Spontaneous symmetry breaking vacuum energy in cosmology*

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The gravitational effect of spontaneous symmetry breaking vacuum energy density is investigated by subtracting the flat space–time contribution from the energy in the curved space–time. We find that the remaining effective energy–momentum tensor is too small to cause the acceleration of the universe, although it satisfies the characteristics of dark energy. However, it could provide a promising explanation to the puzzle of why the gravitational effect produced by the huge symmetry breaking vacuum energy in the electroweak theory has not been observed, as it has a sufficiently small value (smaller than the observed cosmic energy density by a factor of 10^{32}).

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1. Introduction

The observational data of supernovae type Ia (SN Ia) accumulated by the year 1998 have shown that the present universe is accelerating.[1,2] The source for this late-time cosmic acceleration is dubbed dark energy. Despite many years of research,[3–10] its origin has not been identified yet. Dark energy is distinguished from the ordinary matter species in that it has a negative pressure.[3] The simplest candidate for dark energy is the so-called cosmological constant Λ, whose energy density remains a constant. From the viewpoint of particle physics, the cosmological constant appears as the vacuum energy density. The cosmological constant problem had been investigated long before the discovery of dark energy in 1998. In 1989, Weinberg pointed out that possible candidates for vacuum energy density can be the sum of the zero-point energies of all normal modes of fields up to a cut-off scale of the momentum or the vacuum expectation value of the energy density in a certain state that has spontaneous symmetry breaking.[11] If we believe the general relativity up to the Planck scale and take $\Lambda \simeq (8\pi G)^{-1/2}$ as the cut-off scale, the zero-point energy would give $\rho_{\rm vac} \simeq 2 \times 10^{71}$ GeV^4. On the other hand, the vacuum expectation value of the energy density in the electroweak theory would give $\rho_{\rm vac} \simeq -g\ 300^4$ GeV^4, where g is the coefficient of the $(\phi^\dagger\phi)^2$ term in the Higgs Lagrangian. Even for g as small as α^2, with α being the fine structure constant, it would yield $|\rho_{\rm vac}| \simeq 10^6$ GeV^4. Hence, the two candidates are both much larger than the observed value of the total energy density of the present universe, $\rho \simeq 10^{-47}$ GeV^4.[11] This is problematic. Furthermore, a puzzle of nature would be why the gravitational effect produced by such a huge vacuum energy density has not been detected.

Recently, Maggiore proposed a new method to calculate the zero-point fluctuations of quantum fields in cosmology.[12] In the Hamiltonian formulation of the general relativity, the energy associated to an asymptotically flat space–time with metric $g_{\mu\nu}$ is related to Hamiltonian $H_{\rm GR}$ by $E = H_{\rm GR}[g_{\mu\nu}] - H_{\rm GR}[\eta_{\mu\nu}]$, where the subtraction of the flat-space contribution is necessary to get rid of an otherwise divergent boundary term.[13] The classical result indicates that the energy associated to the flat space–time does not gravitate. The definition of the standard Arnouitt–Deser–Misner (ADM) mass is based on such a principle. Maggiore applied this principle to study the effective zero-point energy, proposing that its con-

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tribution to the dynamic of the universe is obtained by computing the vacuum energies of quantum fields in a Friedmann–Robertson–Wallker (FRW) space–time and subtracting from it the flat space–time contribution. Although the remaining energy density after performing the cut-off procedure is not compatible with the present observations of dark energy, Maggiore's concept provides a new credible method to calculate the vacuum energy density. Notice that in Maggiore's paper, he did not prove whether this method is credible in the FRW space–time, which is conformal flat rather than asymptotic flat.

We apply such an idea to study the spontaneous symmetry breaking vacuum energy density in the electroweak theory in the background of a spatially flat FRW space–time, and explore its gravitational effect. In this paper, we do not consider whether such a method is valid in the FRW space–time. Our concern is what result can be found if this method is correct. After subtracting the energy density in the flat space–time, the effective energy–momentum tensor satisfies the characteristic of dark energy. However, we find that the value of such a vacuum energy density is by a factor of 10^{32} smaller than the observed cosmic energy density, and therefore could not be regarded as a candidate for dark energy. Nevertheless, this effective vacuum energy density with such a small value provides a promising interpretation as to why the gravitational effect of the symmetry breaking vacuum energy has not been detected, because its effect on gravity can be neglected while comparing with the observed cosmic energy.

This paper is organized as follows. In Section 2, we calculate the effective symmetry breaking vacuum energy density by using Maggiore's method. In Section 3, we investigate the gravitational effect produced by such a vacuum energy density, and try to explain why the symmetry breaking vacuum energy has not been observed. Section 4 is devoted to the discussion and conclusion.

2. Spontaneous symmetry breaking vacuum energy density

The WMAP five-year data constrain the contribution of the curvature of the present universe to be only (-0.0175)–0.0085,[14] while the total contribution, which contains matter, dark energy, and curvature, is equal to 1. Hence, for simplicity, we consider only the case of a spatially flat FRW space–time. In the preferred coordinate system, the metric takes the form[15,16]

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)\delta_{ik}\mathrm{d}x^i\mathrm{d}x^k. \tag{1}$$

It is convenient to introduce the conformal time $\eta \equiv \int \mathrm{d}t/a(t)$ instead of the cosmic time t. With this new coordinate, metric (1) becomes

$$\mathrm{d}s^2 = a^2(\eta)\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu, \tag{2}$$

and it is obvious that the metric is conformally equivalent to the Minkowski metric $\eta_{\mu\nu}$. Let us consider the effective vacuum energy density with spontaneous symmetry breaking in such a space–time. Since we only concentrate on the present universe, the electroweak phase transition in the early universe will be neglected.

In electroweak theory, the Lagrangian density of the Higgs scalar fields takes the form[17]

$$\mathcal{L} = \frac{1}{2}g^{\mu\nu}\phi^\dagger_{,\mu}\phi_{,\nu} - \frac{1}{2}\mu^2\phi^\dagger\phi - \frac{\lambda}{4}(\phi^\dagger\phi)^2, \tag{3}$$

with $\mu^2 < 0$, and $\lambda > 0$. Putting the Higgs field into the above conformal flat space–time, we obtain the action in the minimally coupled case as

$$S = \int \mathrm{d}^3x\mathrm{d}\eta\bigg[\frac{1}{2}a^2\phi^{\dagger\prime}\phi' - \frac{1}{2}a^2\boldsymbol{\nabla}\phi^\dagger\boldsymbol{\nabla}\phi - \frac{1}{2}\mu^2a^4\phi^\dagger\phi - \frac{\lambda}{4}a^4(\phi^\dagger\phi)^2\bigg], \tag{4}$$

where the prime denotes the derivative with respect to the conformal time η. The condition of $\phi_{;\mu} = \phi_{,\mu}$ for scalar fields has been used. Introducing auxiliary fields $\varphi = a(\eta)\phi$ and $\varphi^\dagger = a(\eta)\phi^\dagger$, we can rewrite the action in Eq. (4) in terms of φ and $\varphi^\dagger$ as

$$S = \int \mathrm{d}^3x\mathrm{d}\eta\bigg[\frac{1}{2}\varphi^{\dagger\prime}\varphi' - \frac{1}{2}\boldsymbol{\nabla}\varphi^\dagger\boldsymbol{\nabla}\varphi - \frac{1}{2}\bigg(\mu^2a^2 - \frac{a''}{a}\bigg)\varphi^\dagger\varphi - \frac{\lambda}{4}(\varphi^\dagger\varphi)^2\bigg] - \frac{1}{2}\int_{\partial\eta}\mathrm{d}^3x\bigg[\bigg(\frac{a'}{a}\varphi^\dagger\varphi\bigg)_{\eta_\mathrm{f}} - \bigg(\frac{a'}{a}\varphi^\dagger\varphi\bigg)_{\eta_\mathrm{i}}\bigg]. \tag{5}$$

The boundary term comes from the total derivative terms. Since $\varphi^\dagger$ and φ are fixed at the boundary, the boundary term contributes only a constant to the Lagrangian and the Hamiltonian. Hence, we will omit this term for a moment, then the action of fields ϕ and $\phi^\dagger$ in the curved FRW space–time is equivalent to the one of φ and $\varphi^\dagger$ in the flat Minkowski space–time. Thus, the Lagrangian density of the new system can be written as

$$\widetilde{\mathcal{L}} = \frac{1}{2}\varphi^{\dagger\prime}\varphi' - \frac{1}{2}\boldsymbol{\nabla}\varphi^\dagger\boldsymbol{\nabla}\varphi$$

$$-\frac{1}{2}\left(\mu^2a^2-\frac{a''}{a}\right)\varphi^\dagger\varphi-\frac{\lambda}{4}(\varphi^\dagger\varphi)^2, \quad (6)$$

and the Hamiltonian density is

$$\begin{aligned}\widetilde{\mathcal{H}} &= \frac{\partial\widetilde{\mathcal{L}}}{\partial\varphi'}\varphi+\varphi^\dagger\frac{\partial\widetilde{\mathcal{L}}}{\partial\varphi^{\dagger\prime}}-\widetilde{\mathcal{L}}\\ &= \frac{1}{2}\varphi^{\dagger\prime}\varphi'+\frac{1}{2}\boldsymbol{\nabla}\varphi^\dagger\boldsymbol{\nabla}\varphi\\ &\quad+\frac{1}{2}\left(\mu^2a^2-\frac{a''}{a}\right)\varphi^\dagger\varphi+\frac{\lambda}{4}(\varphi^\dagger\varphi)^2. \quad (7)\end{aligned}$$

The vacuum is defined as the eigenstate of the Hamiltonian with the lowest possible energy. In the case under consideration, the Hamiltonian depends explicitly on time and thus does not possess time-independent eigenvectors. Nevertheless, given a particular moment of time η_0, we can still define the instantaneous vacuum $|0_{\eta_0}\rangle$ as the lowest-energy state of Hamiltonian $H(\eta_0)$. The classical Hamiltonian has an extreme at the moment η_0 if $\varphi'|_{\eta_0}=\varphi^{\dagger\prime}|_{\eta_0}=0$, $\boldsymbol{\nabla}\varphi|_{\eta_0}=\boldsymbol{\nabla}\varphi^\dagger|_{\eta_0}=0$, and

$$\frac{\partial V}{\partial\varphi}\Big|_{\eta_0}=\frac{\partial V}{\partial\varphi^\dagger}\Big|_{\eta_0}=0,$$

with

$$V(\varphi^\dagger,\varphi)=\frac{1}{2}\left(\mu^2a^2-\frac{a''}{a}\right)\varphi^\dagger\varphi+\frac{\lambda}{4}(\varphi^\dagger\varphi)^2.$$

For $\mu^2<0$ and $\lambda>0$, the solution of the Higgs fields satisfies

$$\varphi^\dagger(\eta_0)\varphi(\eta_0)=\frac{-\mu^2a(\eta_0)^2+a''(\eta_0)/a(\eta_0)}{\lambda}. \quad (8)$$

Consequently, the vacuum expectation value of the energy density in the classical approximation is given by

$$\widetilde{\mathcal{H}}_{\min}(\eta_0)=-\frac{[-\mu^2a(\eta_0)^2+a''(\eta_0)/a(\eta_0)]^2}{4\lambda}. \quad (9)$$

Thus the physical vacuum energy regarded as the total minimum energy of the system (measured in the FRW space–time) is

$$E_{\rm vac}(\eta_0)=\int \mathrm{d}^3x\sqrt{-g(\eta_0)}\Big[-\frac{(-\mu^2+\frac{a''(\eta_0)}{a(\eta_0)^3})^2}{4\lambda}\Big], \quad (10)$$

which leads to a vacuum energy density

$$\rho_{\rm vac}(\eta_0)=-\frac{(-\mu^2+a''(\eta_0)/a(\eta_0)^3)^2}{4\lambda}. \quad (11)$$

Here the term $\mathrm{d}^3x\sqrt{-g(\eta_0)}$ is the physical volume element. Obviously, this conclusion is appropriate for arbitrary moment η.

Following Maggiore's viewpoint, the vacuum energy in the flat space–time has no contribution to the dynamic of the gravitational system, which should be subtracted from the vacuum energy density in the FRW space–time. The vacuum energy density in the electroweak theory in the Minkowski space–time is $\rho'_{\rm vac}=-\mu^4/4\lambda$. Hence, we obtain the effective vacuum energy density in cosmology as

$$\rho_{\rm eff}=\rho_{\rm vac}-\rho'_{\rm vac}=\frac{2\mu^2a''/a^3-(a''/a^3)^2}{4\lambda}. \quad (12)$$

This illustrates that the effective vacuum energy density $\rho_{\rm eff}$ depends on the values of parameters μ^2 and λ. Since the Higgs particle has not been found yet, these two values remain unclear. However, the value of $\sqrt{-\mu^2/\lambda}$, which is the vacuum expectation value of the Higgs field in the electroweak theory, has been obtained as $\langle\phi\rangle\simeq 300$ GeV.[11] If we assume that the value of the term $(a''/a^3)^2$ is very small and neglect it, the value of $\rho_{\rm eff}$ could be calculated with given $a(t)$. The rationality of such an assumption will be discussed in the next section. Note that the constant caused by the boundary term still exists now.

In particle physics, it is licit to adding term V_0 to the Lagrangian density in Eq. (3) with V_0 a constant; the physical effect in the scattering processes would not be modified. Clearly, the contribution of V_0 has been canceled in the present approach, and hence the effective vacuum energy density result is independent of V_0.

The energy–momentum tensor of the scalar fields in the curved space–time is[19,20]

$$T_{\mu\nu}=\phi^\dagger_{,\mu}\phi_{,\nu}-g_{\mu\nu}\left(\frac{1}{2}g^{\rho\sigma}\phi^\dagger_{,\rho}\phi_{,\sigma}-V(\phi^\dagger,\phi)\right). \quad (13)$$

For the vacuum state, which implies $\varphi'|_{\eta_0}=\varphi^{\dagger\prime}|_{\eta_0}=0$, $\boldsymbol{\nabla}\varphi|_{\eta_0}=\boldsymbol{\nabla}\varphi^\dagger|_{\eta_0}=0$, and

$$\varphi^\dagger\varphi\big|_{\eta_0}=\frac{-\mu^2a^2+a''/a}{\lambda},$$

obtaining the non-vanishing components $T^i_i=T^0_0=\rho_{\rm vac}$ and $T'^i_{\ i}=T'^0_{\ 0}=\rho'_{\rm vac}$ is straightforward. The boundary constant has no contribution to $T_{\mu\nu}$. Subtracting the flat space–time contribution from the energy–momentum tensor in the curved space–time, we obtain $\rho_{\rm eff}=-p_{\rm eff}$. Consequently, the energy–momentum tensor of the vacuum with symmetry breaking satisfies the characteristic of dark energy. However, the expression of such an energy density (Eq. (12)) is dependent on time, which can not be regarded as the cosmological constant.

3. Gravitational effect

In this section, we apply the vacuum energy–momentum tensor to the expanding universe, and consider its physical effect on the gravitational system. The logic applied here is that with the Hubble parameter provided by the observation, we know the value of the total energy density in the universe, and then we can consider the contribution of the spontaneous symmetry breaking vacuum energy to the dark energy. Since the Hubble parameter is obtained experimentally, the structure of space–time is decided, and therefore we need not seek the space–time geometry by solving the Einstein equation. First, let us check if the Einstein equation can be satisfied by regarding the spontaneous symmetry breaking vacuum energy as the dark energy. The Einstein equation $G_{\mu\nu} \equiv R_{\mu\nu} - (1/2)g_{\mu\nu}R = 8\pi G T_{\mu\nu}$ in the spatially flat FRW space–time gives

$$H^2 = \frac{8\pi G}{3}\rho, \tag{14}$$

$$\dot{\rho} + 3H(\rho + p) = 0, \tag{15}$$

where $H \equiv \dot{a}/a$ is the Hubble parameter, and the dot represents a derivative with respect to the cosmic time t. We ignore the contribution of the radiation, and regard the present universe as an ideal fluid which contains the non-relativistic matter (include the dark matter), satisfying $p_{\rm M} = 0$ and the dark energy satisfying $p_{\rm DE} = \omega_{\rm DE}\rho_{\rm DE}$. From the combined analysis of SN Ia, CMB, and BAO, we obtain the bound for the WMAP group as $-1.097 < \omega_{\rm DE} < -0.858$.[14] Hence, we assume $\omega_{\rm DE} = -1$, then equations (14) and (15) take the form

$$H^2 = \frac{8\pi G}{3}(\rho_{\rm M} + \rho_{\rm DE}), \tag{16}$$

$$\dot{\rho}_{\rm M} + \dot{\rho}_{\rm DE} + 3H\rho_{\rm M} = 0. \tag{17}$$

Notice that the matter and the dark energy do not satisfy the energy–momentum conservation Eq. (15), respectively. Instead, we only require the summation of them to satisfy the total conservation Eq. (17). In other words, we permit the transformation between the matter and the dark energy, since the particles can be created from the vacuum in quantum field theory in the curved space–time.[21,22] From Eqs. (16) and (17), we obtain

$$\dot{H} = 4\pi G\rho_{\rm DE} - \frac{3}{2}H^2 = \frac{3}{2}H^2(\Omega_{\rm DE} - 1), \tag{18}$$

where $\Omega_{\rm DE} \equiv \rho_{\rm DE}/(\rho_{\rm DE} + \rho_{\rm M})$. If the expression of $\rho_{\rm DE}$ is given, we can obtain the exact solution of $H(t)$ by solving Eq. (18). On the other hand, we can calculate the present value of $\dot{H}_0$. The present Hubble parameter is recognized as $H_0 = 2.1332h \times 10^{-42}$ GeV with $h = 0.72 \pm 0.08$.[23] The combined data analysis has provided the following constraint for the present density parameter of dark energy: $\Omega_{\rm DE(0)} = 0.726 \pm 0.015$.[14] Thus, we have $\dot{H}_0 = (3/2)H_0^2(\Omega_{\rm DE(0)} - 1) = -9.6955 \times 10^{-85}$ GeV^2.

We now consider whether $\rho_{\rm DE}$ could be the symmetry breaking vacuum energy density $\rho_{\rm eff}$. From Eq. (12), we know energy density $\rho_{\rm eff}$ is negative. Combining Eqs. (16) and (17), we obtain

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho_{\rm DE} + \rho_{\rm M} + 3p_{\rm DE}). \tag{19}$$

If the density of dark energy $\rho_{\rm DE}$ is equal to $\rho_{\rm eff}$, $\ddot{a}$ is evidently negative and indicates a decelerate expansion of the present universe. This is of course problematic, as we will show that the contribution of $\rho_{\rm eff}$ to $\rho_{\rm DE}$ is small enough to be ignored. It is worth pointing out that the term $(a''/a^3)^2$ in Eq. (12) is assumed to be sufficiently small and to be neglected, so we have $\rho_{\rm eff} = (2\mu^2 a''/a^3)/4\lambda$. In fact, using relations $\mathrm{d}\eta = \mathrm{d}t/a(t)$ and $R = 6(\ddot{a}/a + \dot{a}^2/a^2)$, we obtain

$$\rho_{\rm eff} = \frac{\mu^2}{2\lambda}\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right). \tag{20}$$

Substituting $H \equiv \dot{a}/a$ into Eq. (20), we obtain

$$\rho_{\rm eff} = \frac{\mu^2}{2\lambda}(2H^2 + \dot{H}). \tag{21}$$

Note that $\rho_{\rm eff}$ is proportional to $H^2(t)$, since $\dot{H} = (3/2)H^2(\Omega_{\rm DE} - 1)$. Using $\sqrt{-\mu^2/\lambda} \simeq 300$ GeV, $H_0 = 2.1332h \times 10^{-42}$ GeV, and $\dot{H}_0 = -9.6955 \times 10^{-85}$ GeV^2, we obtain $\rho_{\rm eff} \simeq -1.69 \times 10^{-79}$ GeV^4. This is smaller than the observed cosmic energy density $\rho \simeq 10^{-47}$ GeV^4 by a factor of 10^{32}, therefore it could be ignored. Hence, although the symmetry breaking vacuum energy may have a contribution to the dark energy, such a contribution is too small to explain the observed value of the dark energy.

A simple calculation shows that $a''/a^3 = 2H^2 + \dot{H} = 3.7485 \times 10^{-84}$ GeV^2, which is very small. It is reasonable to neglect the term $(a''/a^3)^2$ when calculating $\rho_{\rm eff}$, unless the mass of the Higgs particle $m = \sqrt{-2\mu^2}$ is $\simeq 10^{-42}$ GeV. This is clearly impossible, since a Higgs particle with such a small mass should be found easily.

On the other hand, despite the fact that our result cannot be used to solve the dark energy problem, the small value of $\rho_{\rm eff}$ can be used to explain the puzzle of why symmetry breaking vacuum energy has not

been observed. If the vacuum energy density in the flat space–time has not been subtracted, such an energy density with $|\rho_{\rm vac}| \simeq 10^6$ GeV^4 (for the coupling constant of α^2), which is larger than the observed cosmic energy density $\rho \simeq 10^{-47}$ GeV^4 by a factor of 10^{53}, should produce a remarkable effect on gravity. The gravitational effect of the vacuum energy is sufficient to determine the evolution of the universe, and the contribution of the observed matter could be neglected. However, if we use the new concept based on the Hamiltonian formulation of the general relativity to calculate the symmetry breaking vacuum energy, then the contribution of the vacuum energy in the flat space–time is subtracted, by a small value, which is smaller than the observed cosmic energy density by a factor of 10^{32}. Obviously, the gravitational contribution of the symmetry breaking vacuum energy is small and hard to detect.

There is also another way, which does not require the subtraction of the flat space–time contribution, to solve such a problem. As mentioned above, we can correct the Lagrangian of the Higgs fields by adding a term V_0. The absolute energy $-V_0$ would have a gravitational effect. We can obtain the value of V_0 by arguing that the term $-V_0$ cancels $\rho_{\rm vac}$ (or $\rho'_{\rm vac}$) and means that the symmetry breaking vacuum energy is never observed.[11] However, the principle for deciding the absolute value of V_0 is still absent, and we know of no reason why V_0 should have such a value. Adding a constant term V_0 to the electroweak action bears a strong resemblance to adding the cosmological constant term to the Einstein–Hilbert action. As in the dark energy problem, it is necessary to seek the origin of the constant term. Moreover, adding a constant to the Lagrangian of matter fields can be interpreted as changing the zero point of the potential term. Hence, if we calculate the vacuum energy without the subtraction of the flat space–time contribution, the result depends on the zero point of the potential. This means that the gravitational effect will be altered by changing the zero point of the potential, which violates our intuition. Alternatively, our explanation does not depend on the value of V_0 since it has been canceled. This does not require an unknown principle to determine V_0 and does not depend on the selection of the zero point of the potential.

4. Discussion and conclusion

In this paper, we have investigated the spontaneous symmetry breaking vacuum energy density in the electroweak theory in the spatially flat FRW space–time by using a new method which subtracts the flat space–time contribution from the energy in the curved space–time. As discussed above, such a method leads to a remaining effective energy density, which is independent of the selection of the zero point of the potential term in the Lagrangian of matter fields, and therefore consistent with our intuition. The effective energy density is too small to cause the acceleration of the universe, and therefore cannot be a promising candidate for dark energy, although it could have a contribution to the dark energy.

Actually, the legality of applying Maggiore's method to study the zero point energy is ambiguous. First, as pointed out earlier, the FRW space–time is conformal flat rather than asymptotic flat. Second, when we subtract the flat space–time contribution from the zero point energy in the FRW space–time, the residual value is still infinite. Hence, in Maggiore's paper, a cut-off procedure is necessary for effective zero point energy. Fortunately, since we do not consider the quantization of the scalar fields, the difficulty of divergence does not exist in our case.

Our result shows that the symmetry breaking vacuum energy density is proportional to $H^2(t)$. The zero-point energy density calculated by using the same principle is also proportional to $H^2(t)$.[12] Thus, the total vacuum energy density of the quantum fields contains the zero-point energy and the symmetry breaking energy, which depends on time. Although the value of the zero-point energy density can be modified by changing the cut-off scale to obtain a suitable value of the total vacuum energy density, the time dependence of the total vacuum energy is not compatible with the present observations of dark energy. Thus, the vacuum energy density calculated by using Maggiore's method cannot solve the dark energy problem.

However, our result is useful in explaining the puzzle of why the gravitational effect of symmetry breaking vacuum energy has not been observed. There are two problems leading the quantum field theory to be in conflict with general relativity. One is why the gravitational effect of the huge zero-point energy density has not been observed, and the other is why the gravitational effect of the huge symmetry breaking vacuum energy density has not been detected. For

the second problem, our result can provide a possible explanation to reconcile the electroweak theory with the general relativity.

References

[1] Riess A G, Filippenko A V, Challis P, Clocchiatti A, Diercks A, Garnavich P M, Gilliland R L, Hogan C J, Jha S, Kirshner R P, Leibundgut B, Phillips M M, Reiss D, Schmidt B P, Schommer R A, Smith R C, Spyromilio J, Stubbs C, Suntzeff N B and Tonry J 1998 *Astron. J.* **116** 1009

[2] Perlmutter S 1999 *Astron. J.* **517** 565

[3] Peebles P J E and Ratrra B 2003 *Rev. Mod. Phys.* **75** 559

[4] Padmanabhan T 2003 *Phys. Rept.* **380** 235

[5] Copeland E J, Sami M and Tsujikawa S 2006 *Int. J. Mod. Phys.* D **15** 1753

[6] Silvestri A and Trodden M 2009 *Rept. Prog. Phys.* **72** 096901

[7] Lu J B, Wu Y B, Xu L X and Wang Y T 2011 *Chin. Phys.* B **20** 79801

[8] Sun C Y 2009 *Commun. Theor. Phys.* **52** 441

[9] Chen J H and Wang Y J 2008 *Chin. Phys.* B **17** 1184

[10] Caldwell R P and Kamionkowski M 2009 *Ann. Rev. Nucl. Part. Sci.* **59** 397

[11] Weinberg S 1989 *Rev. Mod. Phys.* **61** 1

[12] Maggiore M 2011 *Phys. Rev.* D **83** 063514

[13] Arnowitt R L, Deser S and Misner C W 1962 *Gravitation An Introduction to Current Research* (New York: Wiley)

[14] Komatsu E, Dunkley J, Nolta M R, Bennett C L, Gold B, Hinshaw G, Jarosik N, Larson D, Limon M, Page L, Spergel D N, Halpern M, Hill R S, Kogut A, Meyer S S, Tucker G S, Weiland J L, Wollack E and Wright E L 2009 *Astrophys. J. Suppl.* **180** 330

[15] Robertson H P 1935 *Ap. J.* **82** 284

[16] Walker A G 1936 *Proc. Lond. Math. Soc.* **42** 90

[17] Higgs P W 1964 *Phys. Rev. Lett.* **12** 123

[18] Weinberg S 1967 *Phys. Rev. Lett.* **42** 1264

[19] Parker L and Fulling S A 1974 *Phys. Rev.* D **9** 341

[20] Fulling S A and Parker L 1974 *Ann. Phys.* **87** 176

[21] Hawking S W 1975 *Commun. Math. Phys.* **43** 199

[22] Unruh W G 1976 *Phys. Rev.* D **14** 870

[23] Freedamn W L 2001 *Astrophys. J.* **47** 553