Entanglement evolution of three-qubit mixed states in multipartite cavity–reservoir systems*

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We analyze the multipartite entanglement evolution of three-qubit mixed states composed of a GHZ state and a W state. For a composite system consisting of three cavities interacting with independent reservoirs, it is shown that the entanglement evolution is restricted by a set of monogamy relations. Furthermore, as quantified by the negativity, the entanglement dynamical property of the mixed entangled state of cavity photons is investigated. It is found that the three cavity photons can exhibit the phenomenon of entanglement sudden death (ESD). However, compared with the evolution of a generalized three-qubit GHZ state which has the equal initial entanglement, the ESD time of mixed states is later than that of the pure state. Finally, we discuss the entanglement distribution in the multipartite system, and point out the intrinsic relation between the ESD of cavity photons and the entanglement sudden birth of reservoirs.

Keywords: entanglement evolution, three-qubit mixed states, cavity-reservoir systems

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1. Introduction

Quantum entanglement is one of the important nonclassical features of quantum mechanics. In recent decades, it has been rediscovered as a crucial resource in quantum information processing, such as quantum communication and quantum computation.[1] Therefore, it is a fundamental problem to characterize the entanglement property of quantum systems. Till now, although the entanglement in bipartite systems is well understood in many aspects, the corresponding property in multipartite systems is far from clear, even for three-qubit quantum states.[2]

In three-qubit pure states, the two-qubit and genuine three-qubit entanglement can be quantified by the concurrences[3] and the three-tangle,[4] respectively, which constitute a good hierarchy structure for the entanglement characterization. However, for three-qubit mixed states, the case is much more complicated and different. Lohmayer *et al.* analyzed the entanglement property of a three-qubit mixed state composed of a GHZ state and a W state.[5] They pointed out that, in some region of the mix probability, the mixed state is entangled but there is no concurrence and three-tangle. Compared with the entanglement in pure states, this is a new type of entangled state which exhibits in the qubit-block form. Furthermore, based on a purification scenario, it is elaborated that the entanglement actually comes from the genuine multipartite entanglement between the system and its environment.[6] However, for a deep understanding of the three-qubit entanglement, it is desirable to investigate further the entanglement property in the mixed states and compare it with that of pure states.

Entanglement dynamical property is very important in practical quantum information processing. This is because that entanglement always decays due

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to the unwanted interaction between the system and its environment. Theoretical studies have revealed that entanglement does not always decay in an asymptotic way and it can disappear in a finite time, which is referred to as entanglement sudden death (ESD)[7−10] and has been detected in photon and atom systems experimentally[11,12] (see also a review paper[13] and the references therein).

Recently, Lopez *et al.* analyzed the entanglement dynamics of two cavities interacting with independent reservoirs.[14] They showed that when the cavity entanglement experiences the ESD phenomenon, the reservoir entanglement suddenly and necessarily appears. Moreover, it is shown that the entanglement evolution is restricted by a monogamy relation and the genuine multipartite entanglement is involved in the dynamical procedure.[15,16] However, these papers only addressed the evolution with bipartite initial states, and the case of multipartite initial states is awaited for further studies. In three-qubit mixed states, the qubit-block entanglement[5] is a new kind of entanglement type and its dynamical property is yet to be considered.

In this paper, we analyze the entanglement dynamical property of a class of three-qubit mixed states composed of a GHZ state and a W state. For a composite system consisting of three cavities with independent reservoirs, we first show that the entanglement evolution obeys a set of monogamy relations. Then, we investigate the entanglement evolution of three cavity photons, which can exhibit the ESD property. This property is further compared with that of a generalized GHZ state. Finally, we discuss the entanglement distribution in the multipartite system and give the relation of entanglement between the cavity photons and the reservoirs.

2. Entanglement monogamy relations in multipartite cavity–reservoir systems

Before deriving the monogamy relation in the dynamical evolution, we first introduce the cavity–reservoir system. The interaction between a single cavity and its N-mode reservoir is characterized by the Hamiltonian[14]

$$\hat{H} = \hbar\omega\hat{a}^{\dagger}\hat{a} + \hbar\sum_{k=1}^{N}\omega_k\hat{b}_k^{\dagger}\hat{b}_k + \hbar\sum_{k=1}^{N}g_k(\hat{a}\hat{b}_k^{\dagger} + \hat{b}_k\hat{a}^{\dagger}). \quad (1)$$

When a cavity mode contains a single photon and its corresponding reservoir is in the vacuum state, the quantum state is $|\phi_0\rangle = |1_c\rangle \otimes |0_r\rangle$, in which $|0_r\rangle = \prod_{k=1}^{N}|0_k\rangle_r$. In the limit of $N \to \infty$, the dynamical evolution given by the Hamiltonian in Eq. (1) leads to the output state

$$|\phi_t\rangle = \xi(t)|1_c\rangle|0_r\rangle + \chi(t)|0_c\rangle|1_r\rangle, \quad (2)$$

where $|1_r\rangle = (1/\chi(t))\sum_{k=1}^{N}\lambda_k(t)|1_k\rangle_r$ with the amplitude converging to $\chi(t) = [1-\exp(-\kappa t)]^{1/2}$.[14] Thus, the cavity and reservoir evolve as an effective two-qubit system.

We consider three cavities being affected by the dissipation of three independent reservoirs. The initial state of three cavity photons and the corresponding reservoirs is

$$\rho_{c_1c_2c_3r_1r_2r_3}(0) = \rho_{c_1c_2c_3} \otimes |000\rangle\langle 000|_{r_1r_2r_3}, \quad (3)$$

where the reservoirs are in the vacuum state and the cavity photons are in the new type of entangled state[5]

$$\rho_{c_1c_2c_3}(0) = p|\mathrm{GHZ}\rangle\langle\mathrm{GHZ}| + (1-p)|\mathrm{W}\rangle\langle\mathrm{W}|, \quad (4)$$

where $|\mathrm{GHZ}\rangle = (|000\rangle + |111\rangle)/\sqrt{2}$ and $|\mathrm{W}\rangle = (|001\rangle + |010\rangle + |100\rangle)/\sqrt{3}$, respectively. In order to conveniently obtain the output state of the multipartite cavity–reservoir systems, we introduce an ancillary qubit z which can purify the input state in Eq. (3). So, the global initial state is

$$\begin{aligned}|\Psi_0\rangle = &(\sqrt{p}|\mathrm{GHZ}\rangle|0\rangle \\ &+ \sqrt{1-p}|\mathrm{W}\rangle|1\rangle)_{c_1c_2c_3z}|000\rangle_{r_1r_2r_3}. \quad (5)\end{aligned}$$

Under the time evolution, the global output state has the form

$$\begin{aligned}|\Psi_t\rangle = &\sqrt{p/2}(|000000\rangle + |\phi_t\rangle|\phi_t\rangle|\phi_t\rangle)_{c_1r_1c_2r_2c_3r_3}|0\rangle_z \\ &+ \sqrt{(1-p)/3}(|0000\rangle|\phi_t\rangle + |00\rangle|\phi_t\rangle|00\rangle \\ &+ |\phi_t\rangle|0000\rangle)_{c_1r_1c_2r_2c_3r_3}|1\rangle_z, \quad (6)\end{aligned}$$

where $|\phi_t\rangle = \xi(t)|10\rangle + \chi(t)|01\rangle$ with $\xi(t) = \exp(-\kappa t/2)$ and $\chi(t) = (1-\xi(t)^2)^{1/2}$. For the three cavity–reservoir system, its output state is

$$\rho(t)_{c_1r_1c_2r_2c_3r_3} = \mathrm{Tr}_z(|\Psi_t\rangle\langle\Psi_t|). \quad (7)$$

In many-body quantum systems, entanglement is monogamous, which means that entanglement cannot

be freely shared among many parties. For a three-qubit state ρ_{ABC}, Coffman *et al.* proved the relation[4]

$$C^2_{A|BC} \geq C^2_{AB} + C^2_{AC}, \tag{8}$$

where the concurrence $C^2_{A|BC} = 4\min[\det(\rho_A)]$ quantifies the bipartite entanglement in the partition $A|BC$ (the minimum runs over all the pure state decompositions of ρ_{ABC}) and the concurrence C^2_{ij} quantifies the two-qubit entanglement which is defined as $C(\rho_{ij}) = \max(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4})$ with the decreasing nonnegative real numbers λ_i being the eigenvalues of the matrix $R_{ij} = \rho_{ij}(\sigma_y \otimes \sigma_y)\rho^*_{ij}(\sigma_y \otimes \sigma_y)$.[3] The monogamy relation in Eq. (8) is further generalized to the N-qubit quantum state, which has the form $C^2_{A_1|A_2\cdots A_N} \geq C^2_{A_1A_2} + \cdots + C^2_{A_1A_N}$.[17] Moreover, for the two-qubit partition $A_1A_{1'}|A_2A_{2'}\cdots A_NA_{N'}$, a set of monogamy relations are also proved in Ref. [15]. But, these relations are mainly focused on the distribution of two-qubit entanglement, the monogamy relation for multiqubit entanglement still awaits further study.

In the dissipative procedure of the multipartite cavity–reservoir systems, we can derive the following relations

$$\begin{aligned} C^2_{c_1|c_2c_3z}(0) &= C^2_{c_1r_1|c_2r_2c_3r_3z}(t), && \text{(9a)}\\ &\geq C^2_{c_1|c_2r_2c_3r_3z}(t) + C^2_{r_1|c_2r_2c_3r_3z}(t), && \text{(9b)}\\ &\geq C^2_{c_1|c_2c_3}(t) + C^2_{r_1|r_2r_3}(t), && \text{(9c)}\\ &\geq N^2_{c_1|c_2c_3}(t) + N^2_{r_1|r_2r_3}(t), && \text{(9d)} \end{aligned}$$

where the concurrence C^2 and the negativity N^2 quantify the bipartite entanglement in different multi-qubit quantum systems.[18] In expression (9a), we use the property that entanglement is invariant under local unitary (LU) operations and consider the tensor product structure of the dynamical evolution $U(\hat{H}, t) = U_{c_1r_1}(\hat{H}, t) \otimes U_{c_2r_2}(\hat{H}, t) \otimes U_{c_3r_3}(\hat{H}, t)$. In expression (9b), we use the monogamy relation of three-qubit quantum states, where the subsystem $c_2r_2c_3r_3z$ can be regarded as a logic qubit. In expression (9c), we use the property that entanglement does not increase under local operations and classical communication (LOCC), and we trace out the subsystems r_2r_3z and c_2c_3z, respectively. In expression (9d), we use the relation $C(\rho_{AB}) \geq N(\rho_{AB})$,[19,20] in which the negativity is defined as[18]

$$N(\rho_{AB}) = ||\rho^{\mathrm{T}_A}|| - 1, \tag{10}$$

where ρ^{T_A} is the partial transpose with respect to the subsystem A, and the norm is $||R|| = \mathrm{Tr}\sqrt{R^\dagger R}$.

According to Eq. (9), we know that the initial entanglement between the cavity photon c_1 and the other photons c_2c_3 plus the ancillary system z restricts the entanglement evolution in the multipartite cavity–reservoir systems. This entanglement is not less than the sum of concurrences $C^2_{c_1|c_2r_2c_3r_3z}$ and $C^2_{r_1|c_2r_2c_3r_3z}$. Furthermore, this equation gives hierarchy relations of entanglement in three cavity photons and three reservoirs. About the set of monogamy relations, we want to point out two points. The first one is that the monogamy relations are not limited by the initial state in Eq. (5) and they are satisfied only if the initial cavity photons are in a three-qubit quantum state and the reservoirs in the vacuum state. The second one is that the relation $C^2_{c_1|c_2c_3}(0) = C^2_{c_1r_1|c_2r_2c_3r_3}(t)$ is satisfied in the dynamical procedure, however, the similar monogamy relations like Eq. (9) do not hold in general.

The concurrence for multiqubit mixed states is difficult to compute, because its solution is required to run over all the pure state decompositions and take the minimum. Compared with the concurrence, the negativity is computable and we will use this measure to analyze the entanglement evolution in the multipartite cavity–reservoir systems.

3. Entanglement evolution of three-qubit mixed states

In the multipartite cavity–reservoir system, the initial state of the three cavity photons is the mixed state composed of a GHZ state and a W state as shown in Eq. (4). This quantum state has a special entanglement property that, in a certain region of the mix probability, it is entangled but the entanglement cannot be explained as the two-qubit concurrences or the three-qubit tangle.[4–6] It has a new type of entanglement structure and can exhibit in the qubit-block form.[15] In the following we will first analyze its entanglement evolution property in the multipartite cavity–reservoir system, and then compare it with a generalized GHZ state.

In the dissipative environment, the output state of the three cavity photons can be written as

$$\rho(t)_{c_1c_2c_3} = \mathrm{Tr}_{r_1r_2r_3z}(|\Psi_t\rangle\langle\Psi_t|), \tag{11}$$

where $|\Psi_t\rangle$ is the global evolution state given in Eq. (6). We use negativity to characterize the entanglement evolution in the dynamical procedure. For the output state of cavity photons, its negativity is

$$N[\rho(t)_{c_1c_2c_3}] = ||\rho^{\mathrm{T}_{c_1}}(t)|| - 1 = \sum_i |\lambda_i(t)| - 1, \quad (12)$$

where λ_is are the eigenvalues of the matrix $\rho^{\mathrm{T}_{c_1}}$. In the time evolution, the entanglement degradation of cavity photons has two kinds of routes: one is the asymptotic way, the other is the ESD way. The entanglement is nonzero when at least one eigenvalue is negative, and it is zero when all the eigenvalues are nonnegative. After some calculation, we can obtain the eigenvalues of the matrix $\rho^{\mathrm{T}_{c_1}}(t)$, which can be expressed as

$$\begin{aligned}
\lambda_1 &= \frac{1}{2}\,\mathrm{e}^{-3\kappa t}p,\\
\lambda_2 &= \frac{1}{2}\,\mathrm{e}^{-3\kappa t}(\,\mathrm{e}^{\kappa t}-1)p,\\
\lambda_3 &= \frac{1}{2}\,\mathrm{e}^{-3\kappa t}(\,\mathrm{e}^{\kappa t}-1)^2p,\\
\lambda_4 &= \frac{1}{6}\,\mathrm{e}^{-\kappa t}[4-4p+3(1-\,\mathrm{e}^{-\kappa t})^2p],\\
\lambda_5 &= \frac{1}{12}\left[A_1-\,\mathrm{e}^{-3\kappa t}\sqrt{A_2}\right],\\
\lambda_6 &= \frac{1}{12}\left[A_1+\,\mathrm{e}^{-3\kappa t}\sqrt{A_2}\right],\\
\lambda_7 &= \frac{1}{12}\left[B_1-\,\mathrm{e}^{-2\kappa t}\sqrt{B_2}\right],\\
\lambda_8 &= \frac{1}{12}\left[B_1+\,\mathrm{e}^{-2\kappa t}\sqrt{B_2}\right], \qquad (13)
\end{aligned}$$

where the parameter p is the mix probability of the GHZ and W states, κt represents the time evolution (κ is the dissipative constant), and the parameters A_1, A_2, B_1, and B_2 have the forms

$$\begin{aligned}
A_1 &= \mathrm{e}^{-\kappa t}(2-2p+3(1-\,\mathrm{e}^{-\kappa t})p),\\
A_2 &= 18\,\mathrm{e}^{3\kappa t}p(p-2)+36p^2-108\,\mathrm{e}^{\kappa t}p^2\\
&\quad +\,\mathrm{e}^{4\kappa t}(p+2)^2+3\,\mathrm{e}^{2\kappa t}p(8+31p),\\
B_1 &= 3[p+(1-\,\mathrm{e}^{-\kappa t})^3p+(1-\,\mathrm{e}^{-\kappa t})\\
&\quad \times(2-2p+\,\mathrm{e}^{-2\kappa t}p)],\\
B_2 &= 36[\,\mathrm{e}^{4\kappa t}+p^2-\,\mathrm{e}^{3\kappa t}(p+2)-\,\mathrm{e}^{\kappa t}p(p+2)]\\
&\quad +\,\mathrm{e}^{2\kappa t}(68+44p+41p^2).
\end{aligned}$$

In the eight eigenvalues, λ_1, λ_2, λ_3, λ_4, λ_6, and λ_8 are always nonnegative, because they can be written as the product or sum of some nonnegative terms. The eigenvalues λ_5 and λ_7 can be negative, and they are served as the indicators of the evolution routes. Set $\lambda_5 = 0$, we can solve the relation between the parameters p and κt, which can be written as

$$p = \frac{2\,\mathrm{e}^{2\kappa t}(-1+\,\mathrm{e}^{\kappa t})}{3-9\,\mathrm{e}^{\kappa t}+7\,\mathrm{e}^{2\kappa t}+2\,\mathrm{e}^{3\kappa t}}. \quad (14)$$

For a given value of the parameter p, $\lambda_5(p) < 0$ when the time $\kappa t < \kappa t(p)$, and $\lambda_5(p) > 0$ when $\kappa t > \kappa t(p)$. Set $\lambda_7 = 0$, we can obtain the relation

$$p = \frac{9-18\,\mathrm{e}^{\kappa t}+17\,\mathrm{e}^{2\kappa t}-3\sqrt{D}}{8\,\mathrm{e}^{2\kappa t}-9\,\mathrm{e}^{-2\kappa t}(1-4\,\mathrm{e}^{\kappa t}+4\,\mathrm{e}^{2\kappa t}-\,\mathrm{e}^{3\kappa t})}, \quad (15)$$

where the parameter $D = 17-68\,\mathrm{e}^{\kappa t}+102\,\mathrm{e}^{2\kappa t}-76\,\mathrm{e}^{3\kappa t}+25\,\mathrm{e}^{4\kappa t}$. Similarly, given a value of p, $\lambda_7(p) < 0$ when $\kappa t < \kappa t(p)$, and $\lambda_7(p) > 0$ when $\kappa t > \kappa t(p)$.

In Fig. 1, we plot the mix probability p as a function of the time evolution κt when $\lambda_5 = 0$ and $\lambda_7 = 0$, respectively. The two lines divide the whole area into four parts where $\lambda_5 < 0$ and $\lambda_7 < 0$ in region I, $\lambda_5 < 0$ and $\lambda_7 > 0$ in region II, $\lambda_5 > 0$ and $\lambda_7 < 0$ in region III, and, $\lambda_5 > 0$ and $\lambda_7 > 0$ in region IV, respectively. According to the sign of the two eigenvalues, we know that the entanglement of three cavity photons is zero in region IV and it is nonzero in the other three regions. Therefore, the entanglement evolution experiences the ESD when the corresponding mixed probability p ranges in $(0.25, 1)$, and the first ESD time occurs at $\kappa t \simeq 1.091$ and the probability $p \simeq 0.385$. On the other hand, the entanglement evolution is asymptotic when the probabilities $p \leq 0.25$ and $p = 1$.

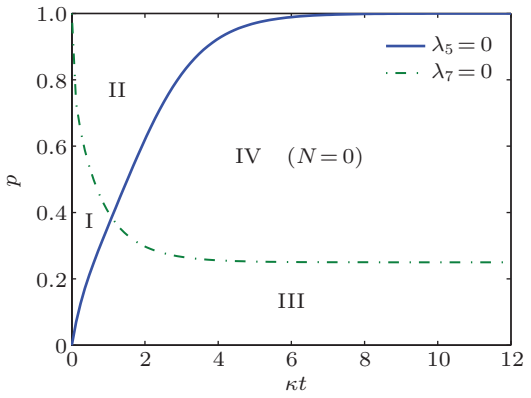


Fig. 1. (colour online) Four entanglement evolution regions described by the signs of eigenvalues λ_5 and λ_7. In the region IV, both eigenvalues are positive, therefore the entanglement of three cavity photons is zero in the dynamical procedure.

In Fig. 2, we plot the negativity of three cavity photons as a function of the mix probability p and

the time evolution κt. When $p = 0$, the initial state is the W state and its entanglement is $N = 2\sqrt{2}/3$. Along with the time evolution, the entanglement decay is asymptotic. With the increase of the parameter p, the initial state entanglement decreases and the entanglement evolution is still asymptotic until the mix probability $p = 0.25$. When $p > 0.25$, the entanglement decays in the ESD way except for $p = 1$. The minimum of the initial entanglement ($N \simeq 0.643$) is located at $p \simeq 0.465$, however, its ESD time is not the least and the minimum of the ESD time corresponds to the value $p \simeq 0.385$. When $p = 1$, the initial state is the GHZ state ($N = 1$) and its evolution is asymptotic.

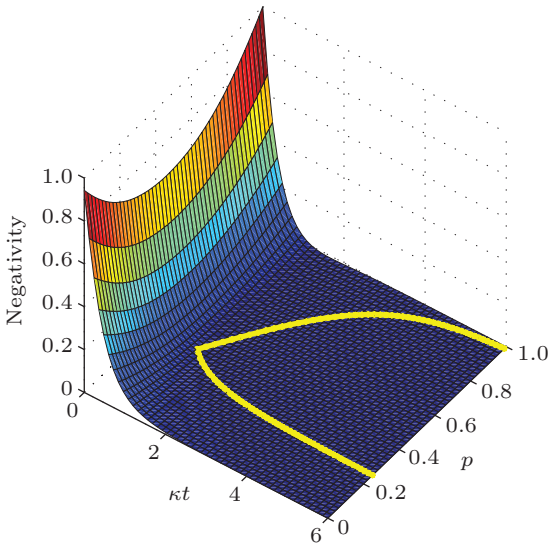


Fig. 2. (colour online) Negativity of three cavity photons as a function of the probability p and the time evolution κt, where the yellow line characterizes the ESD time in the dissipative procedure.

The three cavity photons are initially in the mixed state composed of a GHZ state and a W state whose entanglement structure is quite different from that of pure states. Lohmayer *et al.* pointed out that, when the mix probability p ranges in $[p_c, p_0]$ with $p_c = 7-\sqrt{45} \simeq 0.292$ and $p_0 = 4\sqrt[3]{2}/(3+4\sqrt[3]{2}) \simeq 0.627$, the mixed state is entangled but the entanglement is not the concurrence or the three-tangle.[5] In this range, the mixed state is still genuine three-qubit entanglement and exhibits in the qubit-block form.[6] Therefore, it is desirable to analyze the difference of the entanglement dynamical property between the mixed state and pure states. In the following, we will analyze the entanglement evolution of a generalized GHZ state which is a genuine three-qubit entangled state and has the form

$$|\mathrm{GHZ}^g\rangle = a|000\rangle + b|111\rangle. \tag{16}$$

For the generalized GHZ state, its negativity $N(|\mathrm{GHZ}^g\rangle) = 2ab$. Its output state in the multipartite cavity–reservoir system has the form

$$\rho^g_{c_1c_2c_3}(t) = \mathrm{Tr}_{r_1r_2r_3}(|\Psi^g(t)\rangle\langle\Psi^g(t)|), \tag{17}$$

where $|\Psi^g(t)\rangle_{c_1r_1c_2r_2c_3r_3} = a|000000\rangle + b|\phi_t\rangle|\phi_t\rangle|\phi_t\rangle$ with $|\phi_t\rangle = \xi(t)|10\rangle + \chi(t)|01\rangle$ and $\xi(t) = \exp(-\kappa t/2)$. After some derivation, we can obtain that the negativity of the output state is

$$\begin{aligned} &N(\rho^g_{c_1|c_2c_3}) \\ &= \max\{b\,\mathrm{e}^{-3\kappa t}[\sqrt{F} - b\,\mathrm{e}^{\kappa t}(\,\mathrm{e}^{\kappa t} - 1)], 0\}, \end{aligned} \tag{18}$$

where the parameter $F = 4a^2\exp(3\kappa t) + b^2[2 - 3\exp(\kappa t) + \exp(2\kappa t)]$. Setting $N(\rho^g) = 0$, we can further deduce the relation between the parameters a and κt, which can be written as

$$a(\kappa t) = \sqrt{\frac{(\,\mathrm{e}^{\kappa t} - 1)^3}{(\,\mathrm{e}^{\kappa t} - 1)^3 + \mathrm{e}^{3\kappa t}}}. \tag{19}$$

According to this relation, we can plot the ESD line in the entanglement evolution. The generalized GHZ state can experience the ESD phenomenon when the amplitude $a < \sqrt{2}/2$. When $a \geq \sqrt{2}/2$, the entanglement evolution of quantum state $\rho^g_{c_1|c_2c_3}(t)$ is asymptotic and the negativity is zero only for the limit of $\kappa t \to \infty$.

In Fig. 3, we plot the negativity of the output state $\rho^g_{c_1c_2c_3}(t)$ as a function of the amplitude a and the time evolution κt. The amplitude a ranges in $[0, \sqrt{2}/2]$ which corresponds to the negativity of the generalized GHZ state changing in $[0, 1]$. Along with the time evolution, the entanglement decays in the ESD way. The ESD line (the red line) of the generalized GHZ state is plotted in the figure, where the ESD time increases monotonically with the amplitude a. We also plot the ESD line (the yellow line) of the mixed state composed of a GHZ state and a W state (the mix probability p is scaled with a constant $\sqrt{2}/2$ in order to be compatible with the amplitude a). When the probability $p = 1$, the mixed state becomes the GHZ state and its entanglement evolution is asymptotic, which is the same as that of the generalized GHZ state for the case $a = \sqrt{2}/2$. When $p \leq 0.25$, the evolution of the mixed state is still

asymptotic, however, in the same region of the amplitude a, the entanglement of the generalized GHZ state decays in the ESD way. The mixed state has a special entanglement structure when the mix probability $p \in [p_c, p_0]$. In the case that the generalized GHZ state has an equal initial entanglement to that of the mixed state, we can derive that the parameter a changes in $[0.319, 0.363]$. In this region, the maximal ESD time for the generalized GHZ state is $\kappa t \simeq 0.763$. However, the minimal ESD time of the mixed state is $\kappa t \simeq 1.091$. Therefore, we have the conclusion, with the equal initial entanglement, the ESD time of the mixed state is later than that of the generalized GHZ state.

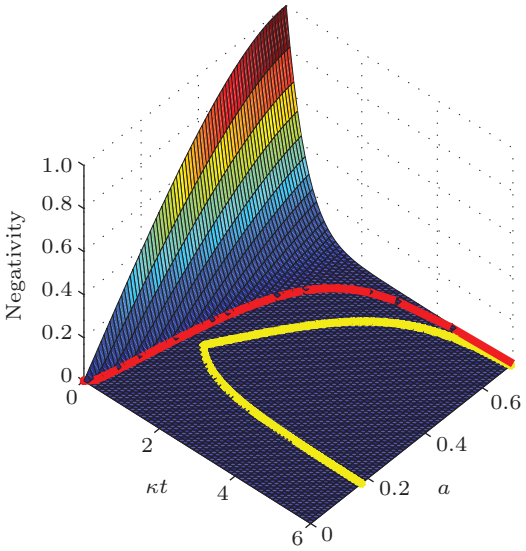


Fig. 3. (colour online) Negativity of the generalized GHZ state, in which the red line is its ESD line and the yellow line is the ESD line of the mixed state composed of the GHZ and W states.

4. Discussion and conclusion

A set of monogamy relations shown in Eq. (13) hold in the entanglement evolution, which restrict the entanglement distribution. Particularly, in the dynamical procedure, the initial entanglement between the cavity photon c_1 and the system c_2c_3z is larger than the sum of entanglement in subsystems $c_1c_2c_3$ and $r_1r_2r_3$. However, the monogamy relation given by the initial entanglement $C^2_{c_1|c_2c_3}(0)$ has not been established. The main reason is that, in this case, the related subsystems cannot be regarded as a logic qubit. Ou pointed out that the monogamy relations induced by concurrences do not hold in general for the higher dimensional objects.[21] However, entanglement monogamy is a fundamental property in many-body systems,[22] and the similar monogamous relation can be satisfied under a well-defined entanglement measure. With a good monogamy relation of entanglement, one can construct multipartite entanglement measures,[4,20,23,24] and have a deep understanding of many-body quantum systems.

In the time evolution, the entanglement of three reservoirs have close relation to that of cavity photons. After some calculation, we can obtain the relation of density matrices between the two subsystems, which can be expressed as

$$\rho_{r_1r_2r_3}(\xi,\chi) = S_{\xi\leftrightarrow\chi}[\,\rho_{c_1c_2c_3}(\xi,\chi)], \tag{20}$$

where S is a transformation interchanging the parameters ξ and χ with $\xi = \exp(-\kappa t/2)$ and $\chi = [1-\exp(-\kappa t)]^{1/2}$. According to this relation, we can know that the entanglement of three cavity photons will transfer completely to the reservoir systems in the limit $\kappa t \to \infty$. Moreover, when the ESD phenomenon of the entangled cavity photons occurs at the time $t_{\rm ESD} = t_0$, the reservoir systems will experience the entanglement sudden birth (ESB) at the time $t_{\rm ESB} = -(1/\kappa t)\ln[1-\exp(-\kappa t_0)]$, which means that the ESD of cavity photons and the ESB of reservoirs are intrinsically related in the multipartite system.

The entanglement evolution of three-qubit mixed states composed of a GHZ state and a W state is analyzed, which has a better dynamical property than that of the generalized GHZ state in the specific parameter regions. For a more general case, consideration of the mix of a generalized GHZ state shown in Eq. (16) and a generalized W state $|W^g\rangle = \alpha|100\rangle + \beta|010\rangle + \gamma|001\rangle$ is deserved.[25] The entanglement evolution has close relation to the type of environment, therefore the entanglement dynamics in non-Markovian environments need to be considered in future.[26–28] Moreover, beside the cavity–reservoir system, other physical systems are also worth being investigated, for example, atom systems,[9,10] quantum dots,[29,30] and spin chains.[31]

In conclusion, we have analyzed the entanglement dynamical property of the mixed state composed of a GHZ state and a W state in multipartite cavity–reservoir systems. As shown in Eq. (9), the entanglement evolution is restricted by a set of monogamy

relations. When the mix probabilities $p \leq 0.25$ and $p = 1$, the entanglement evolution of cavity photons is asymptotic, and, for other regions of the probability, the evolution is in the ESD way. Furthermore, in the specific region $p \in [p_c, p_0]$, the ESD time of the mixed state is later than that of the generalized GHZ state with the equal initial entanglement. Finally, we discuss the entanglement distribution in the multipartite systems and point out the intrinsic relation between the ESD of cavity photons and the ESB of reservoirs.

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