lies in its potential application to the study of amplification/absorption and compression/broadening of optical solitons in inhomogeneous systems.

2. Reduction procedure and soliton solutions

In order to obtain the exact analytical solutions for Eq. (1), we construct the mapping transformation

$$u(z,t) = A(z)U[T(z,t), Z(z)]\exp[\mathrm{i}\phi(z,t)], \quad (2)$$

where the mapping variable, the effective propagation distance, the amplitude, and the phase of the pulse read

$$T = kt + \omega\Lambda(z) + t_0, \quad Z = k^2\Lambda(z), \quad (3)$$

$$A = A_0\exp[-\Gamma(z)], \quad \phi = -\frac{\omega}{2k}t - \frac{\omega^2}{4k^2}\Lambda(z) + \phi_0 \quad (4)$$

with $\Lambda(z) = \int_0^z D(s)\mathrm{d}s$, $\Gamma(z) = \int_0^z G(s)\mathrm{d}s$, then the modified VCNLSE (1) is transformed into the derivative NLSE

$$\mathrm{i}U_Z + U_{TT} + \mathrm{i}(U|U|^2)_T = 0. \quad (5)$$

Note that the GVD parameter $D(z)$ influences the phase and the effective propagation distance.

Further, the constraints for the existence of such solutions are given by

$$F(z) = \frac{k}{3A_0^2}D(z)\exp[2\Gamma(z)], \quad (6)$$

and

$$R(z) = -\frac{\omega}{2k}F(z), \quad (7)$$

from which we know that only two of four parameters $D(z)$, $F(z)$, $R(z)$, and $G(z)$ in Eq. (1) are free parameters. For example, if $D(z)$ and $G(z)$ are chosen to be the free parameters, then $F(z)$ and $R(z)$ will be determined from Eqs. (6) and (7). Thus by choosing the suitable parameters approximate to the real situation, it makes it possible to explain the various soliton controls and dispersion management.

Thus the one-to-one correspondence (2) admits us to obtain abundant solutions. Employing the transformation (2), and the modified inverse scattering transformation (IST),[18] one can obtain a multi-soliton solution with nonvanishing boundary conditions for Eq. (1). Here we only consider one-soliton and two-soliton solutions, which read

$$u_n = A_0\frac{L_nM_n}{\mathrm{M}_n^2}\exp[-\Gamma(z)], \quad (8)$$

where $n = 1, 2$. For one-soliton

$$M_1 = 1 - \mathrm{i}\chi_1\,\mathrm{e}^{\mathrm{i}\alpha_1-\theta_1}, \quad L_1 = 1 - \mathrm{i}\chi_1\,\mathrm{e}^{-\mathrm{i}3\alpha_1-\theta_1},$$

and for two-soliton

$$\begin{aligned} M_2 &= 1 - \mathrm{i}\chi_1\,\mathrm{e}^{\mathrm{i}\alpha_1-\theta_1} - \mathrm{i}\chi_2\,\mathrm{e}^{\mathrm{i}\alpha_2-\theta_2} \\ &\quad - \chi_1\chi_2\frac{\sin^2(\alpha_1-\alpha_2)}{\sin^2(\alpha_1+\alpha_2)}\,\mathrm{e}^{\mathrm{i}(\alpha_1+\alpha_2)-\theta_1-\theta_2}, \\ L_2 &= 1 - \mathrm{i}\chi_1\,\mathrm{e}^{-\mathrm{i}3\alpha_1-\theta_1} - \mathrm{i}\chi_2\,\mathrm{e}^{-\mathrm{i}3\alpha_2-\theta_2} \\ &\quad - \chi_1\chi_2\frac{\sin^2(\alpha_1-\alpha_2)}{\sin^2(\alpha_1+\alpha_2)}\,\mathrm{e}^{\mathrm{i}3(\alpha_1+\alpha_2)-\theta_1-\theta_2} \end{aligned}$$

with

$$\theta_n = kt + (\omega - v_nk^2)\Lambda(z) + \theta_{n0}.$$

$\bar{M}_n$ is the complex conjugate of M_n, α_n are the characteristic parameters of solitons. It is bright soliton for $\chi_n = -1$ or dark soliton for $\chi_n = 1$.

3. Soliton dynamics in dispersion decreasing fiber

Based on solution (8), one can clearly see that the soliton solution is characterized by the amplitude $A_0\exp[-\int_0^z G(s)\mathrm{d}s]$, and the velocity of the soliton is related to $(\omega - v_nk^2)D(z)$. Thus, the analytic solution (8) is of practical importance to characterize the propagating features of the ultrashort soliton pulse with inhomogeneous effects, and we can trap the velocity to control the soliton dynamics by designing appropriate system parameters.

Next we will analyze the mechanism on how the relevant properties of the optical pulse is affected by choosing the dispersion profile of the DDF. The first example: consider a linear DDF

$$\beta_2(z) = \beta_2(0) + \beta_2(0)\left(\frac{1}{W_{\mathrm{eff}}} - 1\right)z, \quad (9)$$

with the input GVD coefficient $\beta_2(0) = -18\ \mathrm{ps}^2\cdot\mathrm{km}^{-1}$ and the effective amplification of the fiber $W_{\mathrm{eff}} = \beta_2(0)/\beta_2(L) = 18$. In the high bit rate and long-distance optical communication systems, there are always multiple pulses. Therefore, it is necessary to study the multi-solitons transmission. For two soliton solutions [$n = 2$ in Eq. (8)], assume $\alpha_2 > \alpha_1$. At distances far away before collision ($z \to -\infty$), $u_2 \simeq u_1(\theta_1) + u_1(\theta_2 + \Delta)$ with

$$\Delta = 2\ln\left|\frac{\sin(\alpha_1-\alpha_2)}{\sin(\alpha_1+\alpha_2)}\right| > 0.$$

It is composed of two well-separated solitons. At remote distances after collision ($z \to \infty$), $u_2 \simeq u_1(\theta_1 + \Delta) + u_1(\theta_2)$, which also consists of two well-separated solitons. These asymptotic behaviors imply that owing to the collision, the soliton of α_2 gets a forward shift Δ, while the soliton of α_1 gets a backward shift $-\Delta$, shown in Figs. 1(a) and 1(b). These shifts are independent of χ_1 and χ_2, regardless of whether they are bright or dark solitons. The interactions between one bright soliton and one dark soliton exhibit two different behaviors. For the interaction between the dark soliton of the smaller α_2 and the bright soliton of the bigger α_1, the bright soliton is squeezed to a sharper peak while passing through the dark soliton. While for the interaction between the dark soliton of the bigger α_2 and the bright soliton of the smaller α_1, a deeper dip in the location of interaction is formed.

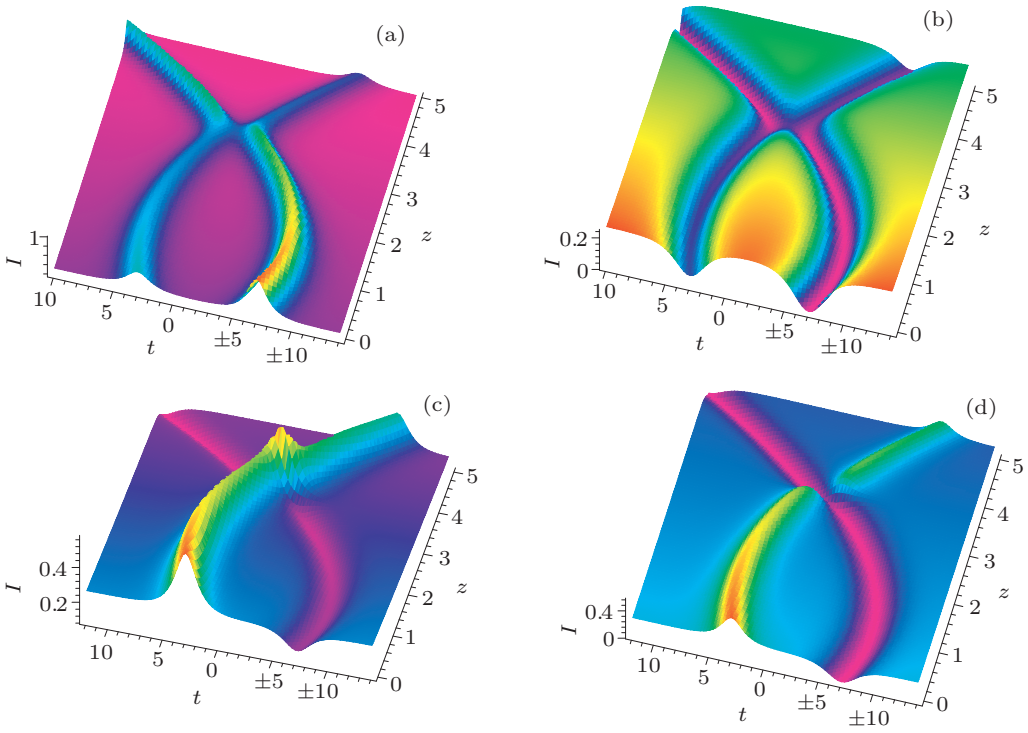


Fig. 1. (colour online) Interaction scenarios between (a) two bright solitons, (b) two dark solitons, (c) and (d) bright and dark solitons. The parameters are chosen as $G = \gamma_0 = 0.05$, $A_0 = \omega = 0.5$, $k = 1$, $v_1 = -v_2 = -\theta_{10} = 2$, $\theta_{20} = 7$, $\alpha_1 = \pi/12$; (a), (b), (d) $\alpha_2 = \pi/6$ and (c) $\alpha_2 = \pi/24$.

As another example, consider a fiber whose dispersion and nonlinearity varies periodically along its length[5,8] such that

$$D(z) = D_0 \sin(\kappa z) \exp(\gamma_0 z), \quad R(z) = R_0 \sin(\kappa z), \tag{10}$$

where D_0 is the parameter related to the initial peak power in the system, R_0 and κ are the parameters used to describe Kerr nonlinearity. For convenience, we take the parameters $D_0 = R_0 = \kappa = 1$. In this situation, from Eq. (6) the gain/loss function $G = \gamma_0/2$, which corresponds to the DDF for $\gamma_0 > 0$. In this case, the interaction scenarios pertaining to in-phase and off-phase injection with equal and unequal amplitudes can be dealt elaborately. Trigonometric functions are physically relevant because they provide for alternating regions of positive and negative dispersion and nonlinearity, which is indicated in improved stability of solitons.

The interactions between bright and bright solitons, and dark and dark solitons have been extensively discussed in Refs. [12] and [13], so here we omit them. In the following, we focus on the control and interaction between one bright and one dark soliton in periodic system (10). Similar to the discussion mentioned in the linear DDF, there also exist the sharper peaks and deeper dips decided by the values of α_1 and α_2 during interaction between one bright and one dark soliton [Figs. 2(a) and 2(c)]. By adjusting the values of θ_{n0}, we can make the bright and dark solitons propagate separately [Figs. 2(b) and 2(d)]. These properties are significant for increasing the bit rate in optical communication.

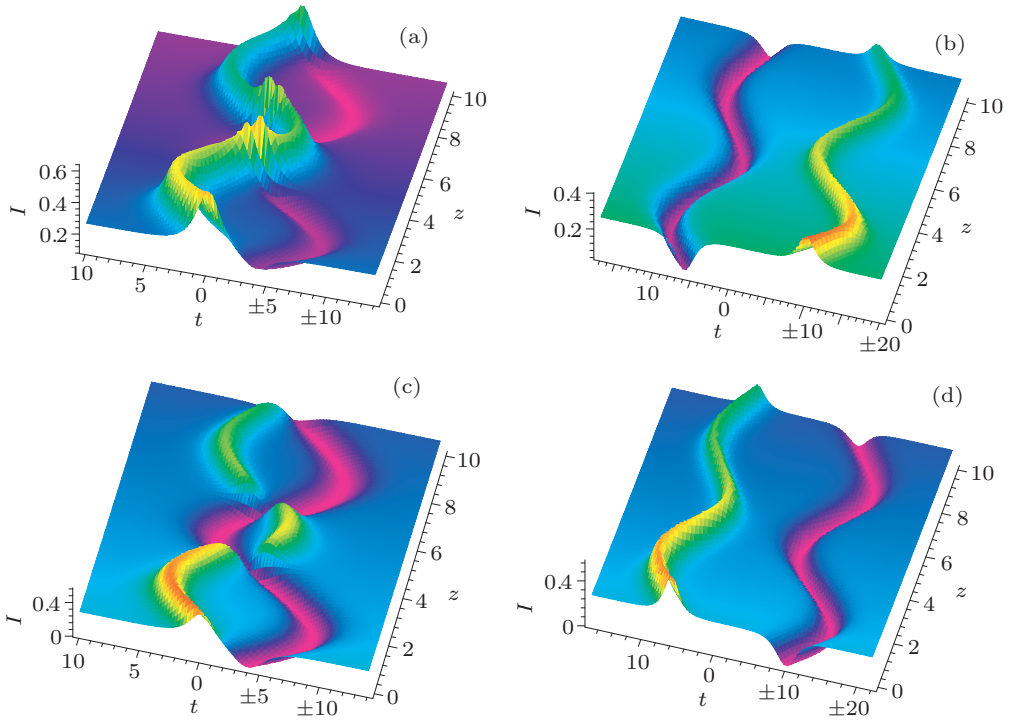


Fig. 2. (colour online) Periodic interaction scenarios between panels (a) and (c) one bright and one dark soliton. Separated propagations for panels (b) and (d) one bright and one dark soliton. The parameters are chosen as $D_0 = R_0 = \kappa = 1$, panels (a) and (c) $\theta_{10} = 0$, $\theta_{20} = 4$; panels (b) and (d) $\theta_{10} = -6$, $\theta_{20} = 8$. Other parameters are the same as those in Figs. 1(c) and 1(d).

4. Conclusions

In summary, we have analytically obtained bright and dark solitons for the modified NLSE with distributed coefficients. Under the parameter condition, we discuss the interaction and control of bright and dark solitons in the DDF. The results show that only two of four parameters $D(z)$, $F(z)$, $R(z)$, and $G(z)$ in Eq. (1) are free parameters, which can control the soliton transmission. The parameters θ_{n0} in solution (8) can modulate the interaction between the neighboring solitons to some extent. During the interaction between one bright and one dark soliton, the parameters α_1 and α_2 can adjust the interaction behaviors, forming the sharper peaks or deeper dips in the locations of interaction. These results obtained in this paper may have potential value for all-optical data-processing schemes and the design of beam compressors and amplifiers. Of course, more practical implementation of these theoretical results might be an interesting task.

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