# A feedback-controlled Brownian ratchet operated by a temperature switch*

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In this paper, we present a model of the Brownian motor in a feedback controlled ratchet, in which the application of the flashing potential depends on the state of the particle to be controlled. We derive an analytical expression for the velocity induced by the feedback ratchet, which is a function of several parameters, including the ratio of the two switching temperatures and the asymmetry parameter of the potential field. The motor shows a current inversion when either parameter is varied.

**Keywords:** feedback, temperature switch, asymmetry parameter

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## 1. Introduction

Noise-induced transport of Brownian motors or ratchets plays an important role in a number of physical systems. Parrondo and de Cisneros[1−5] categorized Brownian motors into three classes according to the nonequilibrium driving forces, i.e., the forced ratchet (motor), the chemical motor, and the thermal motor. The Brownian motor driven by nonuniform temperature[6−8] was studied previously in open-loop ratchets, in which the rectifying potential was applied independently of the state of the system to be controlled. The closed-loop or feedback ratchet for a Brownian motor means that the controller acting on the Brownian particle in a potential field has knowledge of the particle's state. Recent research on the feedback-controlled view of a flashing rachet[4] shows that the controlling action induces a maximum flux proportional to the square-root of the amount of information about the particle's state[9] and a maximum power output proportional to the amount of information.[10] This possible improvement of ratchet performance opens the possibility of improving the technological applications of ratchets[9] and sheds light on the high operating efficiency[11−14] of biological molecular motors.

In this work, we set up a model for the temperature feedback controlled system[15,16] in Section 2. In Section 3, we derive an analytical expression for the particle's mean velocity $\langle v\rangle$ without load force, and it is expressed as a function of the ratio of the two switching temperatures, the ratio of $V_0$ and $kT$, and the structure parameter of the saw tooth potential field. The directed motion and the current inversion are discussed by plotting $\langle v\rangle$ against the model parameters.

## 2. Model

The feedback controlled ratchet model is constructed as follows. The motion of the Brownian particle satisfies the over damped Langevin equation

$$\gamma\frac{\mathrm{d}x}{\mathrm{d}t}=\xi(t)-\frac{\mathrm{d}V(x)}{\mathrm{d}x}, \tag{1}$$

where $\gamma$ is the friction coefficient, which is related to the viscosity coefficient through the Stokes relation $\gamma=6\pi r\eta$, with $r$ being the particle's diameter and the viscosity coefficient being $\eta=\frac{41.17}{T-250}$.[17] $V(x)$ is the saw tooth potential (see Fig. 1), which can be described by a piecewise linear function[4,5]

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$$V(x)=\begin{cases}\dfrac{V_0}{aL}x, & \left(0\le\dfrac{x}{L}<a\right),\\ V_0-\dfrac{V_0}{1-a}\left(\dfrac{x}{L}-a\right), & \left(a\le\dfrac{x}{L}<1\right),\end{cases}\tag{2}$$

where $V_0$ is the maximum of the potential, $L$ is the period of the potential field with $V(x)=V(x+L)$, and $a$ is the asymmetry parameter of the potential field. $\xi(t)$ describes the Gaussian white noise with zero mean $\langle\xi(t)\rangle=0$ and relation $\langle\xi(t)\xi(t+\tau)\rangle=2\gamma kT\delta(\tau)$, where $k$ is the Boltzmann constant, and $kT$ is the strength of the thermal noise. The surrounding temperature of the particle is decided by the following equation:

$$T_c=\Theta\left(V'(x)\right)T'+\left[1-\Theta\left(V'(x)\right)\right]T,\tag{3}$$

where $\Theta(x)$ is the Heaviside function. If $x>0$, $\Theta(x)=1$, else $\Theta(x)=0$, which means that the surrounding temperature of the particle switches from $T'$ to $T$ when it moves from interval $[0,\,aL)$ to interval $[aL,\,L)$. So we use $\gamma'$, $\gamma$ to denote the different friction coefficients and $\xi'$, $\xi$ to denote the white noise of the different intensities in the two intervals of a period. Then we write the corresponding Langevin equations as

$$\begin{aligned}\gamma'\dot{x}&=\xi'(t)-\frac{V_0}{aL}, && \left(0\le\frac{x}{L}<a\right),\\ \gamma\dot{x}&=\xi(t)+\frac{V_0}{(1-a)L}, && \left(a\le\frac{x}{L}<1\right).\end{aligned}\tag{4}$$

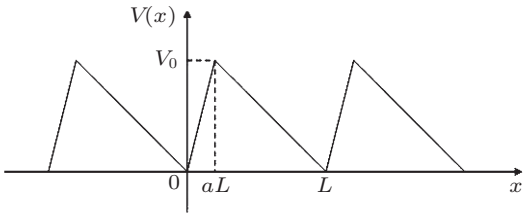



**Fig. 1.** Sketch of the ratchet potential.

Another equivalent form of the dynamic equations corresponding to the Langevin equations (4) is given by the following Fokker–Plank equations:

$$\begin{aligned}\frac{\partial w_1(x,t)}{\partial t}&=-\frac{\partial}{\partial x}\left(-\frac{V_0}{\gamma' aL}+\frac{kT'}{\gamma'}\frac{\partial}{\partial x}\right)w_1(x,t)=-\frac{\partial j_1}{\partial x}, && \left(0\le\frac{x}{L}<a\right),\\ \frac{\partial w_2(x,t)}{\partial t}&=-\frac{\partial}{\partial x}\left[\frac{V_0}{\gamma(1-a)L}+\frac{kT}{\gamma}\frac{\partial}{\partial x}\right]w_2(x,t)=-\frac{\partial j_2}{\partial x}, && \left(a\le\frac{x}{L}<1\right),\end{aligned}\tag{5}$$

where $w_{1,2}(x,t)$ are the joint probability densities when the Brownian particle is located at position $x$ and the potential takes the different piecewise values at time $t$ respectively, and $j_1$, $j_2$ are the probability current densities in the two intervals.

## 3. Results and discussion

In the steady state, we have $j_1=j_2=C$. Thus, the Fokker–Planck equations can be rewritten as

$$\begin{aligned}&\left(-\frac{V_0}{\gamma' aL}+\frac{kT'}{\gamma'}\frac{\partial}{\partial x}\right)w_1(x)=C, && \left(0\le\frac{x}{L}<a\right),\\ &\left[\frac{V_0}{\gamma(1-a)L}+\frac{kT}{\gamma}\frac{\partial}{\partial x}\right]w_2(x)=C, && \left(a\le\frac{x}{L}<1\right).\end{aligned}\tag{6}$$

The solutions can be easily obtained as

$$\begin{aligned}w_1(x)&=\left[w_1(0)+\frac{\gamma' aL}{V_0}C\right]\mathrm{e}^{-\frac{V_0}{aLkT'}x}-\frac{\gamma' aL}{V_0}C,\\ w_2(x)&=\left[w_2(aL)+\frac{\gamma(1-a)L}{V_0}C\right]\mathrm{e}^{-\frac{V_0}{(1-)aLkT}(x-aL)}\\ &\quad-\frac{\gamma(1-a)L}{V_0}C.\end{aligned}\tag{7}$$

Considering the periodic boundary condition $w_1(0)=w_2(L)$, the continuity condition $w_1(aL)=w_2(aL)$, and the normalization condition $\int_0^{aL}w_1(x)\mathrm{d}x+\int_{aL}^{L}w_2(x)\mathrm{d}x=1$, we can determine the three constants as follows:

$$\begin{aligned}w_1(0)&=\frac{\gamma(1-a)L\left(\mathrm{e}^{\frac{V_0}{kT}}-1\right)}{V_0\left[\mathrm{e}^{\frac{V_0}{k}\left(\frac{1}{T}-\frac{1}{T'}\right)}-1\right]}C\\ &\quad+\frac{\gamma' aL\,\mathrm{e}^{\frac{V_0}{kT}}\left(1-\mathrm{e}^{-\frac{V_0}{kT'}}\right)}{V_0\left[\mathrm{e}^{\frac{V_0}{k}\left(\frac{1}{T}-\frac{1}{T'}\right)}-1\right]}C,\\ w_2(aL)&=\frac{\gamma' aL\left(1-\mathrm{e}^{-\frac{V_0}{kT'}}\right)}{V_0\left[\mathrm{e}^{\frac{V_0}{k}\left(\frac{1}{T}-\frac{1}{T'}\right)}-1\right]}C\\ &\quad+\frac{\gamma(1-a)L\,\mathrm{e}^{-\frac{V_0}{kT'}}\left(\mathrm{e}^{\frac{V_0}{kT}}-1\right)}{V_0\left[\mathrm{e}^{\frac{V_0}{k}\left(\frac{1}{T}-\frac{1}{T'}\right)}-1\right]}C,\\ C&=\frac{V_0^2\left[\mathrm{e}^{\frac{V_0}{k}\left(\frac{1}{T}-\frac{1}{T'}\right)}-1\right]}{D+E+F},\end{aligned}\tag{8}$$

with

$$
\begin{aligned}
D &= a(1-a)L^2\left(1-\mathrm{e}^{-\frac{V_0}{kT'}}\right)\left(\mathrm{e}^{\frac{V_0}{kT}}-1\right)\\
&\quad\times(\gamma kT'+\gamma' kT),\\
E &= L^2\left[\gamma kT(1-a)^2\left(1+\frac{V_0}{kT}-\mathrm{e}^{\frac{V_0}{kT}}\right)\right.\\
&\quad\left.+\gamma' kT'a^2\left(1-\frac{V_0}{kT'}-\mathrm{e}^{-\frac{V_0}{kT'}}\right)\right]\\
&\quad\times\left[\mathrm{e}^{\frac{V_0}{k}\left(\frac{1}{T}-\frac{1}{T'}\right)}-1\right],\\
F &= a^2L^2\gamma' kT'\,\mathrm{e}^{\frac{V_0}{kT}}\left(1-\mathrm{e}^{-\frac{V_0}{kT'}}\right)^2\\
&\quad+(1-a)^2L^2\gamma kT\,\mathrm{e}^{-\frac{V_0}{kT'}}\left(\mathrm{e}^{\frac{V_0}{kT}}-1\right)^2. \qquad (9)
\end{aligned}
$$

Taking the ensemble average of Eq. (1) and integrating the probability current over the whole period, we obtain the relationship between the mean velocity and the probability current as

$$
\begin{aligned}
\langle\dot{x}\rangle &= \int_0^L j(x,t)\mathrm{d}x-\left(\frac{kT}{\gamma}-\frac{kT'}{\gamma'}\right)\\
&\quad\times\left(\frac{kT}{\gamma}-\frac{kT'}{\gamma'}\right)[w(0)-w(aL)]. \qquad (10)
\end{aligned}
$$

Thus, the analytical expression for the particle's mean velocity can be derived from Eqs. (8) and (10) as

$$
\langle\dot{x}\rangle=\frac{V_0^2\left[\mathrm{e}^{\frac{V_0}{k}\left(\frac{1}{T}-\frac{1}{T'}\right)}-1\right]}{D+E+F}\left\{1-\frac{\left(\frac{kT}{\gamma}-\frac{kT'}{\gamma'}\right)[\gamma(1-a)+\gamma' a]\left(1-\mathrm{e}^{-\frac{V_0}{kT}}\right)\left(\mathrm{e}^{\frac{V_0}{kT}}-1\right)}{V_0\left[\exp\left(\frac{V_0}{k}\left(\frac{1}{T}-\frac{1}{T'}\right)\right)-1\right]}\right\}. \qquad (11)
$$

As the previous studies introduced $T'/T$ as the temperature gradient,[18] we introduce

$$
\alpha=\frac{V_0}{kT}, \qquad (12)
$$

$$
\beta=\frac{T'}{T}. \qquad (13)
$$

Here $\alpha$ represents the constraining effect of the potential field to the thermal motion, $\beta$ describes the temperature variety, $\beta>1$ represents a temperature rise, and $\beta=1$ means that no variety is induced by the control action, in which case there will be no directional motion of the particle. Thus, equation (11) can be rewritten as

$$
\langle\dot{x}\rangle=\frac{\alpha^2 kT\left(\mathrm{e}^{\alpha-\frac{\alpha}{\beta}}-1\right)}{D^*+E^*+F^*}\left\{1-\frac{(1-\beta)[(1+\beta)T-250]\left(\frac{1-a}{T-250}+\frac{a}{\beta T-250}\right)(\mathrm{e}^{\alpha}-1)\left(1-\mathrm{e}^{-\frac{\alpha}{\beta}}\right)}{\alpha\left(\mathrm{e}^{\alpha-\frac{\alpha}{\beta}}-1\right)}\right\}, \qquad (14)
$$

with

$$
\begin{aligned}
D^* &= La(1-a)\left(1-\mathrm{e}^{-\frac{\beta}{\alpha}}\right)(\mathrm{e}^{\alpha}-1)\left(\frac{775.64\varepsilon\beta}{T-250}+\frac{775.64\varepsilon}{\beta T-250}\right),\\
E^* &= L\left[\frac{775.64\varepsilon}{T-250}(1-a)^2(1+\alpha-\mathrm{e}^{\alpha})+\frac{775.64\varepsilon\beta}{\beta T-250}a^2\left(1-\frac{\alpha}{\beta}-\mathrm{e}^{-\frac{\alpha}{\beta}}\right)\right]\left(\mathrm{e}^{\alpha-\frac{\alpha}{\beta}}-1\right),\\
F^* &= \frac{775.64\varepsilon\beta}{\beta T-250}a^2\,\mathrm{e}^{\alpha}\left(1-\mathrm{e}^{-\frac{\alpha}{\beta}}\right)^2+\frac{775.64\varepsilon}{T-250}(1-a)^2\,\mathrm{e}^{-\frac{\alpha}{\beta}}(\mathrm{e}^{\alpha}-1)^2. \qquad (15)
\end{aligned}
$$

In the following, $T=300$ K, $L=10$ nm, $\alpha$ varies from 0.1 to 5, $\beta$ varies from 0.91 to 1.2334, correspondingly, $T'$ varies from 273 K to nearly 373 K. We mainly restrict ourselves to the above range, which is the range where our evaluation is valid in principle. We will explore Eq. (15) by plotting it against $\alpha$ and $\beta$.

We present several results for the feedback-controlled system. The current is induced basically due to the temperature gradient and the modulation. The current does vanish when the temperature gradient is zero or the modulation is not available. From a broad point of view, this effect is not new, but the effect we discussed is perhaps more elementary.

In Fig. 2, we show the dependences of $\langle v\rangle$ on $\alpha$ and $\beta$ for different values of $a$. From Figs. 2(a)–2(d), we can find that when $\beta=1$, there is no net current. This can be easily understood by taking the two temperatures as two baths. If the two baths are at the same temperature, the Brownian motor can not extract work from them. When $\alpha=0$, there is no net current. For large $T$ or small $V_0$, the broken inversion symmetry is not strong enough to induce a net current.

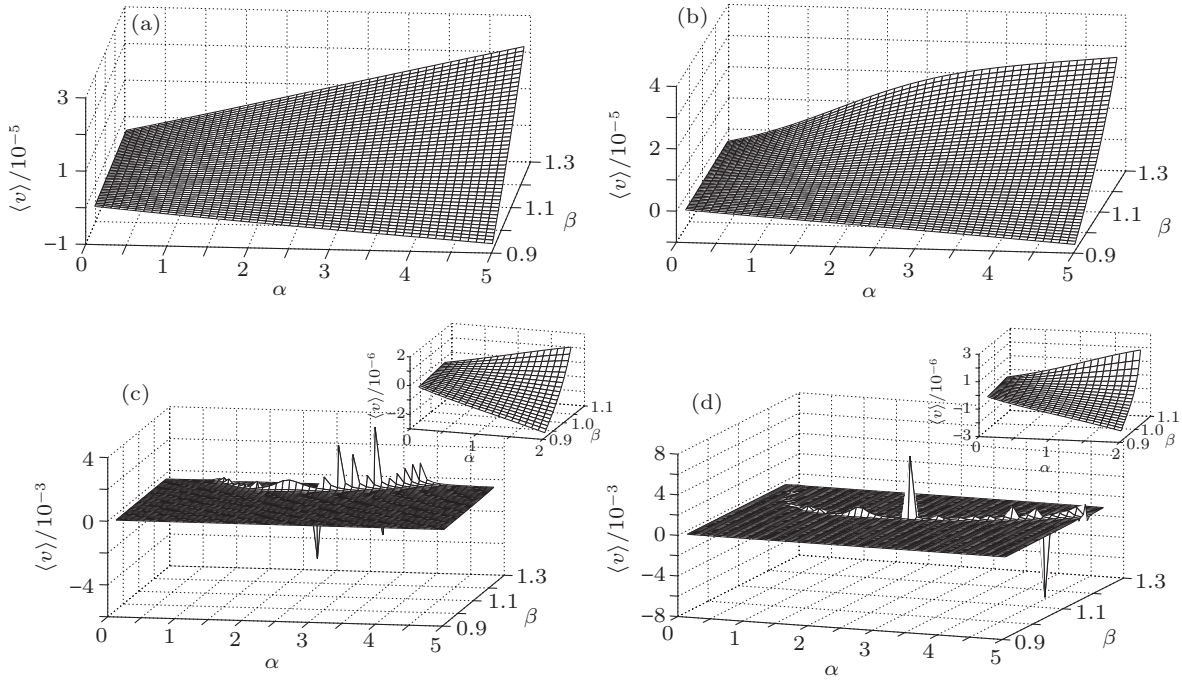



**Fig. 2.** Mean velocity $\langle v\rangle$ as a function of the asymmetry parameters $\alpha$ and $\beta$ for (a) $a = 0.3$, (b) $a = 0.4$, (c) $a = 0.5$, (d) $a = 0.7$. The inset shows the negative current in detail.

As shown in Figs. 2(a) and 2(b), when $a < 0.5$, the flux grows as $\alpha$ increases. The fact that the flux finally reaches a constant value for large potential heights is a direct consequence of the overdamped nature of the ratchet. We note that enlarging the value of $\alpha$ corresponds to effectively diminish the intensity of the white noise that accounts for the thermal fluctuation. For $\alpha \neq 0$, the current is inverted when $\beta$ increases from 0.91 to 1.2334. The motor inverts its velocity when the temperature gradient is inverted, as indicated by the previous studies.[19] Some important thermodynamic requirements are fulfilled, i.e., there is no current when $\beta = 1$ or $\alpha = 0$. It is worth noting that for a large value of $a$, the current is small.

As shown in Figs. 2(c) and 2(d), when $a \geq 0.5$ and $\beta < 1$, the current is negative and increases as $\alpha$ increases in absolute value (see the inset of the figure). When $a \geq 0.5$ and $\beta > 1$, the current first reaches a positive maximum, then decreases to 0, and then continually decreases to a minimum. The entanglement between the dependence on $\alpha$, $\beta$ and $a$ is apparent. The inversion asymmetry and $V_0$ of the potential compete against the feedback, which determines the current and its direction.

# 4. Conclusion

We have studied a feedback-controlled strategy for flashing ratchets, in which the controlled actions are performed by temperature switches. The analytical expression for the particle's mean velocity is obtained. The dependences of the mean velocity on three parameters, $a$, $\alpha$, and $\beta$, are discussed. For $a < 0.5$, the current is enhanced with the increasing $\alpha$, and inverts when the temperature gradient is inverted. For $a \geq 0.5$ and $\beta < 1$, the current is enhanced with the increasing $\alpha$. For $a \geq 0.5$ and $\beta > 1$, the current varies in a complicated way with the increasing $\alpha$ and inverts with the increasing $\alpha$. Regardless of the particular properties of the temperature feedback-controlled ratchet, it is an unrealistic model for biological molecular motors.

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