Squeeze-swapping by Bell measurement studied in terms of the entangled state representation*

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By virtue of the entangled state representation (Hong-Yi Fan and J R Klauder 1994 *Phys. Rev.* A **49** 704) and the two-mode squeezing operator's natural representation (Hong-Yi Fan and Yue Fan 1996 *Phys. Rev.* A **54** 958) we propose the squeeze-swapping mechanism which can generate quantum entanglement and new squeezed states of continuum variables.

Keywords: squeeze-swapping, entangled state representation of continuum variables, two-mode squeezed state, Schmidt decomposition

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1. Introduction

Quantum entanglement plays an essential role in quantum communication.[1,2] For the purpose of this kind of communication the entangled particles must be distributed among distant users. During the distribution process, the entanglement of the particles may decrease. To avoid this problem, an effective method has been proposed which is entanglement swapping,[3−6] which can generate entanglement between two distant particles that have never correlated before. However, in the literature most papers are concerned with discrete entangled states, so in this work we shall show how to engender entangled states of continuum variables by swapping. On the other hand, squeezed states have potential uses in quantum communication, moreover, the two-mode squeezed state is itself an entangled state, so in this work we shall propose the squeeze-swapping mechanism that can generate new squeezed states of continuum variables. We shall tackle this topic with the recently developed entangled state representation.[7,8]

The rest of this paper is arranged as follows. In Section 2, we shall show the entanglement swapping mechanism by Bell measurement, which will be studied in the entangled state representation in manifest fashion. In Section 3, we shall focus on the case of Bell measurement for a couple of two-mode squeezed states, which will also result in quantum swapping, and in Section 4, we shall show that two-mode squeezing for a couple of two-mode squeezed states will yield a new squeeze-swapping mechanism. A brief summary is given in Section 5.

2. Entanglement swapping by Bell measurement in the entangled state representation

To realize quantum entanglement swapping there are usually three spatially separate users and two of them share one pair of entangled particles with the third. Then the third user will operate a joint measurement such as Bell-state measurement on the two particles he possesses. Corresponding to the measurement result, the two particles possessed by the two

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spatially separate users will collapse into an entangled state without any entanglement before the joint measurement, so the realization of the Bell-state measurement is of key importance for the entanglement swapping. In the previous literature quantum swapping is restricted to discrete states, and the reason may lie in lacking well-behaved representation to expose the entanglement. In the following we shall tackle the continuum case by virtue of the recently constructed entangled state representation, because, as one can see later, only in this representation can the swapping be demonstrated clearly and directly.

For continuum quantum states case, the Bell measurement operator can be selected as $|\tau\rangle\langle\tau|$, where $|\tau\rangle$ is the bipartite entangled state ($\tau=(\tau_1+\mathrm{i}\tau_2)/\sqrt{2}$) in a complex number

$$|\tau\rangle_{14}=\exp\left[-\frac{1}{2}|\tau|^2+\tau a_1^\dagger-\tau^* a_4^\dagger+a_1^\dagger a_4^\dagger\right]|00\rangle_{14}\,, \quad (1)$$

this state is the common eigenvector of relative coordinate X_1-X_4 and the total momentum P_1+P_4 for the particle 1 and particle 4,

$$\begin{aligned}(X_1-X_4)\,|\tau\rangle_{14}&=\tau_1\,|\tau\rangle_{14}\,,\\ (P_1+P_4)\,|\tau\rangle_{14}&=\tau_2\,|\tau\rangle_{14}\,.\end{aligned} \quad (2)$$

It was Einstein *et al.*[9] who first used this fact to imagine quantum entanglement.

Assuming particles 1 and 2 compose an entangled state $|\eta\rangle_{12}$,

$$|\eta\rangle_{12}=\exp\left[-\frac{1}{2}|\eta|^2+\eta a_1^\dagger-\eta^* a_2^\dagger+a_1^\dagger a_2^\dagger\right]|00\rangle_{12}\,, \quad (3)$$

while particles 3 and 4 are entangled state $|\eta'\rangle_{34}$

$$|\eta'\rangle_{34}=\exp\left[-\frac{1}{2}|\eta'|^2+\eta' a_3^\dagger-\eta'^* a_4^\dagger+a_3^\dagger a_4^\dagger\right]|00\rangle_{34}\,, \quad (4)$$

too. Now Alice makes a Bell measurement for particles 1 and 4 by the projection operator $|\tau\rangle_{14\,14}\langle\tau|$, then what is the result of this measurement? To put it another way, we want to know is there any entanglement between particles 2 and 3?

We make the Schmidt decomposition for $|\eta\rangle_{12}$

$$\begin{aligned}&\left|\eta=\frac{1}{\sqrt{2}}(\eta_1+\mathrm{i}\eta_2)\right\rangle_{12}\\ &=\exp\left(\frac{-\mathrm{i}\eta_1\eta_2}{2}\right)\int_{-\infty}^{\infty}\mathrm{d}q\,|q\rangle_1\otimes|q-\eta_1\rangle_2\,\mathrm{e}^{\mathrm{i}\eta_2 q},\end{aligned} \quad (5)$$

and $|\eta'\rangle_{34}$

$$\begin{aligned}&\left|\eta'=\frac{1}{\sqrt{2}}(\eta_1'+\mathrm{i}\eta_2')\right\rangle_{34}\\ &=\exp\left(\frac{-\mathrm{i}\eta_1'\eta_2'}{2}\right)\int_{-\infty}^{\infty}\mathrm{d}q'\,|q'\rangle_3\otimes|q'-\eta_1'\rangle_4\,\mathrm{e}^{\mathrm{i}\eta_2' q'},\end{aligned} \quad (6)$$

where

$$|q\rangle_i=\frac{1}{\pi^{1/4}}\exp\left[-\frac{q^2}{2}+\sqrt{2}qa^\dagger-\frac{a^{\dagger 2}}{2}\right]|0\rangle_i \quad (7)$$

is the coordinate eigenvector. On the other hand, we decompose

$$\begin{aligned}&\left|\tau=\frac{1}{\sqrt{2}}(\tau_1+\mathrm{i}\tau_2)\right\rangle_{14}\\ &=\exp\left(\frac{-\mathrm{i}\tau_1\tau_2}{2}\right)\int_{-\infty}^{\infty}\mathrm{d}x\,|x\rangle_1\otimes|x-\tau_1\rangle_4\,\mathrm{e}^{\mathrm{i}\tau_2 x}.\end{aligned} \quad (8)$$

Now we can calculate

$$\begin{aligned}&{}_{14}\langle\tau|\eta\rangle_{12}\otimes|\eta'\rangle_{34}\\ &=\exp\left(\frac{\mathrm{i}\tau_1\tau_2}{2}\right)\int_{-\infty}^{\infty}\mathrm{d}x_4\,\langle x-\tau_1|\otimes_1\langle x|\exp(-\mathrm{i}\tau_2 x)\exp\left(\frac{-\mathrm{i}\eta_1\eta_2}{2}\right)\\ &\quad\times\int_{-\infty}^{\infty}\mathrm{d}q\,|q\rangle_1\otimes|q-\eta_1\rangle_2\exp(\mathrm{i}\eta_2 q)\exp\left(\frac{-\mathrm{i}\eta_1'\eta_2'}{2}\right)\int_{-\infty}^{\infty}\mathrm{d}q'\,|q'\rangle_3\otimes|q'-\eta_1'\rangle_4\,\mathrm{e}^{\mathrm{i}\eta_2' q'}\\ &=\exp\left(\frac{\mathrm{i}\tau_1\tau_2}{2}\right)\int_{-\infty}^{\infty}\mathrm{d}x\,\mathrm{e}^{-\mathrm{i}\tau_2 x}\exp\left(\frac{-\mathrm{i}\eta_1\eta_2}{2}\right)\int_{-\infty}^{\infty}\mathrm{d}q\delta(x-q)\,|q-\eta_1\rangle_2\,\mathrm{e}^{\mathrm{i}\eta_2 q}\exp\left(\frac{-\mathrm{i}\eta_1'\eta_2'}{2}\right)\\ &\quad\times\int_{-\infty}^{\infty}\mathrm{d}q'\delta(x-\tau_1-q'+\eta_1')\,|q'\rangle_3\,\mathrm{e}^{\mathrm{i}\eta_2' q'}\\ &=\exp\left(\frac{\mathrm{i}\tau_1\tau_2}{2}\right)\int_{-\infty}^{\infty}\mathrm{d}x\,\mathrm{e}^{-\mathrm{i}\tau_2 x}\exp\left(\frac{-\mathrm{i}\eta_1\eta_2}{2}\right)|x-\eta_1\rangle_2\,\mathrm{e}^{\mathrm{i}\eta_2 x}\exp\left(\frac{-\mathrm{i}\eta_1'\eta_2'}{2}\right)\\ &\quad\times\int_{-\infty}^{\infty}\mathrm{d}q'\delta(x-\tau_1-q'+\eta_1')\,|q'\rangle_3\,\mathrm{e}^{\mathrm{i}\eta_2' q'}\\ &=\exp\left(\frac{\mathrm{i}\tau_1\tau_2}{2}\right)\exp\left(\frac{-\mathrm{i}\eta_1\eta_2}{2}\right)\exp\left(\frac{-\mathrm{i}\eta_1'\eta_2'}{2}\right)\int_{-\infty}^{\infty}\mathrm{d}q'\,|q'\rangle_3\otimes|q'+\tau_1-\eta_1'-\eta_1\rangle_2\end{aligned}$$

$$
\begin{aligned}
&\times \exp\left[-\mathrm{i}(\tau_2-\eta_2)(q'+\tau_1-\eta_1')\right]\exp(\mathrm{i}\eta_2' q')\\
=\ &\exp\left(\frac{\mathrm{i}}{2}(\tau_1\tau_2-\eta_1\eta_2-\eta_1'\eta_2')\right)\int_{-\infty}^{\infty}\mathrm{d}q'\,|q'\rangle_3\otimes|q'+\tau_1-\eta_1'-\eta_1\rangle_2\\
&\times\exp\left[\mathrm{i}(\eta_2+\eta_2'-\tau_2)q'\right]\exp[\mathrm{i}(\eta_2-\tau_2)(\tau_1-\eta_1')]\\
=\ &\exp\left\{\frac{\mathrm{i}}{2}\left[\tau_1(\eta_2-\eta_2')+\tau_2(\eta_1'-\eta_1)+\eta_1\eta_2'-\eta_1'\eta_2\right]\right\}\exp\left[\frac{\mathrm{i}}{2}(\tau_1-\eta_1'-\eta_1)(\eta_2+\eta_2'-\tau_2)\right]\\
&\times\int_{-\infty}^{\infty}\mathrm{d}q'\,|q'\rangle_3\otimes|q'+\tau_1-\eta_1'-\eta_1\rangle_2\exp\left[\mathrm{i}(\eta_2+\eta_2'-\tau_2)q'\right].
\end{aligned}\tag{9}
$$

Comparing Eq. (9) with the standard form of the bipartite entangled state Eq. (1) we see particles 2 and 3 compose a new entangled state

$$
\begin{aligned}
&{}_{14}\langle\tau|\,\eta\rangle_{12}\otimes|\eta'\rangle_{34}\\
=\ &\exp\left\{\frac{\mathrm{i}}{2}\left[\tau_1(\eta_2-\eta_2')+\tau_2(\eta_1'-\eta_1)+\eta_1\eta_2'-\eta_1'\eta_2\right]\right\}\\
&\times\left|\xi=\frac{1}{\sqrt{2}}\left[(\eta_1'+\eta_1-\tau_1)+\mathrm{i}\,(\eta_2+\eta_2'-\tau_2)\right]\right\rangle_{32},
\end{aligned}
$$

where

$$
\begin{aligned}
|\xi\rangle_{32}=\ &\exp\left[\frac{\mathrm{i}}{2}(\tau_1-\eta_1'-\eta_1)(\eta_2+\eta_2'-\tau_2)\right]\\
&\times\int_{-\infty}^{\infty}\mathrm{d}q'\,|q'\rangle_3\otimes|q'+\tau_1-\eta_1'-\eta_1\rangle_2\\
&\times\exp\left[\mathrm{i}(\eta_2+\eta_2'-\tau_2)q'\right].
\end{aligned}
$$

Thus through the entangled state representation we can clearly see the new entanglement between particles 2 and 3.

3. Bell measurement for a couple of two-mode squeezed states

When particles 1 and 2 compose a two-mode squeezed state

$$
S_{12}\,|00\rangle_{12}=\mathrm{sech}^{1/2}\lambda\exp\left(a_1^\dagger a_2^\dagger\tanh\lambda\right)|00\rangle_{12},\tag{10}
$$

where the squeezing operator S_{12} has its natural representation

$$
\begin{aligned}
&S_{12}=\exp\left[\lambda\left(a_1a_2-a_1^\dagger a_2^\dagger\right)\right]=\mu\int\frac{\mathrm{d}^2\xi}{\pi}\,|\mu\eta\rangle\,\langle\eta|,\\
&\mu=\mathrm{e}^{\lambda},
\end{aligned}\tag{11}
$$

while particles 3 and 4 compose another two-mode squeezed state

$$
S_{34}\,|00\rangle_{34}=\mathrm{sech}^{1/2}\sigma\exp\left(a_3^\dagger a_4^\dagger\tanh\sigma\right)|00\rangle_{34}.\tag{12}
$$

We want to know what is the Bell measurement result for these two two-mode squeezed states.

By making a Bell measurement for particles 1 and 4 by the projection operator $|\tau\rangle_{1414}\langle\tau|$ on the above two two-mode squeezed states, the direct product form of $S_{12}\,|00\rangle_{12}\otimes S_{34}\,|00\rangle_{34}$, we have

$$
\begin{aligned}
&{}_{14}\langle\tau|\,S_{12}\,|00\rangle_{12}\,S_{34}\,|00\rangle_{34}\\
=\ &{}_{14}\langle00|\exp\left[-\frac{|\tau|^2}{2}+\tau^*a_1-\tau a_4+a_1a_4\right]\\
&\times\mathrm{sech}^{1/2}\lambda\,\mathrm{sech}^{1/2}\sigma\exp\left(a_1^\dagger a_2^\dagger\tanh\lambda\right)|00\rangle_{12}\\
&\times\exp\left(a_3^\dagger a_4^\dagger\tanh\sigma\right)|00\rangle_{34}.
\end{aligned}\tag{13}
$$

Then by using the completeness relation of the coherent state

$$
\begin{aligned}
&\int\frac{\mathrm{d}^2z}{\pi}\,|z\rangle\,\langle z|=\int\frac{\mathrm{d}^2z}{\pi}:\mathrm{e}^{-(z-a)\left(z^*-a^\dagger\right)}:=1,\\
&|z\rangle=\exp\left(-\frac{|z|^2}{2}+za^\dagger\right)|0\rangle,
\end{aligned}\tag{14}
$$

and the technique of integration within an ordered product of operators (IWOP),[10–12] as well as the normal ordered form of $|0\rangle\langle0|=:\mathrm{e}^{-a^\dagger a}:$ we can derive the following operator identity

$$
\begin{aligned}
&\exp\left(F_1a_1+F_4a_4+\tau a_4a_1\right)\\
&\times\exp\left(\lambda a_2^\dagger a_1^\dagger+\sigma a_3^\dagger a_4^\dagger+\sum_i G_ia_i^\dagger\right)\\
=\ &\exp\left(F_1a_1+F_4a_4+\tau a_4a_1\right)\int\prod_{i=1}^{4}\frac{\mathrm{d}^2z_i}{\pi}\,|z_i\rangle\,\langle z_i|\\
&\times\exp\left(\lambda a_2^\dagger a_1^\dagger+\sigma a_3^\dagger a_4^\dagger+\sum_i G_ia_i^\dagger\right)\\
=\ &:\exp\Big[F_1(a_1+G_1)+\sum_i G_ia_i^\dagger+\lambda F_1a_2^\dagger\\
&+(\tau G_1+F_4)G_4+\tau(G_1a_4+G_4a_1)+F_4a_4\\
&+\lambda\tau G_4a_2^\dagger+\lambda\sigma\tau a_3^\dagger a_2^\dagger+\lambda a_1^\dagger a_2^\dagger+\tau a_4a_1\\
&+\lambda\tau a_4a_2^\dagger+\sigma\tau a_3^\dagger a_1+\sigma a_3^\dagger a_4^\dagger\Big]:.
\end{aligned}\tag{15}
$$

Then using Eq. (15) we can rewrite Eq. (13) as

$$
\begin{aligned}
&{}_{14}\langle\tau|S_{12}\,|00\rangle_{12}\,S_{34}\,|00\rangle \\
&= \operatorname{sech}^{1/2}\lambda \operatorname{sech}^{1/2}\sigma_{14}\,\langle 00|:\exp\Big[-\frac{|\tau|^2}{2}+\tau^* a_1+\tanh\lambda\tau^* a_2^\dagger - a_4+\tanh\lambda\tanh\sigma a_3^\dagger a_2^\dagger \\
&\quad +\tanh\lambda a_1^\dagger a_2^\dagger + a_4 a_1+\tanh\lambda a_4 a_2^\dagger+\tanh\sigma a_3^\dagger a_1+\tanh\sigma a_3^\dagger a_4^\dagger\Big]:|00\rangle_{12}\,|00\rangle_{34} \\
&= {}_{14}\langle 00|\operatorname{sech}^{1/2}\lambda\operatorname{sech}^{1/2}\sigma\exp\left(-\frac{|\tau|^2}{2}+\tanh\lambda\tau^* a_2^\dagger+\tanh\lambda\tanh\sigma a_3^\dagger a_2^\dagger\right)|00\rangle_{12}\,|00\rangle_{34} \\
&= \operatorname{sech}^{1/2}\lambda\operatorname{sech}^{1/2}\sigma\exp\left(-\frac{|\tau|^2}{2}+\tanh\lambda\tau^* a_2^\dagger+\tanh\lambda\tanh\sigma a_3^\dagger a_2^\dagger\right)|00\rangle_{23}\,. \qquad (16)
\end{aligned}
$$

Comparing Eq. (16) with Eq. (10) we see that equation (16) is a displacement-squeezed state in 2–3 modes. Thus the Bell measurement for a pair of two-mode squeezed states can also engender quantum swapping.

4. Two-mode squeezing for a couple of two-mode squeezed states

Now we examine if the squeeze operation can generate quantum swapping and we name it the squeeze-swapping. Operating the squeezing operator $S_{14}(\lambda)$ on a pair of two-mode squeezed states and using the following operator identity

$$
\mathrm{e}^A\,\mathrm{e}^B = \mathrm{e}^B\,\mathrm{e}^A\,\mathrm{e}^{[A,B]},\qquad [A,[A,B]]=0,\qquad [B,[A,B]]=0, \qquad (17)
$$

we calculate

$$
\begin{aligned}
W &\equiv S_{14}(\lambda)\operatorname{sech}^{1/2}\sigma\exp\left(a_1^\dagger a_2^\dagger\tanh\sigma\right)|00\rangle_{12}\otimes\operatorname{sech}^{1/2}\sigma'\exp\left(a_3^\dagger a_4^\dagger\tanh\sigma'\right)|00\rangle_{34} \\
&= \operatorname{sech}^{1/2}\sigma\operatorname{sech}^{1/2}\sigma' S_{14}(\lambda)\exp\left(a_1^\dagger a_2^\dagger\tanh\sigma\right)\exp\left(a_3^\dagger a_4^\dagger\tanh\sigma'\right)S_{14}(\lambda)^{-1}S_{14}(\lambda)\,|00\rangle_{12}\,|00\rangle_{34} \\
&= \operatorname{sech}^{1/2}\sigma\operatorname{sech}^{1/2}\sigma'\exp\left[\left(a_1^\dagger\cosh\lambda+a_4\sinh\lambda\right)a_2^\dagger\tanh\sigma+a_3^\dagger\left(a_4^\dagger\cosh\lambda+a_1\sinh\lambda\right)\tanh\sigma'\right] \\
&\quad\times S_{14}(\lambda)\,|00\rangle_{12}\,|00\rangle_{34} \\
&= \operatorname{sech}^{1/2}\sigma\operatorname{sech}^{1/2}\sigma'\exp\left[\left(a_1^\dagger\cosh\lambda+a_4\sinh\lambda\right)a_2^\dagger\tanh\sigma\right] \\
&\quad\times\exp\left[a_3^\dagger\left(a_4^\dagger\cosh\lambda+a_1\sinh\lambda\right)\tanh\sigma'\right]\exp\left(a_1^\dagger a_4^\dagger\tanh\lambda\right)|0000\rangle \\
&= \operatorname{sech}^{1/2}\sigma\operatorname{sech}^{1/2}\sigma'\exp\left[a_1^\dagger a_2^\dagger\cosh\lambda\tanh\sigma\right]\exp\left[a_4 a_2^\dagger\sinh\lambda\tanh\sigma\right]\exp\left[a_3^\dagger a_4^\dagger\cosh\lambda\tanh\sigma'\right] \\
&\quad\times\exp[a_3^\dagger a_1\sinh\lambda\tanh\sigma']\exp\left(a_1^\dagger a_4^\dagger\tanh\lambda\right)|0000\rangle\,, \qquad (18)
\end{aligned}
$$

then by using Eq. (17) we have

$$
\begin{aligned}
&\exp[a_3^\dagger a_1\sinh\lambda\tanh\sigma']\exp\left(a_1^\dagger a_4^\dagger\tanh\lambda\right)|0000\rangle \\
&= \exp\left(a_1^\dagger a_4^\dagger\tanh\lambda\right)\exp[a_3^\dagger a_4^\dagger\tanh\lambda\sinh\lambda\tanh\sigma']\exp[a_3^\dagger a_1\sinh\lambda\tanh\sigma']\,|0000\rangle \\
&= \exp\left(a_1^\dagger a_4^\dagger\tanh\lambda\right)\exp[a_3^\dagger a_4^\dagger\tanh\lambda\sinh\lambda\tanh\sigma']\,|0000\rangle\,, \qquad (19)
\end{aligned}
$$

then substituting Eq. (19) into Eq. (18) we obtain

$$
\begin{aligned}
W =& \operatorname{sech}^{1/2}\sigma\operatorname{sech}^{1/2}\sigma'\exp\left[a_1^\dagger a_2^\dagger\cosh\lambda\tanh\sigma\right]\exp[a_4 a_2^\dagger\sinh\lambda\tanh\sigma]\exp\left[a_3^\dagger a_4^\dagger\cosh\lambda\tanh\sigma'\right] \\
&\times\exp\left(a_1^\dagger a_4^\dagger\tanh\lambda\right)\exp[a_3^\dagger a_4^\dagger\sinh\lambda\tanh\lambda\tanh\sigma']\,|0000\rangle \\
=& \operatorname{sech}^{1/2}\sigma\operatorname{sech}^{1/2}\sigma'\exp\left[a_1^\dagger a_2^\dagger\cosh\lambda\tanh\sigma\right]\exp[a_4 a_2^\dagger\sinh\lambda\tanh\sigma] \\
&\times\exp\left(a_1^\dagger a_4^\dagger\tanh\lambda\right)\exp\left[a_3^\dagger a_4^\dagger\cosh\lambda\left(1+\tanh^2\lambda\right)\tanh\sigma'\right]|0000\rangle
\end{aligned}
$$

$$
\begin{aligned}
&= \operatorname{sech}^{1/2}\sigma \operatorname{sech}^{1/2}\sigma' \exp\left[a_1^\dagger a_2^\dagger \cosh\lambda \tanh\sigma\right] \exp\left[a_1^\dagger a_4^\dagger \tanh\lambda\right] \exp\left[a_1^\dagger a_2^\dagger \sinh\lambda \tanh\lambda \tanh\sigma\right] \\
&\quad \times \exp[a_4 a_2^\dagger \sinh\lambda \tanh\sigma] \exp\left[a_3^\dagger a_4^\dagger \cosh\lambda \left(1+\tanh^2\lambda\right) \tanh\sigma'\right] |0000\rangle \\
&= \operatorname{sech}^{1/2}\sigma \operatorname{sech}^{1/2}\sigma' \exp\left[a_1^\dagger a_2^\dagger \cosh\lambda \left(1+\tanh^2\lambda\right) \tanh\sigma\right] \exp\left(a_1^\dagger a_4^\dagger \tanh\lambda\right) \\
&\quad \times \exp\left[a_3^\dagger a_4^\dagger \cosh\lambda \left(1+\tanh^2\lambda\right) \tanh\sigma'\right] \\
&\quad \times \exp\left[a_2^\dagger a_3^\dagger \tanh\sigma \tanh\sigma' \sinh\lambda \cosh\lambda \left(1+\tanh^2\lambda\right)\right] |0000\rangle\,, \qquad (20)
\end{aligned}
$$

we see that the 1–4 squeezing let the original state $\exp(a_1^\dagger a_2^\dagger \tanh\sigma)\,|00\rangle_{12}$ and $\exp(a_3^\dagger a_4^\dagger \tanh\sigma')\,|00\rangle_{34}$ to $\exp[a_1^\dagger a_2^\dagger \tanh\sigma \cosh\lambda \left(1+\tanh^2\lambda\right)]\,|00\rangle_{12}$, and $\exp[a_3^\dagger a_4^\dagger \tanh\sigma' \cosh\lambda \left(1+\tanh^2\lambda\right)]\,|00\rangle_{34}$ respectively. Since $\cosh\lambda \left(1+\tanh^2\lambda\right) \geq 1$, the squeezing of 1–2 modes and of 3–4 modes increases, moreover, the modes 2 and 3 which have never correlated before now also become entangled, so the squeeze-operation can also generate quantum swapping.

5. Summary

In summary, by virtue of the entangled state representation and the two-mode squeezing operator's natural representation we proposed the squeeze-swapping mechanism that can generate new squeezed states of continuum variables. This mechanism can be extended to multimode entanglement cases when multimode entangled state representations are established.

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