Absorption of a massless scalar wave from a Schwarzschild black hole surrounded by quintessence*

Liao Hao(廖 浩), Chen Ju-Hua(陈菊华)†, and Wang Yong-Jiu(王永久)

College of Physics and Information Science, Hunan Normal University, Changsha 410081*, China*

(Received 6 February 2012; revised manuscript received 14 March 2012)

By using the partial wave method, we investigate the absorption of a massless scalar wave from a Schwarzschild black hole surrounded by quintessence. We obtained the expression of absorption cross section

$$\sigma_{\rm abs}(\omega)=\frac{\pi}{\omega^2}\sum_{l=0}^{\infty}(2l+1)|T_{\omega l}(\omega)|^2=\frac{\pi}{\omega^2}\sum_{l=0}^{\infty}(2l+1)\Gamma_{\omega l}(\omega).$$

Then we numerically carry out the absorption cross section and we find that the larger the angular momentum quantum number l is, the smaller the corresponding maximum value of the partial absorption cross section is, and that the total absorption cross section tends to the geometric-optical limit $\sigma_{\rm abs}^{\rm hf}\approx\pi b_c^2$. We also find that higher value of ω_q (state parameter of quintessence) corresponds to the higher value of absorption cross section $\sigma_{\rm abs}$.

Keywords: quintessence, absorption cross section, massless scalar wave

PACS: 04.70.–s, 04.40.–b, 04.50.Gh **DOI:** 10.1088/1674-1056/21/8/080402

1. Introduction

It is well known that general relativity and quantum mechanics are incompatible in their current forms. However, after Hawking found in the 1970s that a black hole can emit, as well as scatter, absorb and radiate, and that the evaporation rate is proportional to the total absorption cross section, a lot of scholars[1] have been interested in the absorption of quantum fields by a black hole. By using a numerical method, Sanchez[2,3] found that the absorption cross section of a massless scalar wave oscillates around the geometry-optical limit. Unruh[4] showed that the scattering cross section for the fermion is exactly 1/8 of that for the scalar wave in the low-energy limit. By numerically solving the single-particle Dirac equation in Painlevé–Gullstrand coordinates, Chris Doran *et al.*[5] studied the absorption of massive spin-half particles by a small Schwarzschild black hole and they found oscillations around the classical limit whose precise form depends on the particle mass. Crispino *et al.*[6] numerically computed the absorption cross section of electromagnetic waves for an arbitrary frequency and found that its high-frequency behavior is very similar to the case of a massless scalar field in a Schwarzschild background.

In the past ten years people were interested in observing the black hole or finding the fingerprint of the black hole. A lot of scholars focused on quasinormal modes of the black hole and the Hawking effects, etc. At the same time the other fingerprint, the quantum field scattering by the black hole, became a very hot issue. Oliveria *et al.*[7] extended to study the absorption of planar in a draining bathtub, the absorption cross section of sound waves with arbitrary frequency in the canonical acoustic hole spacetime,[8] and electromagnetic absorption cross section from a Reissner–Nordström black hole.[9] Recently, absorption cross sections (or gray body factors) have been of interest in the context of higher-dimensional spacetimes[10−13] and an effective string model.[14] More recently, Décanini *et al.*[15,16] used Regge pole techniques and the complex angular momentum approach to investigate the absorption problem for a massless scalar field propagating in static and

*Project supported by the National Natural Science Foundation of China (Grant No. 10873004), the State Key Development Program for Basic Research of China (Grant No. 2010CB832803), and the Program for Changjiang Scholars and Innovative Research Team in University (Grant No. IRT0964).

†Corresponding author. E-mail: jhchen@hunnu.edu.cn

© 2012 Chinese Physical Society and IOP Publishing Ltd

spherically symmetric black holes of arbitrary dimension endowed with a photon sphere; they obtained an elegant and powerful resummation of the absorption cross section and extracted the physical information encoded in the sum of the partial wave contributions. On the other hand, a large number of astronomical observations, such as type Ia supernovae, CMB, and the large scale structure, convincingly showed the accelerating expansion of the universe. The expansion implies the valuable contribution of matter with negative pressure to the evolution of the universe. The origin of the accelerating expansion is regarded to be that the universe is dominated by an exotic component with the negative pressure called "dark energy" which constitutes 70 percent of the energy density of the universe. There are several candidates for dark energy: the first is the cosmological constant,[17,18] and the second is the so-called dynamic candidates such as: Phantom,[19−22] quintessence,[23−25] K-essence,[26,27] and quintom.[28−30]

We all know that a lot of scholars have investigated the quasinormal modes,[31,32] spacetime structure,[33] and gravitational effects[34−38] in a Schwarzschild black hole surrounded by quintessence. In this paper we focus on investigating the absorption of a massless scalar wave from a Schwarzschild black hole surrounded by quintessence. The outline of this paper is as follows: in Section 2, we briefly review the spacetime structure of the Schwarzschild black hole surrounded by quintessence. In Section 3, we introduce the partial wave method for the absorption cross section from the black hole. In Section 4, we show numerical results and discussions.

2. The partial wave method

2.1. Massless scalar field equation

The spherically symmetric metric for a Schwarzschild black hole surrounded by the quintessence[38] is

$$\begin{aligned} \mathrm{d}s^2 = &-\left(1-\frac{2M}{r}-\frac{c}{r^{3\omega_{\mathrm{q}}+1}}\right)\mathrm{d}t^2 \\ &+\left(1-\frac{2M}{r}-\frac{c}{r^{3\omega_{\mathrm{q}}+1}}\right)^{-1}\mathrm{d}r^2 \\ &+r^2(\mathrm{d}\theta^2+\sin^2\theta\,\mathrm{d}\varphi^2), \end{aligned} \tag{1}$$

where M is the mass of the black hole, ω_{q} is the quintessential state parameter and parameter c, which is related to ρ_{q} (the density of quintessential), is the normalization factor.

From the quantum field theory we know that massless scalar particle follows the Klein–Gordon (KG) equation. In the curved spacetime, the general KG equation of the massless scalar field Ψ is

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\Psi)=0. \tag{2}$$

In this case, the KG equation can be written as the following form

$$\frac{1}{f}\frac{\partial^2\Psi}{\partial t^2}-\frac{1}{r^2}\frac{\partial}{\partial r}\left(fr^2\frac{\partial\Psi}{\partial r}\right)+\frac{1}{r^2}\nabla^2\Psi=0, \tag{3}$$

where

$$f=1-\frac{2M}{r}-\frac{c}{r^{3\omega_q+1}}. \tag{4}$$

For Eq. (3), the solutions of the positive-frequency is

$$\Psi_{\omega lm}=\frac{\psi_{\omega l}(r)}{r}\,\mathrm{Y}_{lm}\,\mathrm{e}^{-\mathrm{i}\omega t}, \tag{5}$$

where Y_{lm} are scalar spherical harmonic functions, l and m are angular momentum quantum numbers.

Substituting Eq. (5) into Eq. (3), we obtain the radial equation

$$f\frac{\mathrm{d}}{\mathrm{d}r}\left(f\frac{\mathrm{d}\psi_{\omega l}(r)}{\mathrm{d}r}\right)+\left[\omega^2-V_{\mathrm{eff}}(r)\right]\psi_{\omega l}(r)=0, \tag{6}$$

where

$$V_{\mathrm{eff}}=f\left(\frac{f'}{r}+\frac{l(l+1)}{r^2}\right), \tag{7}$$

and $f'=\mathrm{d}f/\mathrm{d}r$. The effective potential V_{eff} can be written as

$$\begin{aligned} V_{\mathrm{eff}}(r) = &\left(1-\frac{2M}{r}-\frac{c}{r^{3\omega_q+1}}\right)\left(\frac{l(l+1)}{r^2}\right. \\ &\left.+\frac{2Mr^{-2}+(1+3\omega_q)cr^{-2-3\omega_q}}{r}\right). \end{aligned} \tag{8}$$

In Fig. 1, we plot the effective potential V_{eff} with different l for the Schwarzschild black hole surrounded by quintessence. The effective potential V_{eff} acts as the scattering barrier when the scalar field is scattered by the black hole. From this figure we can see that the scattering potential V_{eff} depends on the values of l and r and the scattering barrier increases with the angular momentum quantum number l. The peak values of the potential barrier get higher and the locations of the peak points move to the right side as l increases. It is obvious that $V_{\mathrm{eff}}=0$ at horizon $r=r_+$ and it tends to zero at infinite ($r\to\infty$).

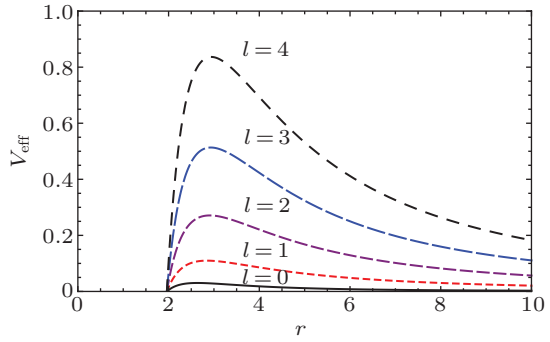


Fig. 1. (colour online) The scattering potential $V_{\rm eff}(r)$ of the Schwarzschild black hole surrounded by quintessence for different values of the angular momentum l by fixed parameters $M = 1$, $c = 0.01$, $\omega_{\rm q} = -2/3$. From this figure, we can see that $V_{\rm eff}(r)$ vanishes at the black hole horizon r_+ and it will tend to zero as $r \to \infty$.

If we introduce the tortoise coordinate x, which is defined by $\mathrm{d}/\mathrm{d}x = f\,\mathrm{d}/\mathrm{d}r$, equation (6) can be written as

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}\psi_{\omega l}(x) + [\omega^2 - V_{\rm eff}(x)]\psi_{\omega l}(x) = 0. \tag{9}$$

In Fig. 2, we present the potential $V_{\rm eff}$ with l from 1 to 4 as a function of the tortoise coordinate x. From this figure we can obviously see that the potential acts as a typical scattering barrier. It is obvious that the larger the value of l is, the higher the scattering barrier is.

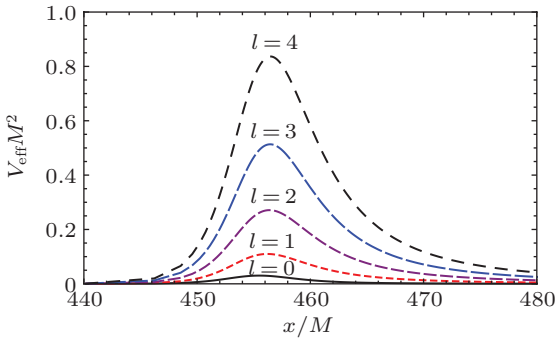


Fig. 2. (colour online) The scattering potential $V_{\rm eff}(x)$ of a Schwarzschild black hole surrounded by quintessence for different values of the angular momentum l by fixed parameters $M = 1$, $c = 0.01$, $\omega_{\rm q} = -2/3$ in the tortoise coordinate x. From this figure we can obviously see that the potential acts as a typical scattering barrier.

Taking into account the properties of the effective scattering potential, we can take the asymptotic solutions of Eq. (9) as follows:

$$\psi_{\omega l} \approx \begin{cases} T_{\omega l}(\omega)\,\mathrm{e}^{-\mathrm{i}\omega x}, & \text{for } x \to -\infty, \\ \omega x[(-\mathrm{i})^{l+1}\,\mathrm{h}_l^{(1)*}(\omega x) + (\mathrm{i})^{l+1} R_{\omega l}(\omega)\,\mathrm{h}_l^{(1)}(\omega x)], & \text{for } x \to +\infty, \end{cases} \tag{10}$$

where $\mathrm{h}_l^{(1)}(\omega x)$ are the spherical Bessel functions of the third kind.[39] $|T_{\omega l}|^2$ and $|R_{\omega l}|^2$ are the transmission and reflection coefficients, respectively, which satisfy the usual probability conservation equation $|T_{\omega l}|^2 + |R_{\omega l}|^2 = 1$. Considering the asymptotic form of the spherical Bessel function of the third kind, we have

$$\mathrm{h}_l^{(1)}(\omega x) = \mathrm{j}_l(\omega x) + \mathrm{i}\mathrm{n}_l(\omega x) \approx (-\mathrm{i})^{l+1}\,\mathrm{e}^{\mathrm{i}\omega x}/(\omega x), \tag{11}$$

when $|\omega x| \gg l(l+1)/2$. So in the region $|\omega x| \gg l(l+1)/2$, we can rewrite Eq. (10) as

$$\psi_{\omega l} \approx \begin{cases} T_{\omega l}(\omega)\,\mathrm{e}^{-\mathrm{i}\omega x}, & \text{for } x \to -\infty, \\ \mathrm{e}^{-\mathrm{i}\omega x} + R_{\omega l}\,\mathrm{e}^{+\mathrm{i}\omega x}, & \text{for } x \to +\infty. \end{cases} \tag{12}$$

2.2. Absorption cross section

Because the Schwarzschild black hole surrounded by quintessence is asymptotically flat, we consider the incident wave propagates along the z direction

$$v(x^\mu) = C\,\mathrm{e}^{-\mathrm{i}\omega(z-t)}, \tag{13}$$

where C is a normalization constant. So the incident current density can be written as

$$S_{\rm I} = 2|C|^2\omega, \tag{14}$$

then, we expand Eq. (13) in terms of Legendre functions as[40]

$$v(x^\mu) = C\sum_{l=0}^{\infty} \mathrm{i}^l(2l+1)\mathrm{j}_l(\omega r)\,\mathrm{P}_l(\cos\theta)\,\mathrm{e}^{-\mathrm{i}\omega t}. \tag{15}$$

In this expression, $\mathrm{P}_l(\cos\theta)$ and $\mathrm{j}_l(\omega r)$ are the Legendre polynomials and the spherical Bessel functions, respectively, and $\mathrm{j}_l(\omega r)$ has an asymptotic form:

$$\mathrm{j}_l \approx \frac{1}{x}\sin\left(x - \frac{l\pi}{2}\right) = \frac{l^{l+1}}{2x}[\mathrm{e}^{-\mathrm{i}x} - (-1)^l\,\mathrm{e}^{\mathrm{i}x}], \tag{16}$$

as $x \gg l(l+1)/2$ this expression is valid, so we can write Eq. (15) as

$$v(x^\mu) \approx \mathrm{i}C\sum_{l=0}^{\infty}\frac{(-1)^l(2l+1)}{2\omega}\frac{\mathrm{e}^{-\mathrm{i}\omega r} + R_{\omega l}\,\mathrm{e}^{\mathrm{i}\omega r}}{r} \times \mathrm{P}_l(\cos\theta)\,\mathrm{e}^{-\mathrm{i}\omega t}. \tag{17}$$

Because the flux associated with the mode Eq. (14) is

$$J = \oint_{r\to\infty} \mathrm{d}\Omega r^2 S^r(v(x_\nu), v(x_\nu)) \tag{18}$$

and

$$\int_0^\pi \mathrm{P}_l(\cos\theta)\,\mathrm{P}_{l'}(\cos\theta)\sin\theta\,\mathrm{d}\theta = \frac{2}{2l+1}\delta_{ll'}, \tag{19}$$

we obtain

$$J = -\frac{2\pi|C|^2}{\omega}\sum_{l=0}^{\infty}(2l+1)(1-|R_{\omega l}|^2). \tag{20}$$

The definition of the absorption cross section is

$$\sigma_{\mathrm{abs}} = -\frac{J}{S_{\mathrm{I}}}, \tag{21}$$

so the total absorption can be written immediately as

$$\begin{aligned}\sigma_{\mathrm{abs}}(\omega) &= \sum_{l=0}^{\infty}\sigma^l_{\mathrm{abs}}(\omega)\\ &= \frac{\pi}{\omega^2}\sum_{l=0}^{\infty}(2l+1)|T_{\omega l}(\omega)|^2.\end{aligned} \tag{22}$$

Following the Décanini *et al.*[15,16] method, it is worthwhile recalling that the transmission coefficients $T_{\omega l}(\omega)$ permit us to construct the grey-body factors (the absorption probabilities by the Schwarzschild black hole for scalar particles with energy ω and angular momentum l. They are given by $|T_{\omega l}(\omega)|^2 = \Gamma_{\omega l}(\omega)$ and, for the massless scalar field considered here, we can express the absorption cross section in terms of them in the following form

$$\sigma_{\mathrm{abs}}(\omega) = \sum_{l=0}^{\infty}\sigma^l_{\mathrm{abs}}(\omega) = \frac{\pi}{\omega^2}\sum_{l=0}^{\infty}(2l+1)\Gamma_{\omega l}(\omega), \tag{23}$$

where $\Gamma_{\omega l}(\omega)$ for $l = 0, 1, \ldots$ are the grey-body factors defined by Eqs. (8) and (9), so we can obtain the partial cross section

$$\sigma^l_{\mathrm{abs}}(\omega) = \frac{\pi}{\omega^2}(2l+1)\Gamma_{\omega l}(\omega). \tag{24}$$

And we note that Sánchez[3,41,42] had obtained a fit of the numerical absorption cross section of the Schwarzschild black hole using sinc-function $\sigma_{\mathrm{abs}} = -8\pi\,\mathrm{e}^{-\pi}\sigma_{\mathrm{geo}} \times \mathrm{sinc}[2\pi(3\sqrt{3}M)\omega]$, where $\mathrm{sinc}(x) = \sin x/x$ and $\sigma_{\mathrm{geo}} = 27\pi M^2$. Moreover, this formula permits us to interpret naturally the period of the maxima of the fluctuations in terms of constructive interferences of the "surface wave" trapped near the photon sphere.[43,44] Décanini[15,16] extended this formula which describes numerically and physically the fluctuations of the high energy absorption cross section. They pointed out that it can also be actually used for rather low frequency. The Regge poles of the S matrix, as the associated quasinormal complex frequencies, are due to tunneling near the top of the effective scattering potential (8). So we obtain the following formula for $\omega > 0$ and $l \in N$,[45,46]

$$\Gamma_{\omega l}(\omega) = \frac{1}{1+\exp\left[-2\pi\dfrac{\omega^2 - V_l(x_{\mathrm{p}})}{\sqrt{-2V_l''(x_{\mathrm{p}})}}\right]}, \tag{25}$$

where the coefficient $\Gamma_{\omega l}(\omega)$ is the absorption probability for a particle with energy ω and angular momentum l, and x_{p} is the value of polar coordinate x corresponding to the peak of the effective potential. We assume that r_+ is a simple root of $f(r)$ and that we have $f(r) > 0$ for $r > r_+$. We also assume that there exists a value $r_{\mathrm{c}} \in [r_+, +\infty]$ for

$$f'(r_{\mathrm{c}}) - \frac{2}{r_{\mathrm{c}}}f(r_{\mathrm{c}}) = 0, \tag{26}$$

$$f''(r_{\mathrm{c}}) - \frac{2}{r_{\mathrm{c}}^2}f(r_{\mathrm{c}}) > 0. \tag{27}$$

That is to say the spacetime considered is an asymptotically flat black hole with an event horizon at r_+, its exterior corresponding to $r > r_+$, and the assumptions on r_{c} imply the existence of a photon sphere located at r_{c}, which is the support of unstable circular null geodesics (see Ref. [16] for more details). We also note that the geometrical cross section of the black hole $\sigma_{\mathrm{geo}} = \pi b_{\mathrm{c}}^2/\Gamma(2)$ and the sinc function involves the orbital period $b_{\mathrm{c}} = r_{\mathrm{c}}/\sqrt{f(r_{\mathrm{c}})}$ of a massless particle orbiting the black hole on the photon sphere. Andersson in Ref. [47] gave the following formula

$$V_l(x_{\mathrm{p}}) = \frac{f(r_{\mathrm{c}})}{r_{\mathrm{c}}^2}(l+1/2)^2, \tag{28}$$

$$V_l''(x_{\mathrm{p}}) = -2[\frac{\eta_{\mathrm{c}}2f(r_{\mathrm{c}})}{r_{\mathrm{c}}^2}]^2(l+1/2)^2, \tag{29}$$

where the parameter η_{c} is given by

$$\eta_{\mathrm{c}} = \frac{1}{2}\sqrt{4f(r_{\mathrm{c}}) - 2f''(r_{\mathrm{c}})}. \tag{30}$$

3. Numerical results and discussions

Recently, Décanini *et al.*[15,16] have investigated the absorption cross section for a massless scalar field propagating in a static and spherically symmetric black hole with arbitrary dimensions endowed with a photon sphere. In this section we use their numerical

method to compute the absorption cross section for a Schwarzschild black hole surrounded by quintessence.

The total absorption cross section is the summation of the partial absorption cross section $\sigma^{l}_{\rm abs}$ for $l = 0$ up to $l = 5$. We can see that the total absorption cross section $\sigma^{\rm total}_{\rm abs}$ oscillates around the limit of geometrical optics. At the intermediate regime $\omega M \sim 0.6$, the contribution from the partial absorption sections creates a regular oscillation pattern. Each maximum in the oscillations of the total absorption cross sections is linked to the maximum of a particular partial wave. When the wavelength of the incoming wave is much smaller than the black hole horizon (i.e., $\omega M \gg 1$), the absorption cross section tends to the geometric-optical limit of $\sigma^{\rm hf}_{\rm abs} = \pi b^2_{\rm c} \approx 25$. This is verified by the total absorption cross section for the massless scalar field which was computed by Sanchez[3] in the last century, and also, is verified[6] for the electromagnetic wave absorption cross section and for the Fermion absorption cross section in the Schwarzschild black hole.[48]

In Fig. 3, we plot the total absorption cross section of a massless scalar wave from a Schwarzschild black hole surrounded by quintessence with $\omega_{\rm q} = -3/6$, $-4/6$, and $-5/6$, respectively. The total absorption cross sections contribute from the angular momentum $l = 0$ up to $l = 5$ for each one. We can see that the total absorption cross section $\sigma^{\rm total}_{\rm abs}$ oscillates around the limit of geometrical optics. By comparing three curves in Fig. 3, we can see that the absorption cross section decreases as the quintessence state parameter $\omega_{\rm q}$ decreases. That is to say the dark energy (quintessence) takes effect on the scattering of a massless scalar wave from the black hole. This similar effect has been found for an electromagnetic field[9] and the massless scalar field[49] from a Reissner–Nordström black hole.

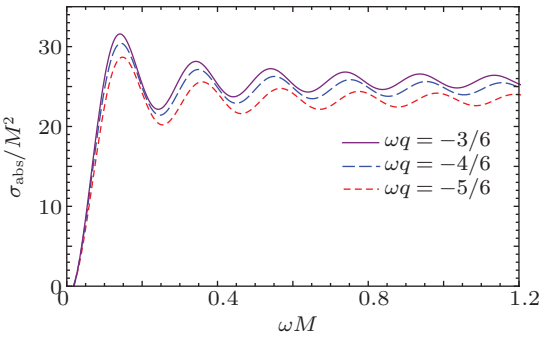


Fig. 3. (colour online) The variation of the total absorption cross section with the state parameter $\omega_{\rm q} = -3/6$, $-4/6$, $-5/6$ of quintessence by fixed parameter $c = 0.01$.

Just as Crispino *et al.*[49] have pointed out: "In principle, highly accurate measurements of, for example, the gravitational wave flux scattered by a black hole could one day be used to estimate the black hole's charge. A more immediate possibility is that scattering and absorption patterns may be observed with black hole analog systems created in the laboratory. Even if experimental verification is not forthcoming, we hope that studies of wave scattering by black holes will continue to improve our understanding of how black holes interact with their environments." We all know that, in order to explain the cosmological accelerating expansion, the physics regarded that the universe is dominated by an exotic component with negative pressure called "dark energy" which constitutes 70 percent of the energy density of the universe. So we hope to gain some knowledge about the dark energy through investigating the absorption cross section from a black hole.

References

[1] Futterman J A H, Handler F A and Matzner R A 1988 *Scattering from Black Holes* (Cambridge: Cambridge University Press) p. 254

[2] Sánchez N 1977 *Phys. Rev.* D **16** 937

[3] Sánchez N 1978 *Phys. Rev.* D **18** 1030

[4] Unruh W G 1976 *Phys. Rev.* D **14** 3251

[5] Doran C, Lasenby A, Dolan Sam R and Hinder I 2005 *Phys. Rev.* D **71** 124020

[6] Crispino Luís C B, Oliveira Ednilton S, Higuchi A and Matsas G E A 2007 *Phys. Rev.* D **75** 104012

[7] Oliveira Ednilton S, Dolan Sam R and Crispino Luís C B 2010 *Phys. Rev.* D **81** 124013

[8] Crispino Luís C B, Oliveira Ednilton S and Matras George E A 2007 *Phys. Rev.* D **76** 107502

[9] Crispino Luís C B and Olivera E S 2008 *Phys. Rev.* D **78** 024011

[10] Chen J H, Liao H and Wang Y J 2011 *Phys. Lett.* B **705** 124

[11] Das S R, Gibbons G and Mathur S D 1997 *Phys. Rev. Lett.* **78** 417

[12] Jung E, Kim S H and Park D K 2004 *Phys. Lett.* B **602** 105

[13] Crispino Luís C B, Higuchi A and Matsas G E A 2001 *Phys. Rev.* D **63** 124008

[14] Gubser S S 1997 *Phys. Rev.* D **56** 7854

[15] Décanini Y, Esposito-Farése G and Folacci A 2011 *Phys. Rev.* D **83** 044032

[16] Décanini Y, Folacci A and Raffaelli B 2010 *Phys. Rev.* D **81** 104039

[17] Padmanabham T 2003 *Phys. Rep.* **380** 23

[18] Alcaniz J S 2004 *Phys. Rev.* D **69** 083501

[19] Caldwell R R and Rahul D 2002 *Phys. Lett.* B **545** 23

[20] Chimento L P and Lazkoz R 2003 *Phys. Rev. Lett.* **91** 211301

[21] Caldwell R R, Rahul D and Paul J S 1998 *Phys. Rev. Lett.* **80** 1582
[22] Sahniv L and Wang L M 2000 *Phys. Rev.* D **62** 103517
[23] Capozziello S, Cardone V F, Piedipalumbo E and Rubano C 2006 *Class. Quantum Grav.* **23** 1205
[24] Vikman A 2005 *Phys. Rev.* D **71** 023515
[25] Rogerio R and Joshua A F 2006 *arXiv*: astro-ph/06112421V1
[26] Chiba T, Takahiro O and Masahide Y 2000 *Phys. Rev.* D **62** 023511
[27] Scherrer R J 2004 *Phys. Rev. Lett.* **93** 011301
[28] Wei H 2005 *Class. Quantum Grav.* **22** 3189
[29] Zhao G B, Xia T Q, Li M Z, Feng B and Zhang X M 2005 *Phys. Rev.* D **72** 123515
[30] Feng B, Xia T Q and Li M Z 2006 *Phys. Lett.* B **20** 2075
[31] Zhang Y, Gui Y X and Yu F 2009 *Chin. Phys. Lett.* **26** 030401
[32] Zhang Y and Gui Y X 2006 *Class. Quantum Grav.* **23** 6141
[33] Chen J H and Wang Y J 2010 *Int. J. Mod. Phys.* A **25** 1439
[34] Tian G H, Wang S K and Zhao Z 2006 *Chin. Phys.* **15** 1430
[35] Chen J H and Wang Y J 2008 *Chin. Phys.* B **17** 1184
[36] Chen J H and Wang Y J 2008 *Commun. Theor. Phys.* **50** 101
[37] Chen J H and Wang Y J 2010 *Chin. Phys.* B **19** 010401
[38] Chen J H and Wang Y J 2003 *Class. Quantum Grav.* **20** 3897
[39] Abrammowitz M and Stegun I A 1965 *Handbook of Mathematical Functions* (New York: Dover Publications) p. 53
[40] Higuchi A 2001 *Class. Quantum Grav.* **18** L130
[41] Mashhoon B 1973 *Phys. Rev.* D **7** 2807
[42] Fabbri R 1975 *Phys. Rev.* D **12** 933
[43] Misner C W, Thorne K S and Wheeler J A 1973 *Gravitation* (San Francisco: Freeman W H and Company) p. 563
[44] Newton R G 1982 *Scattering Theory of Waves and Particles* (New York: Springer-Verlag) p. 531
[45] Iyer S and Will C M 1987 *Phys. Rev.* D **35** 3621
[46] Iyer S 1987 *Phys. Rev.* D **35** 3632
[47] Andersson N 1995 *Phys. Rev.* D **52** 1808
[48] Doran C, Lasenby A, Dolan Sam R and Hinder I 2005 *Phys. Rev.* D **71** 124020
[49] Crispion Luís C B, Dolan Sam R and Oliveira Ednilton S 2009 *Phys. Rev.* D **79** 064022