# Feshbach resonance management of vector solitons in two-component Bose–Einstein condensates*

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We consider two coupled Gross–Pitaevskii equations describing a two-component Bose–Einstein condensate with time-dependent atomic interactions loaded in an external harmonic potential, and investigate the dynamics of vector solitons. By using a direct method, we construct a novel family of vector soliton solutions, which are the linear combination between dark and bright solitons in each component. Our results show that due to the superposition between dark and bright solitons, such vector solitons possess many novel and interesting properties. The dynamics of vector solitons can be controlled by the Feshbach resonance technique, and the vector solitons can keep the dynamic stability against the variation of the scattering length.

**Keywords:** Bose–Einstein condensates, vector solitons

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## 1. Introduction

The experimental realization of multi-component Bose–Einstein condensates (BECs) has opened the floodgate for investigating the novel and interesting physics behind the interacting quantum fluids.[1,2] In the mean-field approximation, the dynamics of multi-component Bose–Einstein condensates at ultra-low temperature can be accurately described by the coupled Gross–Pitaevskii (GP) equations,[3,4] i.e., the cubic coupled nonlinear Schrödinger equations with a potential term. The coupling between the different condensates has given rise to many novel and interesting effects which do not exist in a pure Bose–Einstein condensate, such as multi-domain walls,[5,6] and multi-mode collective excitations.[7−9] In particular, as a fundamental excitation of the atomic matter waves, the formation of matter-wave solitons in BECs has also become an important subject of theoretical and experimental interest.[10−21]

Matter-wave solitons represent a highly nonlinear wave phenomenon with unique propagation features, and such nonlinear waves mainly arise from atomic interactions.[22] In pure dilute atomic gases, attractive atomic interaction can induce the excitation of a bright matter-wave soliton, which represents dispersionless and coherent transport of quantum particles with a zero background. While the creation of dark solitons is mainly due to the repulsive atomic interactions, due to the presence of inter-species atomic interactions in multi-component BECs, solitons often appear in pairs, such as 'dark–dark', 'bright–bright', and 'bright–dark' solitons.[23−30] In addition, a new type of soliton, symbiotic solitons, has been found in a two-component BECs system.[31,32] In general, these soliton pairs are called 'vector solitons'. It should be pointed out that different types of vector solitons are mainly determined by the relative strength between inter-species and intra-species atomic interactions.

Recently, experiments have demonstrated that the effective atomic interactions strength can be successfully modulated by means of the so-called Feshbach resonance technique.[33,34] With a time-dependent magnetic field $B(t)$, the atomic interactions

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can be continuously tuned from positive to negative. Thus this advanced experimental technique can provide us with a powerful tool to realize the control of quantum gas, for example, controlling the miscibility–immiscibility transition in the mixture of $^{87}$Rb and $^{85}$Rb.[35,36] Motivated by this, the aim of this paper is to investigate the dynamics of vector solitons in two-component Bose–Einstein condensates with tunable atomic interactions. We first obtain a novel family of exact vector solition solutions, which are the linear combination between dark and bright solitons in each component. By modulating the atomic interactions in different ways, we analyze the effects of atomic interactions on the dynamics of such vector solitons.

## 2. The model and vector solitons solution

We start with considering a two-coupled BECs. For the sake of simplicity, we perform our analysis in the zero-temperature mean-field approximation. We assume that two hyperfine states of the same species atom with equivalent particle number $N$ are trapped in a quasi-one-dimensional harmonic potential $V(x)$. Consequently, the evolution of the two-component BECs is governed by a pair of coupled dimensionless GP equations

$$\begin{aligned} 0 = {} & \mathrm{i}\frac{\partial \psi_j}{\partial t} + \frac{1}{2}\frac{\partial^2 \psi_j}{\partial x^2} \\ & + \left(g_{jj}\left|\psi_j\right|^2 + g_{j,3-j}\left|\psi_{j,3-j}\right|^2\right)\psi_j \\ & + \frac{1}{2}\lambda^2 x^2 \psi_j, \end{aligned} \tag{1}$$

where $j = 1, 2$, $\psi_j(x,t)$ denotes the macroscopic wave functions of the $j$-th component, $t$ and $x$ are the temporal and spatial coordinates measured in units $1/\omega_\perp$ and $l_\perp = \sqrt{\hbar/m\omega_\perp}$ with $l_\perp$ and $\omega_\perp$ being the linear oscillator length and the angular frequency in the transverse direction, respectively. The aspect ratio reads $\lambda^2 = \omega^2/\omega_\perp^2$ with $\omega$ being the angular frequency in the axial direction, $m$ is the mass of atoms. The intra-species interactions between atoms reads $g_{jj} = 4a_{jj}N/l_\perp$, while the inter-species interactions $g_{j,3-j} = 4a_{j,3-j}N/l_\perp$, where $a_{jj}$ is the intra-species s-wave scattering lengths of component $j$-th, and $a_{j,3-j}$ the inter-species one. Motivated by the famous experiment on modulating the effective atomic interactions by the Feshbach resonance technique,[33,34] in this paper we assume all the interactional parameters $g_{jj}$ are time-dependent, which can be realized by using a time-dependent magnetic field $B(t)$.

In general, since the presence of inter-species atomic interactions in two-component BECs, solitons often appear in pairs. In different regimes of interactional parameters, the forms of vector solitons are different. The "phase diagram" of vector solitons versus inter-species and intra-species interactions has been discussed in Refs. [27]–[30]. From their results, the vector solitons only exist in the form of dark–dark, bright–bright, and bright–dark solitons. In fact, a new family of vector soliton solutions can exist beyond their results. To this purpose, we consider the soliton solution of each component is a linear combination between dark and bright solitons, and the corresponding ansatz are as follows:

$$\psi_1 = \rho(t)\left[a_1 \tanh\theta + a_2 \operatorname{sech}\theta\, \mathrm{e}^{\mathrm{i}\varphi(t)}\right] \mathrm{e}^{\mathrm{i}\phi(t)}, \tag{2a}$$

$$\psi_2 = \rho(t)\left[c_1 \tanh\theta + c_2 \operatorname{sech}\theta\, \mathrm{e}^{\mathrm{i}\varphi(t)}\right] \mathrm{e}^{\mathrm{i}\phi(t)}, \tag{2b}$$

where $\theta = b_1(t)x + b_2(t)$ and $\phi = \eta_2(t)x^2 + \eta_1(t)x + \eta_0(t)$ with $b_1(t)$, $b_2(t)$, $\eta_2(t)$, $\eta_1(t)$, $\eta_0(t)$, $\rho(t)$, $\varphi(t)$, $\phi(t)$ being undetermined real functions of time $t$. Finally, $a_1, a_2, c_1, c_2$ are assumed to be arbitrary real constants.

Before analyzing the existence of such vector soliton solutions or not, we clarify the detailed significance behind such ansatz. Firstly, some well-known vector soliton solutions, such as the bright–dark or dark–bright soliton solutions, can be successfully recovered from the ansatz in Eq. (2). Secondly, the ansatz in Eq. (2) can be regarded as the interaction between the dark soliton and bright soliton in each component. If $a_1 \gg a_2$, the bright soliton term in $\psi_1$ can be considered as a perturbation on the dark one, while the dark soliton term in $\psi_2$ can be considered as a perturbation on the bright one. Thirdly, since the total phase in the dark and bright terms differ by $\varphi(t)$, these solutions will oscillate along the time direction. Especially, the superposition between the dark and bright terms in the ansatz will make the vector solitons exhibit the properties of breather solitons in space. Therefore, such vector solitons possess many interesting features, which do not exist in the well-known vector solitons.[27–30]

In order to analyze whether such vector solitons exist or not, we first substitute the ansatz (2) into the GP equations (1) and collect the coefficients of $\tanh\theta$ and $\operatorname{sech}\theta$. By separating the real part and the imaginary part for each coefficient, we derive a

family of ordinary differential equations (ODEs) with respect to $b_1(t)$, $b_2(t)$, $\eta_2(t)$, $\eta_1(t)$, $\eta_0(t)$, $\rho(t)$, $\varphi(t)$, $\phi(t)$. Finally, solving these ODEs, we obtain the existence conditions of such vector soliton solutions

$$\frac{1}{g}\frac{\mathrm{d}^2 g}{\mathrm{d}t^2} - \frac{2}{g^2}\left(\frac{\mathrm{d}g}{\mathrm{d}t}\right)^2 + \lambda^2 = 0 \tag{3}$$

with

$$g_{ij} = g > 0, \quad \rho\,(t) = \sqrt{g}, \quad \varphi = \frac{A^2}{2}\int g^2\,\mathrm{d}t, \tag{4a}$$

$$a_1 = A_1, \quad a_2 = -\sqrt{2}A_2, \quad c_1 = A_2, c_2 = \sqrt{2}A_1, \tag{4b}$$

$$b_1(t) = Ag\,(t)\,, \quad b_2(t) = -Ak\int g^2\,\mathrm{d}t, \tag{4c}$$

$$\eta_2(t) = -\frac{1}{2g}\frac{\mathrm{d}g}{\mathrm{d}t}, \quad \eta_1 = kg,$$

$$\eta_0 = \left(A^2 - \frac{k^2}{2}\right)\int g^2\,\mathrm{d}t, \tag{4d}$$

where $A_1$, $A_2$, and $k$ are arbitrary real constants, and $A = \sqrt{A_1^2 + A_2^2}$.

From these restraint conditions, on the one hand, such vector soliton solutions exist only when $g_{ij} = g > 0$ corresponding to the symmetrical GP equations with attractive interactions including the inter-species one. In fact, the actual difference of the scattering lengths in a BEC mixture of two different hyperfine states of the same atomic species is very small,[1,2] so the restraint condition can be regarded as a good approximation to a certain extent. On the other hand, the amplitude and width of the vector soliton solutions are inextricably linked with the nonlinearity coefficient $g$, namely, the dynamics of vector solitons can be conveniently modulated by using the Feshbach resonance technique, which is one of our purposes throughout this paper. Finally, we would like to point out that although the restraint equation of the Riccati type in Eq. (3) has no real significance to the experiment, it plays an a priori role for us to understand the effects of atomic interactions on the dynamics of the matter-wave soliton.

# 3. The dynamics of vector solitons

In this section, we shift our eyes to the dynamics of vector soliton solutions (2). Based on the restraint condition in Eq. (3), we analyze the dynamics of vector solitons in detail including their stability under several different nonlinear coefficients $g$. Our results show that with tunable atomic interactions, the solutions (2) not only display many interesting features, but they will also keep their dynamical stability.

## 3.1. The dynamics of vector solitons under an exponential variable scattering length

From the GP equations (1), when the axial direction trapping frequency $\omega$ takes a complex constant, the trap corresponds to the stationary confining potential with the aspect ratio $\lambda^2 < 0$ while a real frequency $\omega$ corresponds to a stationary expulsive potential with the aspect ratio $\lambda^2 > 0$. So in both cases, the nonlinearity coefficient $g(t)$ can be exactly obtained by solving the equation (3)

$$\lambda^2 > 0 : g(t) = \mathrm{e}^{\gamma t}, \tag{5a}$$

$$\lambda^2 < 0 : g(t) = \frac{\gamma}{c_1 \sin(\gamma t) - c_2 \cos(\gamma t)}, \tag{5b}$$

where we have assumed $\gamma = |\lambda|$. In the case of a confining potential with $\lambda^2 < 0$, the solutions diverge at $t = \arctan(c_2/c_1)/\lambda$ which means that the BECs will collapse. In this case, the stationary confining potential scarcely has real significance to the experiment, so we do not deeply analyze the dynamics of vector solitons in such a confining potential.

For the case of an expulsive potential, figure 1 has shown the time evolution of the vector solitons. With increasing the absolute value of scattering length exponentially, the peaking value of the solitons increases, while the width of it is compressed. Therefore, increasing the scattering length can be used to compress the vector soliton into a higher matter density. An obvious fact shown in Fig. 1 is that on the one hand the vector solitons possess the breathing characteristic along the time direction, which is mainly caused by the superposition between dark and bright terms in equations (2). The difference of phase $\varphi(t)$ in the dark and bright terms leads to the oscillation of amplitude of vector solitons with the oscillation frequency $\Omega = \mathrm{d}\varphi/\mathrm{d}t = g^2A^2/2$, which depends on the nonlinear coefficient $g$. In other words, the faster the scattering length increases, the faster the vector solitons oscillates. On the other hand, the higher density regime of $\psi_1$ coincides with the lower density regime of the $\psi_2$, which represents a microscopic phase separation. Moreover, it is interesting to observe that in the expulsive parabolic potential, the soliton propagates along the longitudinal direction, instead of oscillating in a confining parabolic potential.

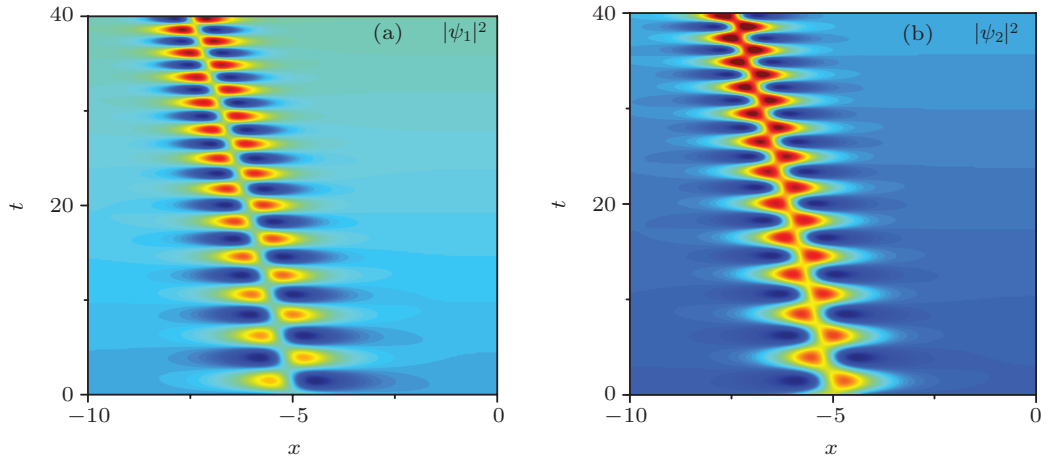



**Fig. 1.** (colour online) Evolutions of vector solitons Eq. (2) in two-component BECs with the nonlinear coefficient $g(t) = \mathrm{e}^{\lambda t}$ in an expulsive potential. Where the parameters are $A_1 = 1.2$, $A_2 = 1$, $k = -0.1$, $\lambda = 0.01$ with $\omega = 2\pi \times 7$ Hz and $\omega_\perp = 2\pi \times 700$ Hz. Panels (a) and (b) have demonstrated that the vector solitons possess the breathing characteristic along the time direction, and the amplitude of vector solitons can be compressed into a higher matter density.

A forthcoming fundamental problem is that an abrupt change in scattering length may obviously decrease the numbers of atoms in the original solitons so that the vector solitons finally vanish, which is undesirable in real experiments or application. In order to investigate the stability of the vector solitons against the variation of the scattering length in the expulsive parabolic potential, we introduce the quantities defined as follows:

$$\begin{aligned}\delta_j &= \int_{-\infty}^{\infty} \left[|\psi_j|^2 - |\psi_j(x = \pm\infty, t)|^2\right] \mathrm{d}x, \\ &= \frac{2}{A}\left(2A_{3-j}^2 - A_j^2\right), \end{aligned} \tag{6}$$

which count the exact number of atoms in the two solitons. Equation (6) displays that $\delta_j$ are independent of the variable scattering lengths and external potential frequency, namely, it does not depend on the time. This shows clearly that the number of atoms in the solitons can remain invariant. So in the propagation process, the vector solitons can keep the dynamic stability against the variation of the scattering length in the expulsive parabolic potential.

Another unavoidable question arises about the possibility of observing the phenomenon of compressing the soliton experimentally in an expulsive potential. In our paper, the choices of the potential and nonlinearity coefficient are motivated by the famous experiment on the formation of a matter-wave bright soliton.[12] A crucial question is that since we consider a BECs loaded in an expulsive potential, the lifetime of the BECs is finite, thus we must ensure that the vector solitons can be observed during the lifetime of the stable BECs. In real experiments, the lifetime of a stable BECs in an expulsive potential is of the order of 1 s which is about 220 in our dimensionless units, so the studies of such vector solitons including their stability is reasonable during the lifetime of a stable BECs in an expulsive potential. More importantly, as this vector soliton has the same feature as a bright soliton in that it exists only when the atomic interaction is attractive, it is reasonable for us to believe that the phenomenon of compressing the vector solitons can be observed experimentally by using the same experimental scheme as the well-known experiment.[12]

### 3.2. The dynamics of a dark–bright soliton pair under a periodical variable scattering length

To better understand the influence of tunable atomic interactions on vector solitons, in this section we focus on the dynamic of a bright–dark soliton pair when $A_1 = 0$. For the nonlinear coefficient $g$, we consider a periodical variable scattering length. As discussed in Refs. [37] and [38], a periodical scattering length can be obtained by means of an alternating magnetic field $B(t)$, which allows one to create a self-confined two-dimensional (2D) BEC without the magnetic trap. So we assume that the interaction parameter $g$ is changed periodically as

$$g\,(t) = 1 + \epsilon \sin(\varpi t), \tag{7}$$

where $\epsilon$ and $\varpi$ are the strength and frequency of the modulation. In this case, the aspect ratio can be obtained by solving the restraint condition in Eq. (3)

$$\lambda^2 = \frac{\epsilon\varpi^2\left(\epsilon\cos^2\varpi t + \sin\varpi t + \epsilon\right)}{(1+\epsilon\sin\varpi t)^2}. \tag{8}$$

Next, we investigate how an alternating magnetic field $B(t)$ can control the dynamics of the vector solitons. From the forms of the solutions (2) and the restraint conditions (4), the amplitude of vector solitons are $\sqrt{2g}A_2$ and $\sqrt{g}A_2$, respectively, and the width of both the bright–dark solitons are equal to $1/gA_2$, which shows that both the amplitude and width vary periodically with time. When the amplitude takes a bigger value, the vector solitons will have narrower widths. At the same time, the strength $\epsilon$ of the alternating magnetic field determines the intense level of oscillation in amplitudes and widths of vector solitons, while the frequency $\varpi$ determines the oscillation speed. These results have been shown in Fig. 2. Similarly, the vector solitons can keep their dynamic stability when the atomic interactions change periodically with time, which can be proved by using the results in Eq. (6).

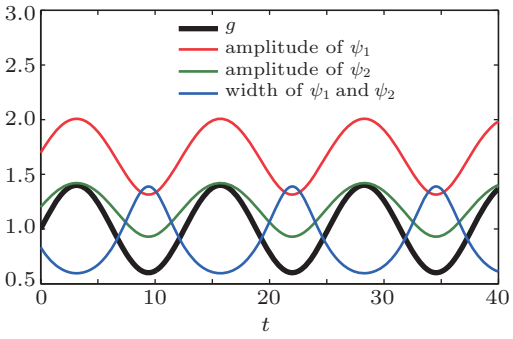



**Fig. 2.** (colour online) The evolution of Eq. (2) in two-component BECs with $g(t) = 1 + \epsilon\sin(\varpi t)$. Where the parameters are $A_1 = 0$, $A_2 = 1.2$, $k = -0.1$, $\epsilon = 0.4$, $\varpi = 0.5$. As shown in the figure, the amplitude and width of the vector solitons vary periodically with time.

## 4. Conclusion

In summary, we have investigated the dynamics of vector solitons in trapped two-component Bose–Einstein condensates with tunable interactions. Based on the two coupled GP equations, we use a direct method to obtain a novel family of exact vector soliton solutions which are in the form of a linear combination between dark and bright solitons. We also analyze the dynamics of such vector solitons by choosing different time-dependent nonlinear coefficients. Our results show that the amplitude and width of the vector solitons can be modulated by using the Feshbach resonance technique. With a time-dependent atomic interaction, the amplitude and width of the vector solitons are proportional and inversely proportional to the scattering length, respectively. Although the amplitude and width of the solitons vary with time, the exact number of atoms in the two solitons remain invariant, namely the vector solitons can keep their dynamic stability against the variation of the scattering length.

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