Generating and reversing spin accumulation by temperature gradient in a quantum dot attached to ferromagnetic leads*

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We propose to generate and reverse the spin accumulation in a quantum dot (QD) by using the temperature difference between the two ferromagnetic leads connected to the dot. The electrons are driven purely by the temperature gradient in the absence of an electric bias and a magnetic field. In the Coulomb blockade regime, we find two ways to reverse the spin accumulation. One is by adjusting the QD energy level with a fixed temperature gradient, and the other is by reversing the temperature gradient direction for a fixed value of the dot level. The spin accumulation in the QD can be enhanced by the magnitudes of both the leads' spin polarization and the asymmetry of the dot–lead coupling strengths. The present device is quite simple, and the obtained results may have practical usage in spintronics or quantum information processing.

Keywords: quantum dot, temperature gradient, spin accumulation, ferromagnetic leads

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1. Introduction

The preparation and manipulation of the single electron spin in a quantum dot (QD) is crucial for both spintronic devices[1] and spin-based quantum information processing.[2,3] Various techniques have been developed to prepare the single spin state in QD, including the spin blockade,[4] the optical pumping,[5−7] and the photoluminescence polarization.[8] The prepared electron spin can be subsequently manipulated using an oscillating magnetic field,[4] ultrafast optical pulses,[5,9] and spin-to-charge conversion techniques.[10,11] Besides the above-mentioned works involving time-dependent fields, the electrical spin control based on spin–orbit interaction[12−14] and spin bias[15−18] has also been intensively investigated in the last few decades. Up to now, an effective spin control method in a simple device is still lacking, and this research topic is still in its infancy.

Recently, the spin Seebeck effect has been observed in a metallic magnet based on the spin detection technique by Uchida *et al.*, where the spin bias was generated by a temperature gradient.[19] This effect can also be used to manipulate and detect the spin-related information in terms of thermal signals, suggesting a way to design and fabricate thermospin quantum devices based on the thermal bias instead of the usual electric bias. After that, much work has been devoted to the investigation of the spin-dependent thermoelectric transport in a single QD attached to ferromagnetic leads.[20−23] It was shown that due to the discretization of the QD energy level and the intradot Coulomb interaction, the spin current can be obtained in a rather simple device with the help of the temperature gradient. Moreover, semiconductor spacers of InAs QD with tunable size and energy levels have been inserted inbetween nickel or cobalt leads.[24−26] In such a device, the spin polarization of the current injected from the ferromagnetic leads and the tunnel magnetoresistance (TMR) can be effectively adjusted by using a gate near the QD, which opens new possible applications in spintronics. Its new characteristics, such as the anomalies of the TMR caused by the intradot Coulomb repul-

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sion energy, were explained in subsequent theoretical work.[27]

In view of the above advances, we investigate the possibility of generating and reversing the spin accumulation in a QD with the help of ferromagnetic leads and a thermal bias. The spin accumulation denotes the difference between the spin-up and spin-down electron occupation numbers. The manipulation of this quantity is at the heart of spin-based quantum information processing and spintronics. From the application point of view, a magnitude of spin accumulation exceeding 0.5 is expected, which is obtained in the present paper. Driven only by the thermal bias, the spin accumulation is zero when the magnetic moments of the two leads are arranged in the parallel configuration. While a large spin accumulation emerges when the leads' magnetic moments are in the antiparallel configuration, or when the dot is coupled to one ferromagnetic lead and one non-magnetic lead (FM–QD–NM). In the Coulomb blockade regime, the spin imbalance can be generated and switched by a quite weak temperature gradient in the dot. We emphasize that the present device is very simple and realizable with current technology.

2. Model and method

The system of a QD coupled to the left and the right leads can be described by the following Hamiltonian:[28−30]

$$\begin{aligned} H = & \sum_{k,\sigma,\beta=\mathrm{L,R}} \varepsilon_{k\beta\sigma} c^{\dagger}_{k\beta\sigma} c_{k\beta\sigma} \\ & + \sum_{\sigma} \varepsilon_d d^{\dagger}_{\sigma} d_{\sigma} + U d^{\dagger}_{\uparrow} d_{\uparrow} d^{\dagger}_{\downarrow} d_{\downarrow} \\ & + \sum_{k,\sigma,\beta} (t_{\beta} c^{\dagger}_{k\beta\sigma} d_{\sigma} + \mathrm{H.c.}), \end{aligned} \tag{1}$$

where $c^{\dagger}_{k\beta\sigma}$ ($c_{k\beta\sigma}$) is the creation (annihilation) operator of the electron with momentum k, spin σ ($\sigma =\uparrow,\downarrow$ or $\sigma = \pm 1$), and energy $\varepsilon_{k\beta\sigma}$ in lead β; $d^{\dagger}_{\sigma}$ (d_{σ}) creates (annihilates) an electron of energy ε_d in the dot; U denotes the intradot Coulomb interaction; and t_{β} describes the dot–lead tunneling coupling (its energy-dependence is neglected for the sake of simplicity). The ferromagnetism of the lead is described by the spin-dependent density of states $\rho_{\beta\sigma}$, based on which the lead's spin asymmetry factor is defined as $p_{\beta} = (\rho_{\beta\uparrow} - \rho_{\beta\downarrow})/(\rho_{\beta\uparrow} + \rho_{\beta\downarrow})$.[31]

To find the dot average occupation numbers, we use the master equation technique[18,32,33]

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t} n_{\sigma} = & \, \Gamma^{+}_{\sigma}[(1-n_{\sigma})(1-n_{-\sigma})] - \Gamma^{-}_{\sigma}(n_{\sigma} - n_d) \\ & + \tilde{\Gamma}^{+}_{\sigma}(n_{\sigma} - n_d) - \tilde{\Gamma}^{-}_{\sigma} n_d, \end{aligned} \tag{2}$$

where $n_{\sigma} = \langle d^{\dagger}_{\sigma} d_{\sigma} \rangle$ and $n_d = \langle d^{\dagger}_{\uparrow} d_{\uparrow} d^{\dagger}_{\downarrow} d_{\downarrow} \rangle$ are the single and the double average occupation numbers on the dot, respectively. The total tunneling rates are

$$\Gamma^{\pm}_{\sigma} = \Gamma^{\pm}_{\mathrm{L}\sigma} + \Gamma^{\pm}_{\mathrm{R}\sigma} = \Gamma_{\mathrm{L}\sigma} f^{\pm}_{\mathrm{L}} + \Gamma_{\mathrm{R}\sigma} f^{\pm}_{\mathrm{R}}, \tag{3}$$

and

$$\tilde{\Gamma}^{\pm}_{\sigma} = \tilde{\Gamma}^{\pm}_{\mathrm{L}\sigma} + \tilde{\Gamma}^{\pm}_{\mathrm{R}\sigma} = \tilde{\Gamma}_{\mathrm{L}\sigma} \tilde{f}^{\pm}_{\mathrm{L}} + \tilde{\Gamma}_{\mathrm{R}\sigma} \tilde{f}^{\pm}_{\mathrm{R}}, \tag{4}$$

with

$$\begin{aligned} f^{+}_{\beta} &= 1/\{1 + \exp[(\epsilon_d - \mu_{\beta})/k_{\mathrm{B}} T_{\beta}]\}, \\ \tilde{f}^{+}_{\beta} &= 1/\{1 + \exp[(\epsilon_d + U - \mu_{\beta})/k_{\mathrm{B}} T_{\beta}]\}, \\ f^{-}_{\beta} &= 1 - f^{+}_{\beta}, \\ \tilde{f}^{-}_{\beta} &= 1 - \tilde{f}^{+}_{\beta}, \end{aligned}$$

where k_{B} and μ_{β} are the Boltzmann constant and the electrochemical potential of lead β, respectively, and T_{β} is the temperature of lead β. In the present paper, we set $\mu_{\mathrm{L}} = \mu_{\mathrm{R}} = 0$, $T_{\mathrm{L}} = T + \Delta T$, and $T_{\mathrm{R}} = T$, where ΔT is the temperature difference between the two leads. The line-width functions in the above expressions are given by $\Gamma_{\beta\sigma} = 2\pi |t_{\beta}|^2 \rho_{\beta\sigma}(\epsilon_d)$ and $\tilde{\Gamma}_{\beta\sigma} = 2\pi |t_{\beta}|^2 \rho_{\beta\sigma}(\epsilon_d + U)$. Using the wide-band approximation that $\rho_{\beta\sigma}$ is a constant and featureless, we have $\Gamma_{\beta\sigma} = \tilde{\Gamma}_{\beta\sigma}$. Then n_d in Eq. (2) is canceled out, resulting in the following simplified rate equation for the occupation numbers:[32]

$$\frac{\mathrm{d}}{\mathrm{d}t} n_{\sigma} = \Gamma^{+}_{\sigma}(1 - n_{\sigma} - n_{-\sigma}) - \Gamma^{-}_{\sigma} n_{\sigma} + \tilde{\Gamma}^{+}_{\sigma} n_{-\sigma}. \tag{5}$$

The line-width functions can be written in terms of the spin asymmetry factors as $\Gamma_{\beta\sigma} = \tilde{\Gamma}_{\beta\sigma} = \gamma_{\beta}(1 + \sigma p_{\beta})$, where $\gamma_{\beta} \ll k_{\mathrm{B}} T$ is a parameter. In the stationary regime ($\mathrm{d}n_{\sigma}/\mathrm{d}t = 0$), the occupation numbers are derived from Eq. (5) as

$$n_{\sigma} = \frac{\Gamma^{+}_{\sigma} \Gamma^{-}_{-\sigma} + \Gamma^{+}_{-\sigma} \tilde{\Gamma}^{+}_{\sigma}}{\Pi_{\sigma}}, \tag{6}$$

where

$$\Pi_{\sigma} = (\Gamma^{+}_{\sigma} + \Gamma^{-}_{\sigma})(\Gamma^{+}_{-\sigma} + \Gamma^{-}_{-\sigma}) - (\tilde{\Gamma}^{+}_{\sigma} - \Gamma^{+}_{\sigma})(\tilde{\Gamma}^{+}_{-\sigma} - \Gamma^{+}_{-\sigma}).$$

3. Numerical results

In the following numerical calculations, we choose the intradot Coulomb interaction $U = 1$ as the energy unit and set the constants $e = \hbar = k_{\mathrm{B}} = 1$.

The system temperature in the equilibrium state is fixed at $T = 0.02U$, which is about 350 mK for $U = 15$ meV in experiments.[23,24] Under a zero thermal bias ($T_{\rm L} = T_{\rm R} = T$) and symmetric coupling $\gamma_{\rm L} = \gamma_{\rm R}$, the Fermi functions of the two leads are the same, $f_{\rm L} = f_{\rm R} = f$ and $\tilde{f}_{\rm L} = \tilde{f}_{\rm R} = \tilde{f}$, and then the electron occupation numbers in Eq. (6) are reduced to

$$n_\sigma = \frac{f(1-f+\tilde{f})}{1-(f-\tilde{f})^2}. \tag{7}$$

The spin-up and the spin-down occupation numbers are the same, since there is neither a driving force ($eV = \Delta T = 0$) nor a magnetic field. For the deep dot level case ($\varepsilon_d < -U$), $f = \tilde{f} = 1$ in the low temperature regime, and $n_\sigma = 1$. In the Coulomb blockade regime ($-U < \varepsilon_d < 0$), the Fermi functions are $f = 1$ and $\tilde{f} = 0$, then $n_\sigma = f/(1+f) = 1/2$. In the high dot level regime ($\varepsilon > 0$), we have $f = \tilde{f} = 0$ and $n_\sigma = 0$. Moreover, the occupation numbers are independent of the ferromagnetism of the leads, which are shown in Figs. 1(a) and 1(b) by solid lines.

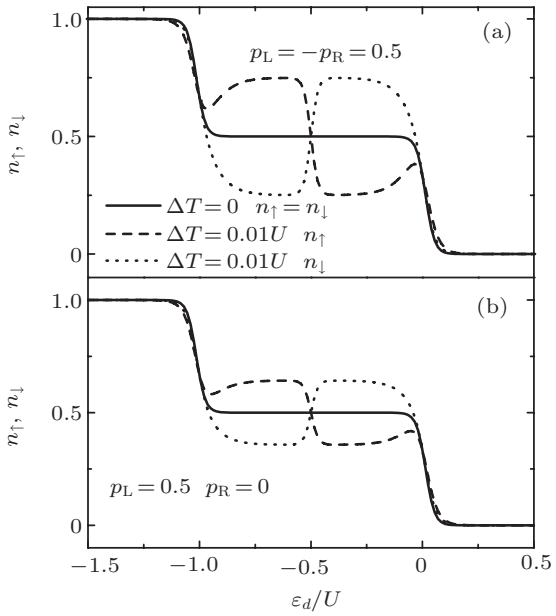


Fig. 1. Spin-up and spin-down electron occupation numbers as a function of the dot level for different values of thermal bias. In panel (a), the leads' magnetic moments are arranged in the antiparallel configuration; in panel (b), the dot is coupled to one ferromagnetic lead and one normal metal lead.

We now turn to the case of finite thermal bias $\Delta T = T_{\rm L} - T_{\rm R}$ and symmetric coupling $\gamma_{\rm L} = \gamma_{\rm R}$. The occupation numbers n_σ for the ferromagnetic leads arranged in parallel configuration $p_{\rm L} = p_{\rm R}$ are given by

$$n_\sigma = \frac{(f_{\rm L}+f_{\rm R})(2-f_{\rm L}-f_{\rm R}+\tilde{f}_{\rm L}+\tilde{f}_{\rm R})}{4-(\tilde{f}_{\rm L}+\tilde{f}_{\rm R}-f_{\rm L}-f_{\rm R})^2}, \tag{8}$$

which is independent of the spin. This is because the ingoing and the outgoing tunneling rates of each spin component are the same, i.e., $\Gamma_{{\rm L}\sigma} = \Gamma_{{\rm R}\sigma}$. Therefore, the spin-up and the spin-down electrons spend the same amount of time in the dot,[32,33] resulting in zero spin accumulation ($n_\uparrow - n_\downarrow = 0$). For the antiparallel configuration ($p_{\rm L} = -p_{\rm R} = p$), the spin accumulation is generated in the Coulomb blockade regime by both the leads' ferromagnetism and the difference between the left and the right Fermi functions. As is seen from the dashed and dotted lines in Fig. 1(a), $n_\uparrow > n_\downarrow$ when $-U < \varepsilon_d < -U/2$, and $n_\uparrow < n_\downarrow$ when $-U/2 < \varepsilon_d < 0$. The spin accumulation is zero when the dot level is located at the electron–hole symmetry point ($\varepsilon_d = -U/2$). Moreover, the total occupation number $n_\uparrow + n_\downarrow$ remains the same as that for the zero thermal bias case (the solid line). This behavior can be explained as follows. Due to the temperature difference, more electrons are excited above the Fermi level $\mu = 0$ in the left lead. In the deep dot level regime ($-U < \varepsilon_d < -U/2$), the electrons enter into the QD from the left lead through the level of $\varepsilon + U$, which is above the Fermi level. Now more spin-up electrons tunnel into the dot than the spin-down ones, as $\Gamma_{{\rm L}\uparrow} > \Gamma_{{\rm L}\downarrow}$. Moreover, the outgoing tunneling rates have $\Gamma_{{\rm R}\uparrow} < \Gamma_{{\rm R}\downarrow}$, which indicates that the spin-up electrons are more difficult to tunnel out of the dot, resulting in the increase of $n_\uparrow$ as shown by the dashed line in Fig. 1(a). On the other hand, the spin-down electrons can easily leave the dot, resulting in the decrease of $n_\downarrow$ (see the dotted line in Fig. 1(a)). When the QD energy level is located in the region of $-U/2 < \varepsilon_d < 0$, the electrons enter into the QD from the right lead through the level of ε_d, which is below the Fermi level. Under this condition, the ingoing and the outgoing tunneling rates of the spin-up and the spin-down electrons are interchanged, resulting in the inverse of the spin accumulation. For $\varepsilon_d = -U/2$, the electrons can tunnel into the QD from either the left lead through $U/2$ or the right lead through $-U/2$. Therefore, the spin-up and spin-down electrons have the same ingoing and outgoing tunneling rates, and the spin accumulation is zero. The above discussion holds true for the FM–QD–NM structure, as shown in Fig. 1(b), but the difference between the spin-up and the spin-down electron occupation numbers is decreased. This is induced by the decreased difference between the ingoing and outgoing tunneling rates of the spin-up and the spin-down electrons.

Figure 2 shows how the magnitude of thermal bias ΔT influences the spin accumulation for fixed values of the leads' spin polarizations. Along the arrow, $\Delta T = 0.02U$, $0.01U$, $0.005U$, $-0.005U$, $-0.01U$, and $-0.02U$ respectively. For the antiparallel configuration, as shown in Fig. 2(a), except for the dot energy level around the electron–hole symmetry point, the magnitude of the spin accumulation is increased with the increase of the thermal bias, as more electrons are excited to participate in the transport. The maximum of the spin accumulation equals to the spin polarization of the leads. Figure 2 indicates that the spin accumulation can be reversed not only by the QD level, which has been shown in Fig. 1, but also by the direction of the thermal bias. The latter can be understood by a similar reasoning as the one discussed above. The behavior of the spin accumulation in the FM–QD–NM device resembles that of the antiparallel configuration shown in Fig. 2(a) but with decreased magnitudes. Moreover, the maximum of the spin accumulation is weakened to be half of that in the antiparallel case because of the lack of the ferromagnetism in the right lead.

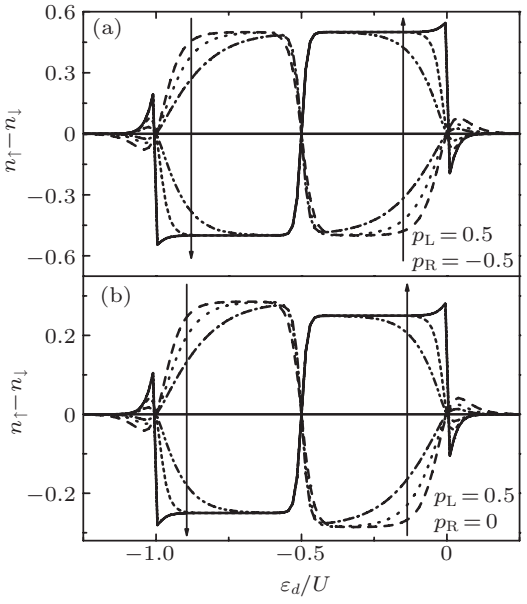


Fig. 2. Spin accumulation $n_\uparrow - n_\downarrow$ varying with the dot energy level for different values of thermal bias. In panel (a), the leads' magnetic moments are arranged in the antiparallel configuration; in panel (b), the dot is coupled to one ferromagnetic lead and one normal metal lead. Along the arrow in the figures, the values of the thermal bias are $\Delta T = 0.02U$, $0.01U$, $0.005U$, -0.005U, -0.01U, and $-0.02U$ respectively.

The spin accumulation is monotonously increased by the increase of the spin polarization of the leads, as shown in Fig. 3, and its maximum remains the same as the magnitude of the spin polarization of the leads (Fig. 3(a)), or half of it in the FM–QD–NM system (Fig. 3(b)).[32] This can be attributed to the property of the tunneling rates. Figure 4 shows that the magnitude of the spin accumulation can be either enhanced or suppressed depending on the asymmetry of the dot–lead coupling $\alpha = \gamma_L/\gamma_R$ and the direction of the thermal bias. Explicitly, the magnitude of the spin accumulation is enhanced when the tunneling rate is larger in the hotter lead, which is shown in Figs. 4(a) and 4(c). While it is weakened if the magnitude of the tunneling rate in the colder lead is increased, as shown in Figs. 4(b) and 4(d). This is reasonable, as the spin imbalance in the QD is enhanced when more electrons enter into the QD easily while fewer electrons are difficult to leave the dot. It is expected that the spin accumulation will be generated in the parallel configuration in the presence of the dot–lead coupling asymmetry, since the ingoing and outgoing tunneling rates are different from each other.

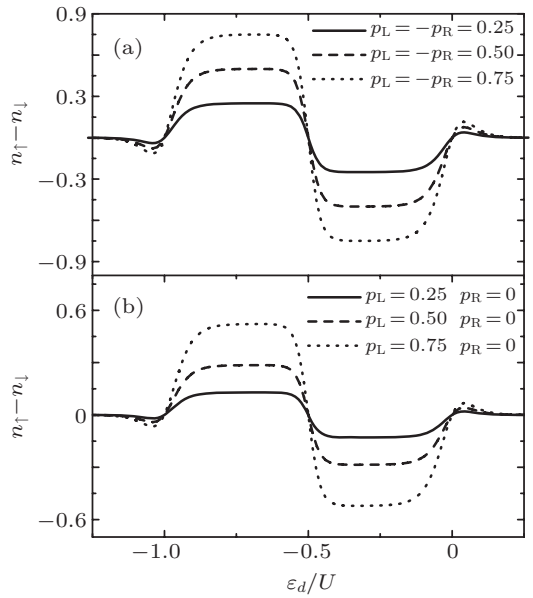


Fig. 3. Spin accumulation $n_\uparrow - n_\downarrow$ as a function of the dot level for different values of the leads' spin polarization p_β. The temperature difference between the two leads is fixed at $0.01U$. In panel (a), the leads' magnetic moments are arranged in the antiparallel configuration; in panel (b), the dot is coupled to one ferromagnetic lead and one normal metal lead.

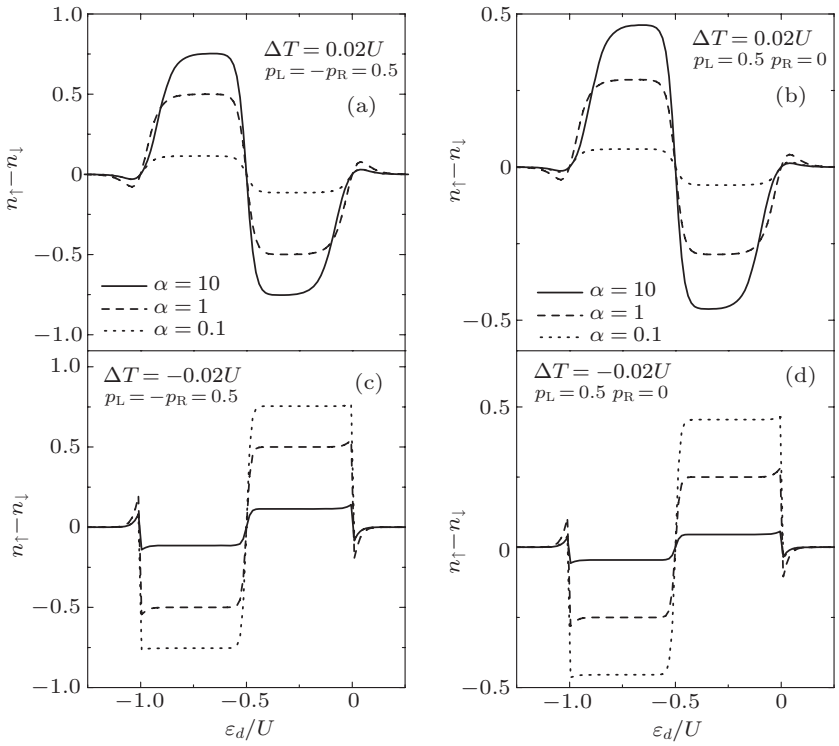


Fig. 4. Spin accumulation as a function of the dot level for different values of asymmetry of the dot–lead coupling $\alpha = \gamma_{\rm L}/\gamma_{\rm R}$. In panels (a) and (b), the leads' magnetic moments are arranged in an antiparallel configuration; in panels (c) and (d), the dot is coupled to one ferromagnetic lead and one normal metal lead.

4. Conclusion

In conclusion, we have studied the spin accumulation in a QD purely under the thermal driving force. Due to the combined effect of temperature gradient and ferromagnetism of the leads, a quite large spin accumulation is generated even under a weak temperature difference. When the dot is symmetrically coupled to the left and the right leads, the maximum of the spin accumulation equals the spin polarization of the leads when arranged in the antiparallel configuration, and is suppressed when the dot is coupled to one ferromagnetic lead and one normal metal lead. The spin accumulation can be reversed by either the dot level or the direction of the thermal bias. The asymmetry of the dot–lead coupling provides another means to tune the magnitude of spin accumulation.

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