Modeling and dynamics analysis of the fractional-order Buck–Boost converter in continuous conduction mode*

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In this paper, the fractional-order mathematical model and the fractional-order state-space averaging model of the Buck–Boost converter in continuous conduction mode (CCM) are established based on the fractional calculus and the Adomian decomposition method. Some dynamical properties of the current-mode controlled fractional-order Buck–Boost converter are analysed. The simulation is accomplished by using SIMULINK. Numerical simulations are presented to verify the analytical results and we find that bifurcation points will be moved backward as α and β vary. At the same time, the simulation results show that the converter goes through different routes to chaos.

Keywords: fractional-order Buck–Boost converter, modeling, bifurcation, numerical simulation

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1. Introduction

The idea of fractional calculus has been known since the development of the regular calculus. It is accepted today as a new tool that extends the descriptive power of the conventional calculus, supporting mathematical models that, in many cases, describe more accurately the dynamic response of actual systems in various applications. Nowadays, the concept is employed in physics, engineering, biology, economics, and other scientific fields.[1−3]

The real objects are generally fractional.[4−6] The importance of fractional-order models is that they yield a more accurate description and give a deeper insight into the physical processes underlying a long range memory behaviour. The main reason for using the integer-order models is due to the lack of a method of solving the fractional differential equations.

In previous studies, the researchers in most cases used the integral-order model of the DC–DC converter. However, the capacitor and the inductor are all fractional in nature. Jonscher[7] demonstrated that the ideal capacitor cannot exist in nature, because an impedance of the form $1/(\mathrm{j}\omega C)$ would violate causality. In fact, the dielectric materials exhibit a fractional behavior, yielding electrical impedances of the form $1/(\mathrm{j}\omega CF)^{\alpha}$, with $\alpha \subset R^{+}$. Westerlund[2] proposed that better capacitor impedance could be $Z(\mathrm{j}\omega) = 1/(\mathrm{j}\omega)^{\alpha}C$, and they experimentally determined the order of the fractional-order capacitor in different dielectrics. Moreover, Westerlund[4] demonstrated that the inductor is also fractional in nature. Petras[8] designed fractional-order Chua's system. Haba *et al.*[9−11] created the fractional-order capacitor. Many real dynamical circuits were better characterized by using non-integer order models based on fractional calculus.[12,13] The modeling of the fractional-order DC–DC converter has become the subject which has important significance in theory and practice.

In this paper, we will analyse the modeling and some dynamical properties of the fractional-order Buck–Boost converter in continuous conduction mode (CCM). Firstly, we will give the fractional-order mathematical model and the fractional-order state-space averaging model of the Buck–Boost converter. Then we will reveal the difference between the integer model and the fractional model. Finally, we will analyse the nonlinear dynamic behaviour of the current-mode controlled fractional-order model, and an illustrative simulation result will be given to demonstrate the cor-

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rectness of the proposed theoretical analysis and the efficiency of the fractional modeling.

2. Fractional calculus and the mathematical modeling of the fractional-order Buck–Boost converter

In this section, first, we introduce some basic concepts of the fractional calculus. The idea of fractional calculus is not new and is, in fact, as old as its integer-order counterpart. Fractional calculus has been used for modeling different physical phenomena[5] and in control theory.[14,15] We can notice that the systems in nature have fractional behaviours, but many of them have very low fractionalities.[15]

The Riemann–Liouville definition which is well known for the fractional differential operator is given as

$$\frac{\mathrm{d}^{\alpha} f(t)}{\mathrm{d}t^{\alpha}} = \frac{1}{\Gamma(n-\alpha)} \frac{\mathrm{d}^{n}}{\mathrm{d}t^{n}} \int_{0}^{t} \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} \mathrm{d}\tau, \quad (1)$$

where $n-1 < \alpha < n$, n is an integer number, and $\Gamma(\bullet)$ is the gamma function.

Another alternative definition of the Riemann–Liouville function was reported by Caputo as follows:

$$\frac{\mathrm{d}^{\alpha} f(t)}{\mathrm{d}t^{\alpha}} = \frac{1}{\Gamma(n-\alpha)} \int_{0}^{t} \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha-n+1}} \mathrm{d}\tau. \quad (2)$$

Then we create the fractional-order mathematical model of the Buck–Boost converter by using the concept of fractional calculus.

Westerlund[2] in 1994 proposed a new linear capacitor model. It is based on Curie's empirical law developed in 1889 which states that the current through a capacitor is

$$i(t) = \frac{V_0}{h_1 t^{m}},$$

where h_1 and m are constant, V_0 is the DC voltage applied at $t = 0$, and $0 < m < 1$.

For a general input voltage $V(t)$, the current is

$$i(t) = C_{\mathrm{f}} \frac{\mathrm{d}^{\alpha} u(t)}{\mathrm{d}t^{\alpha}}, \quad 0 < \alpha < 1, \quad (3)$$

where C_{f} is the capacitance of the capacitor. It is related to the kind of dielectric. Another constant α (order) is related to the loss of the capacitor. Westerlund provided in his work the table of various capacitor dielectric materials with the values of appropriated constant α which had been obtained experimentally.

Westerlund in his work also described the behaviour of a real inductor.[4] For a general current in the inductor, the voltage is

$$u(t) = L_{\mathrm{f}} \frac{\mathrm{d}^{\beta} i(t)}{\mathrm{d}t^{\beta}}, \quad 0 < \beta < 1, \quad (4)$$

where L_{f} is the inductance of the inductor and β (order) is the constant related to the "proximity effect".

The Buck–Boost converter, sometimes called a step-up/down power stage, is an inverting power stage topology. The current-mode Buck–Boost converter is shown in Fig. 1.[16]

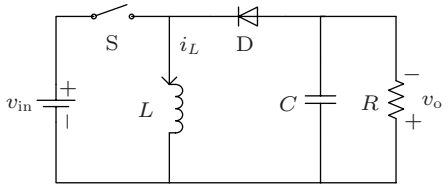


Fig. 1. Buck–Boost converter.

In the CCM, there are two switching modes. 1) When switch S is on, the inductor current passes through the switch, and the diode is reverse-biased with the inductor current i_L rising. 2) When the switch S is turned off, the inductor maintains current flowing in the same direction so that the diode is forward-biased. The state equations of the converter are as follows.

When S turns on,

$$\begin{bmatrix} \dfrac{\mathrm{d}^{\alpha} i_L(t)}{\mathrm{d}t^{\alpha}} \\ \dfrac{\mathrm{d}^{\beta} v_{\mathrm{o}}(t)}{\mathrm{d}t^{\beta}} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -\dfrac{1}{CR} \end{bmatrix} \begin{bmatrix} i_L(t) \\ v_{\mathrm{o}}(t) \end{bmatrix} + \begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix} v_{\mathrm{in}}(t), \quad (5)$$

$$A_1 = \begin{bmatrix} 0 & 0 \\ 0 & -\dfrac{1}{CR} \end{bmatrix}, \quad B_1 = \begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix};$$

When S turns off,

$$\begin{bmatrix} \dfrac{\mathrm{d}^{\alpha} i_L(t)}{\mathrm{d}t^{\alpha}} \\ \dfrac{\mathrm{d}^{\beta} v_{\mathrm{o}}(t)}{\mathrm{d}t^{\beta}} \end{bmatrix} = \begin{bmatrix} 0 & \dfrac{1}{L} \\ -\dfrac{1}{C} & -\dfrac{1}{CR} \end{bmatrix} \begin{bmatrix} i_L(t) \\ v_{\mathrm{o}}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} v_{\mathrm{in}}(t), \quad (6)$$

$$A_2 = \begin{bmatrix} 0 & \dfrac{1}{L} \\ -\dfrac{1}{C} & -\dfrac{1}{CR} \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

In comparison with the integer-order model, the fractional-order model of the Buck–Boost converter in the CCM described by Eqs. (5) and (6) is relevant to α and β.

3. State-space averaging model of fractional-order Buck–Boost converter

According to the operational features of the Buck–Boost converter, the circuit variables in the Buck–Boost converter, such as inductance current i_L and the output voltage $v_{\rm o}$, have all the high frequency switching harmonics. These harmonics can be removed by averaging a circuit variable over one switching period, i.e.,

$$\langle x(t)\rangle_T = \frac{1}{T}\int_t^{t+T} x(\tau)\,{\rm d}\tau, \tag{7}$$

where x is an arbitrary circuit variable of the Buck–Boost converter.

In light of the properties of fractional calculus, we obtain

$$\begin{aligned}\frac{{\rm d}^\alpha\langle x(t)\rangle_T}{{\rm d}t^\alpha} &= \frac{{\rm d}^\alpha\left(\frac{1}{T}\int_t^{t+T}x(\tau)\,{\rm d}\tau\right)}{{\rm d}t^\alpha}\\ &= \frac{1}{T}\int_t^{t+T}\left(\frac{{\rm d}^\alpha x(\tau)}{{\rm d}\tau^\alpha}\right){\rm d}\tau\\ &= \left\langle\frac{{\rm d}^\alpha x(t)}{{\rm d}t^\alpha}\right\rangle,\end{aligned} \tag{8}$$

where α is the order and $0<\alpha<1$.

Above all, we begin to create the fractional state-space averaging model of the Buck–Boost converter in CCM. Then theoretical analysis on this model will be carried out.

The average values of the circuit variables $\langle i_L(t)\rangle$, $\langle v_{\rm o}(t)\rangle$, $\langle v_{\rm in}(t)\rangle$, and $d(t)$ can be described in the following forms:

$$\begin{aligned}&\langle i_L(t)\rangle = I_L+\hat{i}_L(t),\quad \langle v_{\rm o}(t)\rangle = V_{\rm o}+\hat{v}_{\rm o}(t),\\ &\langle v_{\rm in}(t)\rangle = V_{\rm in}+\hat{v}_{\rm in}(t),\quad d(t)=D+\hat{d}(t),\end{aligned} \tag{9}$$

where I_L, $V_{\rm o}$, D, and $V_{\rm in}$ are the DC components of $i_L(t)$, $v_{\rm o}(t)$, $d(t)$, and $v_{\rm in}(t)$ respectively, $\hat{i}_L(t)$, $\hat{v}_{\rm o}(t)$, $\hat{d}(t)$, and $\hat{v}_{\rm in}(t)$ are the AC components respectively. In circuit the AC component of each variable is much smaller than the corresponding amplitude of the DC component (under small-signal assumption), that is, $|\hat{i}_L(t)|$ is far less than I_L, $|\hat{v}_{\rm o}(t)|$ is far less than $V_{\rm o}$, $|\hat{d}(t)|$ is far less than D, and $|\hat{v}_{\rm in}(t)|$ is far less than $V_{\rm in}$.

According to Ref. [16], we can obtain

$$A = d(t)A_1+(1-d(t))A_2,$$

$$A=\begin{bmatrix}0 & \dfrac{1-d(t)}{L}\\ -\dfrac{1-d(t)}{C} & -\dfrac{1}{CR}\end{bmatrix}, \tag{10}$$

$$B = d(t)B_1+(1-d(t))B_2,\quad B=\begin{bmatrix}\dfrac{d(t)}{L}\\ 0\end{bmatrix}. \tag{11}$$

Using the results of Ref. [16], D is the steady state value of $d(t)$, we can obtain the quiescent operation point as follows:

$$\begin{aligned}\begin{bmatrix}I_L\\ V_{\rm o}\end{bmatrix} &= -A^{-1}BV_{\rm in}\\ &= -\begin{bmatrix}0 & \dfrac{1-D}{L}\\ -\dfrac{1-D}{C} & -\dfrac{1}{CR}\end{bmatrix}^{-1}\begin{bmatrix}\dfrac{D}{L}\\ 0\end{bmatrix}V_{\rm in}\\ &= \begin{bmatrix}\dfrac{V_{\rm in}D}{(1-D)^2R}\\ -\dfrac{V_{\rm in}D}{1-D}\end{bmatrix}.\end{aligned} \tag{12}$$

The fractional-order state-space averaging model is

$$\begin{aligned}\begin{bmatrix}\dfrac{{\rm d}^\alpha\langle i_L(t)\rangle_T}{{\rm d}t^\alpha}\\ \dfrac{{\rm d}^\beta\langle v_{\rm o}(t)\rangle_T}{{\rm d}t^\beta}\end{bmatrix} &= \begin{bmatrix}0 & \dfrac{1-d(t)}{L}\\ -\dfrac{1-d(t)}{C} & -\dfrac{1}{CR}\end{bmatrix}\begin{bmatrix}\langle i_L(t)\rangle\\ \langle v_{\rm o}(t)\rangle\end{bmatrix}\\ &\quad+\begin{bmatrix}\dfrac{d(t)}{L}\\ 0\end{bmatrix}\langle v_{\rm in}(t)\rangle\\ &= \begin{bmatrix}0 & \dfrac{1-D-\hat{d}(t)}{L}\\ -\dfrac{1-D-\hat{d}(t)}{C} & -\dfrac{1}{CR}\end{bmatrix}\\ &\quad\times\begin{bmatrix}I_L+\hat{i}_L(t)\\ V_{\rm o}+\hat{v}_{\rm o}(t)\end{bmatrix}\\ &\quad+\begin{bmatrix}\dfrac{D+\hat{d}(t)}{L}\\ 0\end{bmatrix}\left(V_{\rm in}+\hat{v}_{\rm in}(t)\right).\end{aligned} \tag{13}$$

Using the definition of fractional-order derivative given by Caputo[5] and considering the input voltage $V_{\rm in}$ to be a constant, we can obtain the inductor cur-

rent ripple Δi_L in $(0, DT)$ as

$$\Delta i_L = \frac{V_{\rm in}\,[DT]^{\alpha}}{L\alpha\Gamma(\alpha)}, \tag{14}$$

where $\Gamma(\bullet)$ is the gamma function.[5] The inductor current ripple Δi_L is related not only to the inductance, the input voltage $V_{\rm in}$, the duty ratio D, and the switching period T, but also to the order of inductor L. When α is increased, $\Delta i_{\rm L}$ is reduced. Especially when $\alpha = 1$, we can obtain the integral-order model mentioned in Ref. [17].

From Eqs. (12) and (14), we can obtain the peak inductor current in the following form:

$$\begin{aligned} i_{L\,\max} &= I_L + \frac{1}{2}\Delta i_L \\ &= \frac{V_{\rm in}D}{(1-D)^2R} + \frac{V_{\rm in}(DT)^{\alpha}}{2L\alpha\Gamma(\alpha)}. \end{aligned} \tag{15}$$

The continuous-conduction mode of operation occurs when the current through the inductor in the circuit of Fig. 1 is continuous. That means the inductor current is always greater than zero, i.e.,

$$I_L > \frac{1}{2}\Delta i_L. \tag{16}$$

Substituting Eqs. (12) and (14) into inequality (16), we have the following inequality:

$$\frac{L}{RT^{\alpha}} > \frac{D^{\alpha}(1-D)^2}{2D\alpha\Gamma(\alpha)}. \tag{17}$$

In order to make the Buck–Boost converter operate in the CCM, we should consider not only the values of inductor, load resistance R, duty ratio D, and switching period T, but also the value of α. When α increases, it is easier for the Buck–Boost converter to work in the CCM. Especially when $\alpha = 1$, we can obtain the integral-order model mentioned in Ref. [18].

Then we consider the variation of output voltage $v_{\rm o}$. Using the Adomian decomposition method, we can solve the following fractional differential equation:

$$D^{\alpha}x(t) = Ax(t) + f(t), \tag{18}$$

where $0 < \alpha < 1$, $0 < t < T$. According to Eq. (5), we can have the following equation:

$$D^{\beta}v_{\rm o}(t) = -\frac{1}{CR}v_{\rm o}(t), \tag{19}$$

where $A = -1/CR$ and $f(t) = 0$. Using the results of Ref. [19] we can have

$$v_{\rm o}(t) = E_{\beta,1}(At^{\beta})V_{\rm os}, \tag{20}$$

where $E_{\beta}(\bullet)$ is the Mittag–Leffler function[5] and $V_{\rm os}$ is the initial value of the output voltage. When $t = 0$, the switch is on, so

$$v_{\rm o}(0) = E_{\beta,1}(0)V_{\rm os} = V_{\rm os}; \tag{21}$$

when $t = DT$, the output voltage is

$$v_{\rm o}(DT) = E_{\beta,1}\left[-\frac{(DT)^{\beta}}{CR}\right]V_{\rm os}. \tag{22}$$

Then, we can obtain the output voltage ripple $\Delta v_{\rm o}$ as

$$\begin{aligned} \Delta v_{\rm o} &= v_{\rm o}(0) - v_{\rm o}(DT) \\ &= V_{\rm os} - E_{\beta,1}\left[-\frac{(DT)^{\beta}}{CR}\right]V_{\rm os} \\ &= \left\{1 - E_{\beta,1}\left[-\frac{(DT)^{\beta}}{CR}\right]\right\}V_{\rm os}. \end{aligned} \tag{23}$$

The approximate expression of $V_{\rm os}$ is

$$V_{\rm os} = V_{\rm o} + \frac{\Delta v_{\rm o}}{2}. \tag{24}$$

Substituting Eq. (24) into Eq. (23) yields

$$\Delta v_{\rm o} = \left\{1 - E_{\beta,1}\left[-\frac{(DT)^{\beta}}{CR}\right]\right\}\left(V_{\rm o} + \frac{\Delta v_{\rm o}}{2}\right). \tag{25}$$

Substituting Eq. (12) into Eq. (25), we have the following equation:

$$\begin{aligned} \Delta v_{\rm o} &= 2V_{\rm o}\frac{\left\{1 - E_{\beta,1}\left[-\frac{(DT)^{\beta}}{CR}\right]\right\}}{\left\{1 + E_{\beta,1}\left[-\frac{(DT)^{\beta}}{CR}\right]\right\}} \\ &= -\frac{2V_{\rm in}D}{(1-D)}\frac{\left\{1 - E_{\beta,1}\left[-\frac{(DT)^{\beta}}{CR}\right]\right\}}{\left\{1 + E_{\beta,1}\left[-\frac{(DT)^{\beta}}{CR}\right]\right\}}. \end{aligned} \tag{26}$$

The output voltage ripple is related not only to capacitor C, load resistance R, input constant voltage $V_{\rm in}$, duty ratio D, and switching period T, but also to β. When β increases, $\Delta v_{\rm o}$ is reduced. Especially when $\beta = 1$, we can have the integral-order model mentioned in Ref. [17].

Separate the AC component in Eq. (13), then we will obtain the fractional-order AC small signal state equations as

$$\begin{bmatrix} \dfrac{{\rm d}^{\alpha}\hat{i}_L(t)}{{\rm d}t^{\alpha}} \\ \dfrac{{\rm d}^{\beta}\hat{v}_{\rm o}(t)}{{\rm d}t^{\beta}} \end{bmatrix} = \begin{bmatrix} 0 & \dfrac{1-d(t)}{L} \\ -\dfrac{1-d(t)}{C} & -\dfrac{1}{CR} \end{bmatrix} \begin{bmatrix} \hat{i}_L(t) \\ \hat{v}_{\rm o}(t) \end{bmatrix}$$

$$+\begin{bmatrix}\dfrac{d(t)}{L}\\0\end{bmatrix}\hat{v}_{\rm in}(t)+\begin{bmatrix}0&-\dfrac{1}{L}\\\dfrac{1}{C}&0\end{bmatrix}\times\begin{bmatrix}I_L\\V_{\rm o}\end{bmatrix}\hat{d}(t)+\begin{bmatrix}\dfrac{1}{L}\\0\end{bmatrix}V_{\rm in}\hat{d}(t)+\begin{bmatrix}0&-\dfrac{1}{L}\\\dfrac{1}{C}&0\end{bmatrix}\begin{bmatrix}\hat{i}_L(t)\\\hat{v}_{\rm o}(t)\end{bmatrix}\hat{d}(t)+\begin{bmatrix}\dfrac{1}{L}\\0\end{bmatrix}\hat{v}_{\rm in}(t)\hat{d}(t). \quad (27)$$

To neglect the higher order terms $\hat{d}(t)\hat{v}_{\rm o}(t)$, $\hat{d}(t)\hat{i}_L(t)$, $\hat{v}_{\rm in}(t)\hat{d}(t)$, the following state equations can be obtained

$$\begin{bmatrix}\dfrac{{\rm d}^\alpha\hat{i}_L(t)}{{\rm d}t^\alpha}\\\dfrac{{\rm d}^\beta\hat{v}_{\rm o}(t)}{{\rm d}t^\beta}\end{bmatrix}=\begin{bmatrix}0&\dfrac{1-D}{L}\\-\dfrac{1-D}{C}&-\dfrac{1}{CR}\end{bmatrix}\begin{bmatrix}\hat{i}_L(t)\\\hat{v}_{\rm o}(t)\end{bmatrix}+\begin{bmatrix}\dfrac{D}{L}\\0\end{bmatrix}\hat{v}_{\rm in}(t)+\begin{bmatrix}0&-\dfrac{1}{L}\\\dfrac{1}{C}&0\end{bmatrix}\begin{bmatrix}I_L\\V_{\rm o}\end{bmatrix}\hat{d}(t)+\begin{bmatrix}\dfrac{1}{L}\\0\end{bmatrix}V_{\rm in}\hat{d}(t). \quad (28)$$

By using Laplace transform based on the FC, we obtain

$$s^\beta\hat{v}_{\rm o}(s)=-\frac{(1-D)\hat{i}_L(s)}{C}+\frac{\hat{d}(s)I_L}{C}-\frac{\hat{v}_{\rm o}(s)}{CR}, \quad (29)$$

$$s^\alpha\hat{i}_L(s)=\frac{V_{\rm in}\hat{d}(s)+D\hat{v}_{\rm in}(s)+(1-D)\hat{v}_{\rm o}(s)-V_{\rm o}\hat{d}(s)}{L}. \quad (30)$$

The transfer function of $\hat{v}_{\rm o}(t)$ to $\hat{v}_{\rm in}(t)$ is

$$G_{v_{\rm o}v_{\rm in}}(s)=\left.\frac{\hat{v}_{\rm o}(s)}{\hat{v}_{\rm in}(s)}\right|_{\hat{d}(s)=0}=-\frac{D(1-D)}{LCs^{\alpha+\beta}+\dfrac{L}{R}s^\alpha+(1-D)^2}. \quad (31)$$

The transfer function of $\hat{v}_{\rm o}(t)$ to $\hat{d}(t)$ is

$$G_{v_{\rm o}d}(s)=\left.\frac{\hat{v}_{\rm o}(s)}{\hat{d}(s)}\right|_{\hat{v}_{\rm in}(s)=0}=-\frac{V_{\rm in}\left(1-\dfrac{LD}{(1-D)^2}s^\alpha\right)}{LCs^{\alpha+\beta}+\dfrac{L}{R}s^\alpha+(1-D)^2}. \quad (32)$$

The transfer function of $\hat{i}_L(t)$ to $\hat{v}_{\rm in}(t)$ is

$$G_{i_Lv_{\rm in}}(s)=\left.\frac{\hat{i}_L(s)}{\hat{v}_{\rm in}(s)}\right|_{\hat{d}(s)=0}=-\frac{D\left(1-\dfrac{C}{R}s^\alpha\right)}{LCs^{\alpha+\beta}+\dfrac{L}{R}s^\alpha+(1-D)^2}. \quad (33)$$

4. Simulation study and dynamical property analysis of the current-mode controlled fractional-order Buck–Boost converter in CCM

Firstly, the mathematical model of the fractional-order Buck–Boost converter in CCM is built by using SIMULINK based on the fractional differential equation of the fractional-order Buck–Boost converter which has been derived in Section 3. In the simulation we use the following parameters: $f=2.5$ kHZ, $v_{\rm in}=20$ V, $d=0.6$, $R=20\ \Omega$, $L=0.02$ H, and $C=47\ \mu$F. Therefore, the modified Oustaloup's method[20,21] is used to solve the system of fractional differential equations. Because the switching frequency of the converter is $f=2.5$ kHZ and the corresponding rotational frequency is $\omega=2\pi f=1.57\times10^4$ rad/s, we choose the parameters of Oustaloup to be $\omega_{\rm b}=5\times10^{-5}$ rad/s, $\omega_{\rm h}=2\times10^4$ rad/s, $N=8$. When $\alpha=1$ and $\beta=1$, according to Eqs. (15) and (26), we can obtain $I_L=3.75$ A, $\Delta i_L=0.24$ A, $i_{L\,\max}=3.87$ A, $V_{\rm o}=-30$ V, $\Delta v_{\rm o}=-7.62$ V, and $v_{\rm omin}=-33.81$ V. According to Fig. 2, the practical simulation results are $i_{L\,\max}=3.84$ A and $v_{\rm omin}=-33.61$ V. When $\alpha=0.8$ and $\beta=0.95$, according to Eqs. (15) and (26), we obtain $I_L=3.75$ A, $\Delta i_L=1.15$ A, $i_{L\,\max}=4.325$ A, $V_{\rm o}=-30$ V, $\Delta v_{\rm o}=-11.59$ V, and $v_{L\,\min}=-35.795$ V. According to Fig. 2, the practical simulation results are $i_{L\,\max}=4.31$ A and $v_{\rm omin}=-34.7$ V.

Then some dynamical properties of the model will be analysed. We wish to obtain a bifurcation diagram for the current-mode controlled Buck–Boost converter. The operation of the system can be described as shown in Fig. 3.

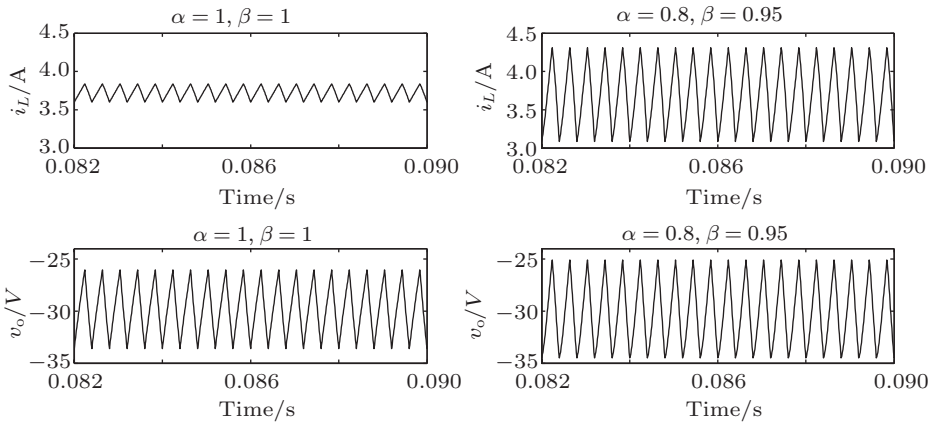


Fig. 2. Time evolutions of $i_L(t)$ and $v_o(t)$.

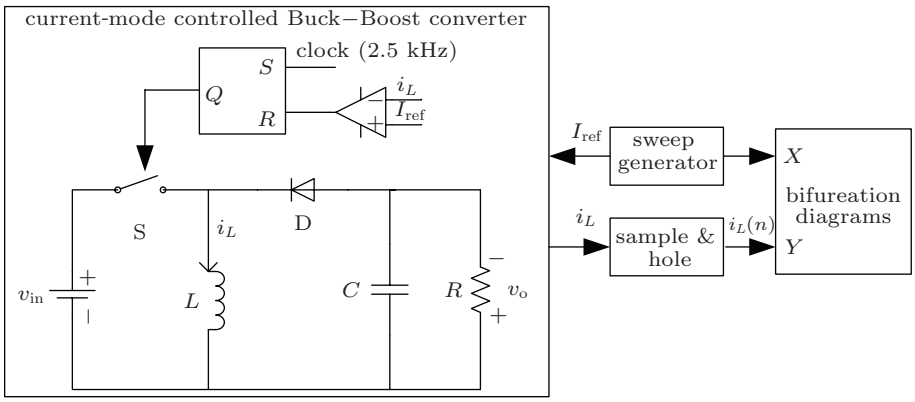


Fig. 3. Block diagram of the system for displaying bifurcation diagrams.

We use reference current $I_{\rm ref}$ as the bifurcation parameter. When $\alpha = 1$ and $\beta = 1$, the bifurcation diagram of the fractional-order Buck–Boost converter in CCM is shown in Fig. 4(a), and when $\alpha = 0.8$ and $\beta = 0.95$, the bifurcation diagram is shown in Fig. 4(b). The fractional-order Buck–Boost converter goes through period-1, period-2, period-4, and eventually to chaos as reference current $I_{\rm ref}$ varies, which is similar to the case in Fig. 4. At the same time, bifurcation points will be moved backward when α and β vary and it shows going through different routes to chaos.

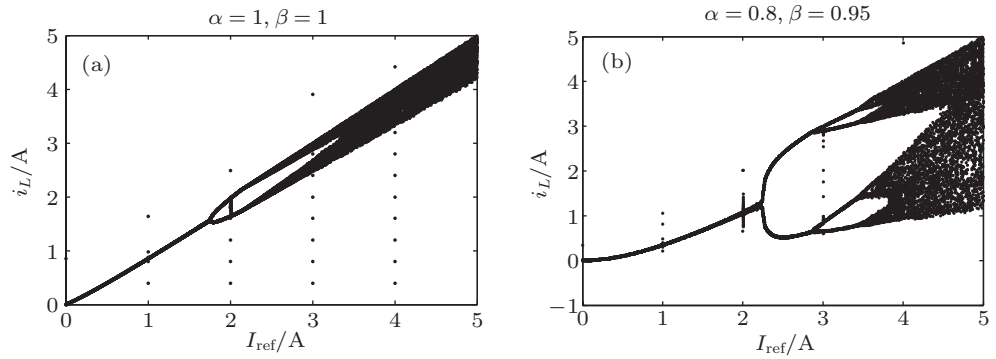


Fig. 4. Bifurcation diagrams for the current-mode controlled Buck–Boost converter. (a) $\alpha = 1$, $\beta = 1$ (b) $\alpha = 0.8$, $\beta = 0.95$

Compared with the integral-order model, the fractional-order model shows the different dynamic behaviours at the same $I_{\rm ref}$.

At $I_{\rm ref} = 2$ A, the phase portrait of period-2 is presented in Fig. 5(a) when $\alpha = 1$ and $\beta = 1$. Figure 5(b) shows the phase portrait of period-1 when $\alpha = 0.8$ and $\beta = 0.95$.

At $I_{\rm ref} = 3$ A, the phase portrait of the converter is shown in Fig. 5(c), and it can be observed that there is a strange attractor in the phase portrait for $\alpha = 1$ and $\beta = 1$, that is, the integral-order Buck–Boost converter is chaotic. However, as shown in Fig. 5(d), when$\alpha = 0.8$ and $\beta = 0.95$, the phase portrait is period-4.

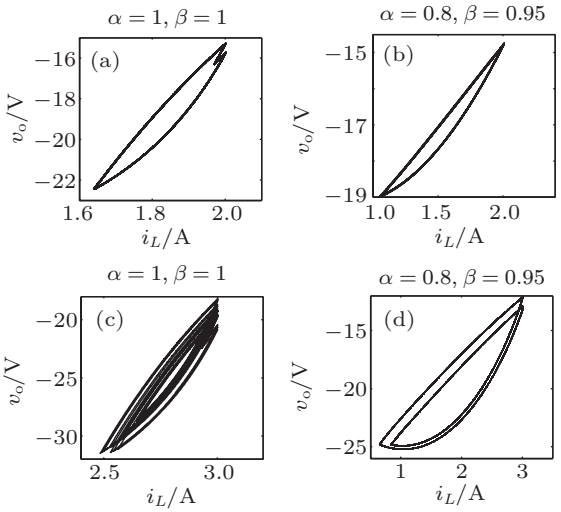


Fig. 5. Phase portraits from Buck–Boost operating under current-mode control: (a) $I_{\rm ref} = 2$ A, $\alpha = 1$, $\beta = 1$, (b) $I_{\rm ref} = 2$ A, $\alpha = 0.8$, $\beta = 0.95$, (c) $I_{\rm ref} = 3$ A, $\alpha = 1$, $\beta = 1$, and (b) $I_{\rm ref} = 3$ A, $\alpha = 0.8$, $\beta = 0.95$.

5. Conclusions

According to the theorem of fractional calculus, we created the mathematical model and the state-space averaging model of the fractional-order Buck–Boost converter. Then we gave the parameter condition of the Buck–Boost converter in CCM. Finally, some dynamical properties of the model were analysed. Through the theoretical analysis, we can obtain the following conclusions: in CCM, the DC component of the output voltage and the inductor current are independent of α and β. The inductor current ripple, the peak inductor current, the output voltage ripple, the initial value of the output voltage and the transfer functions ($G_{v_{\rm o}v_{\rm in}}(s)$, $G_{v_{\rm o}d}(s)$, and $G_{i_L v_{\rm in}}(s)$) are all related to fractional-order α or β. When α increases, the inductor current ripple and the peak inductor current decrease. When other parameters are fixed, the output voltage ripple and the initial value of the output voltage decrease with β increasing. According to simulation results, we find that when α and β vary, bifurcation points will be moved back and the converter goes through the different routes to chaos.

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