Analytical model of signal amplification in silicon waveguides*

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In this paper, an analytical model to investigate the parametric amplification (PA) and the PA + stimulated Raman scattering (SRS) in silicon waveguides is put forward. When two pump signals are employed, the PA bandwidth of the probe signal is so large that the Raman contribution has to be considered. When Raman contribution fraction f is set to be 0, only the PA occurs to amplify the probe signal, and when f is set to be 0.043, the PA and the SRS amplify the probe signal at the same time. The signal amplifications of both single and dual pump schemes are investigated by using this model. With this model, three main affecting factors, i.e., zero dispersion wavelength (ZDWL), third-order dispersion (TOD), and fourth-order dispersion (FOD), are discussed in detail.

Keywords: parametric amplification, stimulated Raman scattering, silicon waveguides, Raman contribution fraction

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1. Introduction

In recent decades, optical amplifiers have played an important role in long distance communication.[1,2] There are many kinds of amplifiers, such as Erbium doped fiber amplifiers (EDFA), semiconductor amplifiers, and fiber Raman amplifiers, which make the optical transmission farther but more expansive. With the development of microelectronics and laser techniques, optical devices of micro or submicro size on a single chip become multifunctional and convenient.[3−6] Unlike long silica fibers, which are usually hundreds of meters long, silicon waveguides that are shorter than several centimeters can provide a gain of more than 10 dB.[7,8] In this sense, they have great potential to be widely manufactured and employed.

As is well known, when an incident light is strong enough, the nonlinear effects in the transmission medium become significant. Besides four-wave mixing (FWM), which realizes the parametric amplification (PA), stimulated Raman scattering (SRS) can also influence the signal gain. The amplification process in silicon is more complicated than that in fiber, because special mechanisms like two-photon absorption (TPA), free carrier absorption (FCA), and free carrier dispersion (FCD) need to be considered. Many types of pump schemes including single and dual pumps with pulse and continuous wave (CW) signals have been investigated in the previous papers.[8,9] There are Raman lasing[6] and amplification[10] in the silicon waveguides as well.

With two pump signals in the PA, a large gain bandwidth of over 200 nm can be obtained.[8] The wavelength range covers the wavelength of the Raman gain peak generated by the pump signals, so the Raman scattering cannot be neglected. An analytical model to investigate PA and PA + SRS cases in the silicon waveguides is presented in this paper. And we show that the SRS must be included in the amplification process. For a convenient investigation based on the silica fibers, the Raman contribution fraction is chosen to be the approximate value with which the theoretical deduction fits the irregular experimental spectrum best.[11] Similar to the previous papers,[12,13] this fraction can also be introduced into the silicon waveguides[14] to depict the importance of Raman gain in the total signal gain, which is a constant of 0.043 on condition that ultrafast pulsed signals are employed as the pumps. In addition, the influences of three affecting factors on the signal gain spectrum are analyzed.

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2. Analytical model

The stimulated Raman scattering (SRS) in silicon is dominated by optical phonons near the Brillouin-zone center, its response function to light is approximated by a Lorentzian function

$$H(\Omega) = \frac{\Omega_{\rm R}^2}{\Omega_{\rm R}^2 - \Omega^2 - 2{\rm i}\Gamma_{\rm R}\Omega}, \tag{1}$$

where $\Gamma_{\rm R}$ is the full width at half maximum (FWHM) of the Raman gain peak, $\Gamma_{\rm R}/\pi \approx 105$ GHz, and $\Omega_{\rm R}$ represents the Raman frequency shift, $\Omega_{\rm R}/2\pi = 15.6$ THz. When the pump signal is strong enough, it will generate a Stokes signal at a longer wavelength and an anti-Stokes signal at the symmetry side. The frequency shift away from the pump frequency is 15.6 THz, as shown in Fig. 1. If the PA gain bandwidth is wide enough, the gain peaks of Stokes and anti-Stokes signals induced by the SRS will be included into the PA bandwidth, which should be suppressed for a wider amplification bandwidth.

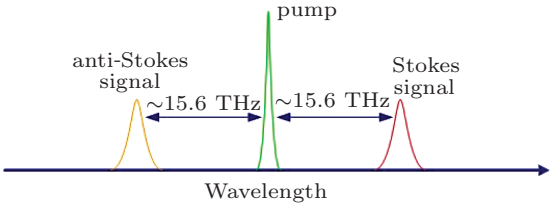


Fig. 1. (colour online) Pump and its corresponding Stokes and anti-Stokes signal waves.

When two pump signals (angular frequencies $\omega_{\rm h}$ and $\omega_{\rm l}$) and one probe signal ($\omega_{\rm s}$) are incident into a silicon waveguide, FWM occurs and generates an idler wave ($\omega_{\rm i} = \omega_{\rm h} + \omega_{\rm l} - \omega_{\rm s}$). Besides this effect, the SRS induced by all signals can also occur. Although the intensities of the pump signals are much stronger than those of the probe and the idler signals, all the induced SRS effects are considered here for objectivity. Apart from the well known linear attenuation, self-phase modulation (SPM), cross-phase modulation (XPM), and chromatic dispersion in silica fiber, a few special effects like TPA, FCA, and FCD in silicon waveguides should be added when modifying the previous model.[12,15] We assume that the four optical waves are linearly co-polarized and propagate along the z axis, their coupled amplitude equations for pump signal fields $|A_{\rm h}|$ and $|A_{\rm l}|$, probe signal $|A_{\rm s}|$, and idler signal $|A_{\rm i}|$ are

$$\begin{aligned}\frac{{\rm d}A_{\rm h}}{{\rm d}z} &= -\frac{(\alpha+\alpha_{\rm h}^{\rm F})}{2}A_{\rm h} + {\rm i}\left(\left(\gamma + {\rm i}\frac{\beta}{2}\right)|A_{\rm h}|^2 A_{\rm h}\right.\\ &\quad + \varphi_1(z)A_{\rm h} + 2\gamma\left\{1 - f + \frac{f}{2}[H(\omega_{\rm s}-\omega_{\rm l})\right.\\ &\quad \left.\left. + H(\omega_{\rm i}-\omega_{\rm l})]\right\}A_{\rm s}A_{\rm i}A_{\rm l}^*\exp({\rm i}\Delta\beta z)\right), \quad (2a)\\ \frac{{\rm d}A_{\rm l}}{{\rm d}z} &= -\frac{(\alpha+\alpha_{\rm l}^{\rm F})}{2}A_{\rm l} + {\rm i}\left(\left(\gamma + {\rm i}\frac{\beta}{2}\right)|A_{\rm l}|^2 A_{\rm l}\right.\\ &\quad + \varphi_2(z)A_{\rm l} + 2\gamma\left\{1 - f + \frac{f}{2}[H(\omega_{\rm s}-\omega_{\rm h})\right.\\ &\quad \left.\left. + H(\omega_{\rm i}-\omega_{\rm h})]\right\}A_{\rm s}A_{\rm i}A_{\rm h}^*\exp({\rm i}\Delta\beta z)\right), \quad (2b)\\ \frac{{\rm d}A_{\rm s}}{{\rm d}z} &= -\frac{(\alpha+\alpha_{\rm s}^{\rm F})}{2}A_{\rm s} + {\rm i}\left(\left(\gamma + {\rm i}\frac{\beta}{2}\right)|A_{\rm s}|^2 A_{\rm s}\right.\\ &\quad + \varphi_{\rm s}(z)A_{\rm s} + 2\gamma\left\{1 - f + \frac{f}{2}[H(\omega_{\rm h}-\omega_{\rm i})\right.\\ &\quad \left.\left. + H(\omega_{\rm l}-\omega_{\rm i})]\right\}A_{\rm h}A_{\rm l}A_{\rm i}^*\exp(-{\rm i}\Delta\beta z)\right), \quad (2c)\\ \frac{{\rm d}A_{\rm i}}{{\rm d}z} &= -\frac{(\alpha+\alpha_{\rm i}^{\rm F})}{2}A_{\rm i} + {\rm i}\left(\left(\gamma + {\rm i}\frac{\beta}{2}\right)|A_{\rm i}|^2 A_{\rm i}\right.\\ &\quad + \varphi_{\rm i}(z)A_{\rm i} + 2\gamma\left\{1 - f + \frac{f}{2}[H(\omega_{\rm h}-\omega_{\rm s})\right.\\ &\quad \left.\left. + H(\omega_{\rm l}-\omega_{\rm s})]\right\}A_{\rm h}A_{\rm l}A_{\rm s}^*\exp(-{\rm i}\Delta\beta z)\right), \quad (2d)\end{aligned}$$

where

$$\begin{aligned}\varphi_1 &= (2-f)\gamma(|A_{\rm l}|^2 + |A_{\rm s}|^2 + |A_{\rm i}|^2)\\ &\quad + f\gamma[H(\omega_{\rm h}-\omega_{\rm l})|A_{\rm l}|^2 + H(\omega_{\rm h}-\omega_{\rm s})|A_{\rm s}|^2\\ &\quad + H(\omega_{\rm h}-\omega_{\rm i})|A_{\rm i}|^2], \quad (3a)\\ \varphi_2 &= (2-f)\gamma(|A_{\rm h}|^2 + |A_{\rm s}|^2 + |A_{\rm i}|^2)\\ &\quad + f\gamma[H(\omega_{\rm l}-\omega_{\rm h})|A_{\rm h}|^2 + H(\omega_{\rm l}-\omega_{\rm s})|A_{\rm s}|^2\\ &\quad + H(\omega_{\rm l}-\omega_{\rm i})|A_{\rm i}|^2], \quad (3b)\\ \varphi_{\rm s} &= (2-f)\gamma(|A_{\rm h}|^2 + |A_{\rm l}|^2 + |A_{\rm i}|^2)\\ &\quad + f\gamma[H(\omega_{\rm s}-\omega_{\rm h})|A_{\rm h}|^2 + H(\omega_{\rm s}-\omega_{\rm l})|A_{\rm l}|^2\\ &\quad + H(\omega_{\rm s}-\omega_{\rm i})|A_{\rm i}|^2], \quad (3c)\\ \varphi_{\rm i} &= (2-f)\gamma(|A_{\rm h}|^2 + |A_{\rm l}|^2 + |A_{\rm s}|^2)\\ &\quad + f\gamma[H(\omega_{\rm i}-\omega_{\rm h})|A_{\rm h}|^2 + H(\omega_{\rm i}-\omega_{\rm l})|A_{\rm l}|^2\\ &\quad + H(\omega_{\rm i}-\omega_{\rm s})|A_{\rm s}|^2]. \quad (3d)\end{aligned}$$

The linear phase mismatch is

$$\Delta\beta = 2\sum_{m=1}^{\infty}\frac{\beta_{2m}}{2m!}[(\Delta\omega_{\rm s})^{2m} - (\Delta\omega_{\rm d})^{2m}], \tag{4}$$

where $\Delta\omega_{\rm s} = \omega_{\rm s} - \omega_{\rm c}$, the center pump frequency is $\omega_{\rm c} = (\omega_{\rm h} + \omega_{\rm l})/2$, and $\Delta\omega_{\rm d} = (\omega_{\rm h} - \omega_{\rm l})/2$. The first terms on the right hand sides of Eqs. (2a)-(2d) are related to the linear attenuation and the FCA loss with coefficients $\alpha = 0.2$ dB/cm and $\alpha_j^{\rm F} = 1.45 \times 10^{-17}(\lambda_j/1550)^2 N$ (j=h, l, s, i). The TPA coefficient is $\beta = 0.75 \times 10^{-11}$ m/W.[7] Raman contri-

bution fraction f in Eqs. (3a)–(3d) is of great importance, it is the criterion of evaluating the SRS contribution to the amplification process. When f is set to be 0, no SRS effect is considered, and only the PA in silicon occurs. When f is set to be 0.043,[14] the SRS and the PA are both considered. So this model can be used to analyze the two cases by changing the value of f.

The generated free carrier density N in silicon meets the following relation (when the pulse width is much smaller than the carrier lifetime, $T_0 \ll \tau$):[7]

$$N(z) \approx \frac{\dfrac{1}{1-\mathrm{e}^{-1/R\tau}}\beta T_0 I^2(z)}{2h\upsilon}, \tag{5}$$

where R is the bit rate of the input pulses (e.g., the pump signals), and $I(z)$ is the total intensity in the effective mode area (EMA) at propagation distance z. At the output end of the waveguide ($z = L$), the amplification gain $G(L)$ can be described as

$$G(L) = 10\lg(|A_\mathrm{s}(L)|^2/|A_\mathrm{s}(0)|^2)\ (\mathrm{dB}). \tag{6}$$

3. Gain characteristics in two pump schemes

When one pump signal is employed, the gain spectra of the probe signal are shown in Fig. 2. A gain more than 4 dB can be obtained at the pump peak power of 2 W. Case 1 involves only the PA process, and case 2 involves the PA + SRS process, the different colours or line styles represent different pump wavelengths used. It can be seen that there are two sharp peaks on each gain plot in case 2, which agrees well with the CW pump scheme in Ref. [9]. As the peaks are on the edges of the gain bandwidth, the influence of SRS is smaller. When the pump signal works at 1560 nm, the bandwidths in cases 1 and 2 are almost the same, and the latter is even larger. When the pump signal works at 1550 nm, the bandwidth in case 1 is 72 nm broader than that in case 2. Therefore, in some situations, the SRS should be considered even in the single pump cases.

The gain spectra with two pump pulse signals are shown in Fig. 3. Case 1 involves only the PA process, and case 2 involves the PA + SRS process, the different colours or line styles represent three combinations of different pump wavelengths.[8] In case 2, four sharp peaks and valleys appear, and the effective bandwidth is decreased by half as a result of the SRS effect. Although a gain over 10.1 dB can be achieved, the gain bandwidth is badly influenced. This phenomenon has already been reported and depicted before in Ref. [9]. When the wavelengths of the pumps are at 1512 nm and 1591 nm, the effective bandwidth difference in the two cases is about 110 nm. From the above analysis, it can be seen that the SRS effect with dual pump signals has a more obvious influence on the gain bandwidth than that with the single pump signal.

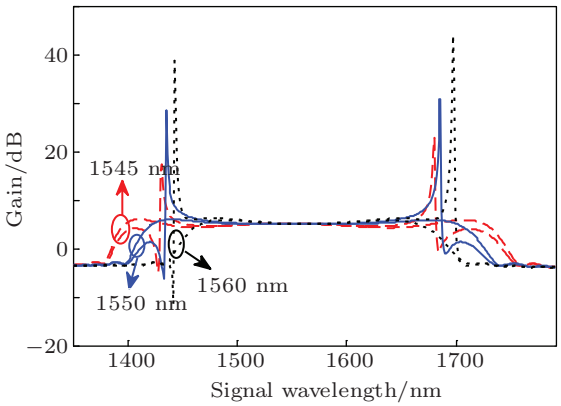


Fig. 2. (colour online) Gain spectra at the pump signal wavelengths of 1545 nm (red dash lines), 1550 nm (blue solid lines), and 1560 nm (black dot lines).

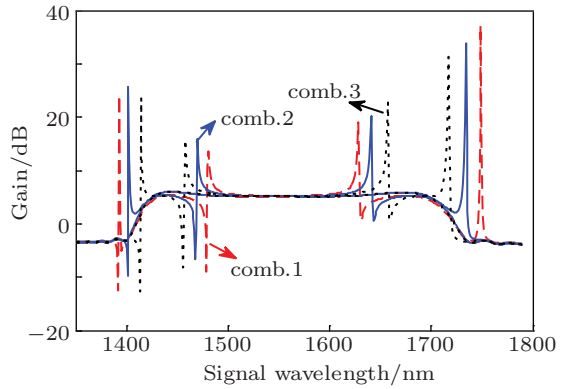


Fig. 3. (colour online) Gain spectra at three combinations of different pump signal wavelengths (red dash lines: pump signals at 1501.4 nm and 1603.5 nm, blue solid lines: 1512 nm and 1591 nm, black dot lines: 1525.7 nm and 1576.7 nm).

4. Three factors affecting gain spectra

Concerning the impact of dispersion parameters, we discuss three main factors relating to parametric and Raman amplification, which have a vital influence on the signal gain spectrum. The gain spectra at different zero dispersion wavelengths (ZDWL) λ_0 of the waveguide are shown below for the two cases. In Fig. 4(a), the gain fluctuation becomes obvious as the pump central wavelength (1551.1 nm) is further away from the ZDWL of the waveguide, and its hatching up

and bottoming up come from the phase mismatch between the optical waves. As shown in Fig. 4(b), with the ZDWL much larger than the central wavelength of the pump signals, very flat gain spectra in the range of 150 nm can be realized, and the central wavelength should be set properly around the ZDWL to get a very small ripple in the gain bandwidth.

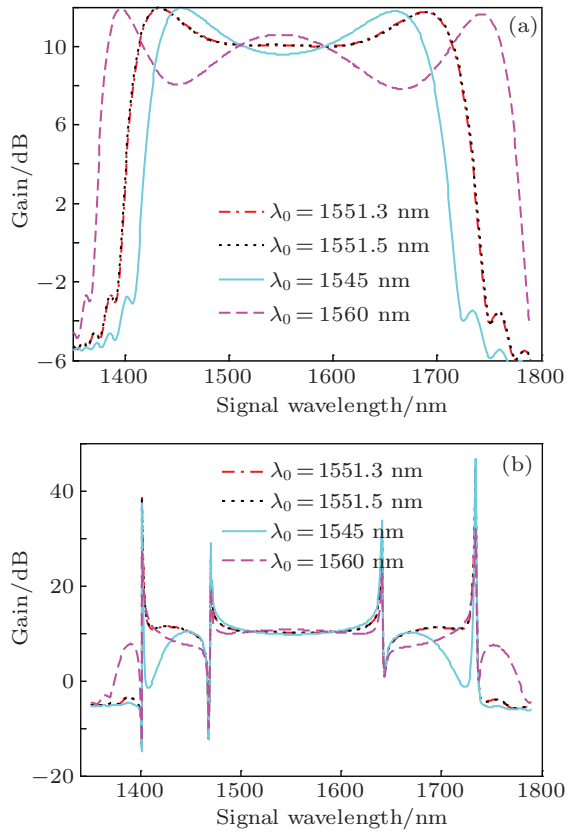


Fig. 4. (colour online) Gain spectra of PA (a) without and (b) with the SRS effect at fixed ZDWLs of 1551.3 nm (red dash-dot lines), 1551.5 nm (black dot lines), 1545 nm (light-blue solid lines), and 1560 nm (pink dash lines).

The gain spectra varying with the third-order dispersion (TOD) and the fourth-order dispersion (FOD) in both cases are shown in Figs. 5 and 6, respectively. The trends and the patterns are very much the same as those in Ref. [8], which in turn verifies the validity of the mentioned model. The TOD has less effect on the gain value but a little obvious effect on the edges of the gain bandwidth in case 1, but in case 2, the gain spectra are nearly independent of the TOD. As shown in Figs. 6(a) and 6(b), when β_4 approaches zero (light-blue solid lines) from the negative side, the gain bandwidth is widely spread in case 1, and each discrete section actually has a flatter gain in case 2. On the pink dash line of β_4=3 × 10^{-53} s^4/m, the PA bandwidth is the narrowest, but the PA + SRS gain spectrum has the smallest ripple in the middle section. The β_4 has an important effect on the

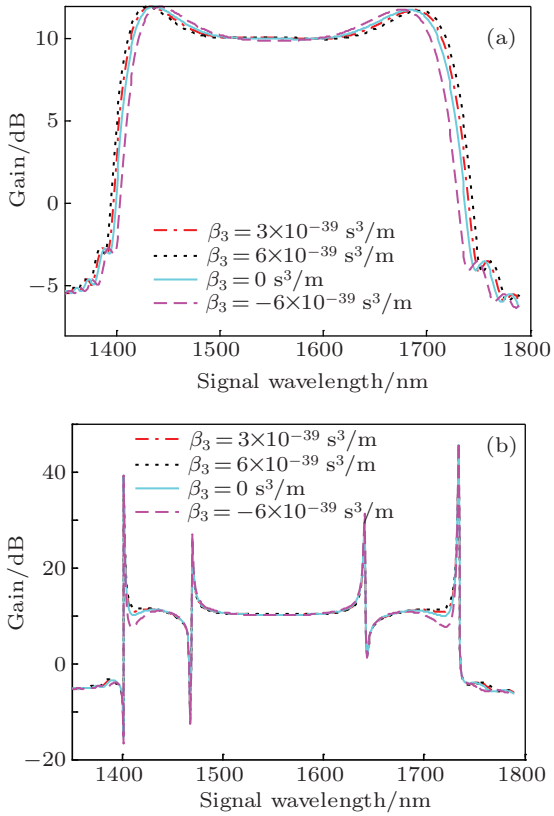


Fig. 5. (colour online) Gain spectra of PA (a) without and (b) with the SRS obtained at different TOD coefficients.

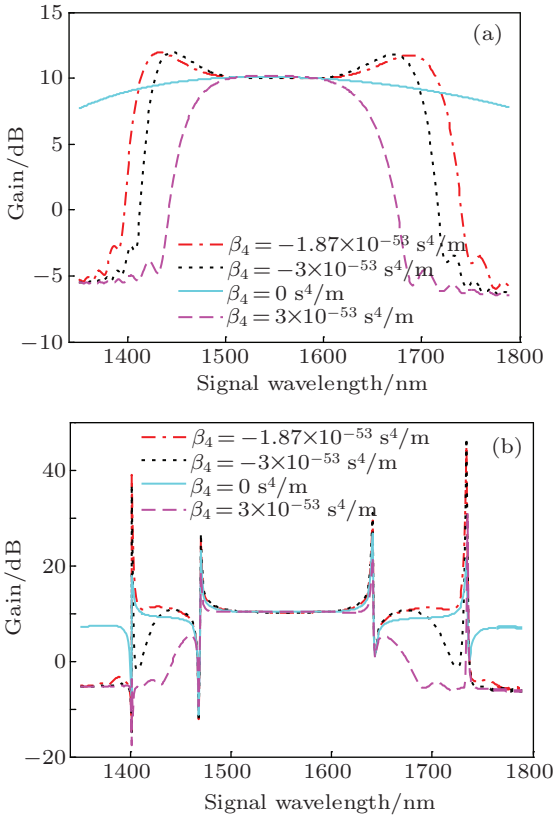


Fig. 6. (colour online) Gain spectra of PA (a) without and (b) with the SRS obtained at different FOD coefficients.

gain characteristics, and with a proper design, a high performance amplifier based on silicon can be obtained.

5. Conclusion

Different from the other analytical methods, a model analyzing PA and PA + SRS by simply setting f to be 0 and 0.043 respectively is presented. Because of a wider gain bandwidth, the SRS has a relatively greater influence on the PA bandwidth with the dual pumps than that with the single pump. The wavelength in both pump schemes has an obvious effect on the gain spectrum and should be carefully chosen to get an optimum bandwidth. Three important factors ZDWL, TOD, and FOD are discussed in detail. A conclusion is reached that with the ZDWL of the waveguide close to the central wavelength of the pump signals and FOD coefficient β_4 approaching zero, a high performance amplifier can be realized. This model can be used to simulate more comprehensive amplification processes if more mechanisms like the stimulated Brillouin scattering and the free carrier dispersion are considered. And by slightly modifying this model based on the amplitude coupled equations, functions of other devices based on silicon can be achieved. It may also be a solution for chip scale information processing and optical communications in integrated optics.

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