# Matter-wave solutions of Bose–Einstein condensates with three-body interaction in linear magnetic and time-dependent laser fields

Etienne Wamba[a)†], Timoléon C. Kofané[a)], and Alidou Mohamadou[b)c)]

[a)] *Laboratory of Mechanics, Department of Physics, Faculty of Science, University of Yaoundé I, P.O. Box* 812, *Yaoundé, Republic of Cameroon*

[b)] *Condensed Matter Laboratory, Department of Physics, Faculty of Science, University of Douala, P.O. Box* 24157, *Douala, Republic of Cameroon*

[c)] *The Abdus Salam International Centre for Theoretical Physics, P.O. Box* 586, *Strada Costiera* 11, *I*-34014, *Trieste, Italy*

(Received 7 December 2011; revised manuscript received 13 February 2012)

We construct, through a further extension of the tanh-function method, the matter-wave solutions of Bose–Einstein condensates (BECs) with a three-body interaction. The BECs are trapped in a potential comprising the linear magnetic and the time-dependent laser fields. The exact solutions obtained include soliton solutions, such as kink and antikink as well as bright, dark, multisolitonic modulated waves. We realize that the motion and the shape of the solitary wave can be manipulated by controlling the strengths of the fields.

**Keywords:** extended tanh-function method, Gross–Pitaevskii equation, cubic–quintic nonlinearity, soliton solutions

**PACS:** 05.45.Yv, 03.75.Lm, 03.75.Kk **DOI:** 10.1088/1674-1056/21/7/070504

## 1. Introduction

The achievement of Einstein's predictions on the condensation of bosons has triggered a tremendous number of works in this new field of research. Many spectacular phenomena on nonlinear excitations of the atomic matter waves,[1,2] like Josephson oscillations, macroscopic quantum tunnelling,[3] as well as the formation and propagation of waves such as bright,[4] dark,[1] grey,[5] and gap[3] solitons, vortices,[6,7] vortex lattices,[8−10] and Faraday waves,[11] have been predicted and observed in experiments. In cosmology, since it is thought that the (unknown) dark matter which constitutes the expanding universe could be BEC, this has also become a relevant subject of investigation.[12] A suitable model, which is based on the description of these various phenomena, is provided by the Gross–Pitaevskii (GP) mean-field theory. Since the BEC is attained in a magneto–optical trap, and due to the nonlinearities arising from the interatomic interactions, most of the time, the Gross–Pitaevskii equation is easily solved by means of numerical methods. However, some exact solitonic solutions of this equation have been found for particular interatomic interactions and trapping potential strengths.[13−19] In this paper, we use a further extension of the tanh-function method to obtain exact solitonic solutions of the cubic–quintic Gross–Pitaevskii equation with a time-dependent potential induced by a bias magnetic field and a laser field.

## 2. Model and solutions

The dynamics of trapped quasi-one-dimensional (1D) cigar-shaped Bose gases in the presence of two- and three-body interatomic interactions can be described in the framework of a variant of the cubic–quintic nonlinear Schrödinger (CQ-NLS) equation. The 1D GP equation is written in the following normalized form:[3,19−22]

$$\begin{aligned} \mathrm{i}\psi_t = & -c\psi_{xx} + x[\lambda + \varepsilon\sin(\omega t)]\psi + g|\psi|^2\psi \\ & + \chi|\psi|^4\psi, \end{aligned} \tag{1}$$

where space coordinate $x$ and time $t$ are measured in the units of $\varsigma = 1$ μm (the characteristic length unit in this type of experiment) and $m\varsigma^2/\hbar$, respectively, with $m$ being the mass per particle.[23,24] The

†Corresponding author. E-mail: wambaetienne@yahoo.fr

© 2012 Chinese Physical Society and IOP Publishing Ltd

parameter of cubic nonlinearity $g$ represents the two-body interatomic interaction coefficient, while the parameter of quintic nonlinearity $\chi$ accounts for the three-body interatomic interaction. The $\psi(x,t)$ is the normalized macroscopic wave function of the condensate. Parameters $\lambda$ and $\varepsilon$ are the strengths of the linear magnetic field and the time-dependent laser field, respectively.[23] The intensity of the laser light field is periodically modulated in time with modulation frequency $\omega$.

Equation (1) includes both the two- and the three-body interactions between the atoms of the condensate. Though in most practical experiments, the three-body interaction term is weak or dropped, it can be roughly of the same magnitude and range as the two-body term.[25] In some regimes, the three-body interaction can even become dominant,[26] for instance, in the cases of higher densities and Efimov resonance. A very useful tool in the experiments with ultracold atomic gases is the control of strengths of interparticle interactions. The widely encountered case concerns the two-body interaction, which is manipulated through an external magnetic field near the Feshbach resonance.[27,28] The recent works have proposed the possibility of controlling the strength of the three-body interaction between atoms independently from the control of the two-body collisions.[29,30] These findings have triggered numerous works on BECs with three-body interactions based on the CQ-NLS.[31−34]

Some exact solutions of the GP equation with bias magnetic and time-dependent laser fields have been obtained in Refs. [35], [24], and [23] through the generalized $(G'/G)$-expansion method, the inverse scattering method, and the extended tanh-function method, respectively. In contrast to the model considered in those works, we here take into account the effect of the three-body atom–atom interaction.

In this paper, we follow the ideas of Ref. [23] and seek the exact solitonic solutions of Eq. (1) with the extended tanh-function method. For this, we need to introduce the sixth-order auxiliary equation

$$u'^2 = c_0 + c_2 u^2 + c_4 u^4 + c_6 u^6, \tag{2}$$

where $u' = \mathrm{d}u/\mathrm{d}\zeta$, $\zeta = \zeta(x,t)$, and $c_0$, $c_2$, $c_4$, $c_6$ are some real constants. Let $a$ and $m$ be two arbitrary real constants. We can verify that equation (2) admits the following solutions. (i) When $c_0 = 0$, $c_2 = m^2$, $c_4 = -m^2$, $c_6 = (1/4)m^2$, then

$$u(\zeta) = \pm\frac{2}{\sqrt{2 + a\exp(2m\,\zeta)}}. \tag{3}$$

(ii) When $c_0 = -m^2$, $c_2 = -m^2$, $c_4 = m^2$, $c_6 = m^2$, then

$$u(\zeta) = \pm\mathrm{i}\frac{\tanh(m\sqrt{2}\,\zeta + a)}{\sqrt{2 - \tanh(m\sqrt{2}\,\zeta + a)^2}}. \tag{4}$$

(iii) When $c_0 = m^2$, $c_2 = m^2$, $c_4 = -m^2$, $c_6 = -m^2$, then

$$u(\zeta) = \pm\frac{\tan(m\sqrt{2}\,\zeta + a)}{\sqrt{2 + \tan(m\sqrt{2}\,\zeta + a)^2}}. \tag{5}$$

(iv) When $c_0 = 0$, $c_2 = m^2$, $c_4 = 0$, $c_6 = -m^2$, then

$$u(\zeta) = \pm\delta\sqrt{\mathrm{sech}(m\,\zeta + a)}, \quad \delta = 1, \mathrm{i}. \tag{6}$$

Let $\zeta(x,t) = p(t)x + q(t)$ and use the transformation

$$\psi(x,t) = [f(t) + h(t)u(\zeta)]\,\mathrm{e}^{\mathrm{i}[k(t)x + \varphi(t)]}. \tag{7}$$

We substitute Eqs. (2) and (7) into Eq. (1), and set the real and the imaginary parts of the resulting equation to zero. Then collecting coefficients of powers of $x^n u^{n'}$ $(n = 0, 1;\ n' = 0, 1, 2, 3, 4, 5)$ and setting each of them to zero, we obtain the following set of partial differential equations:

$$\begin{aligned}
&f' = 0, \quad h' = 0, \quad hp' = 0,\\
&h(q' + 2cpk) = 0,\\
&h[k' + \lambda(t) + \varepsilon\sin(\omega t)] = 0,\\
&f[k' + \lambda(t) + \varepsilon\sin(\omega t)] = 0,\\
&h(2cp^2c_4 - gh^2 - 10\chi f^2 h^2) = 0,\\
&fh^2(-3g - 10\chi f^2) = 0,\\
&h(3cp^2c_6 - \chi h^4) = 0,\\
&-5\chi f h^4 = 0,\\
&f(\varphi' + ck^2 + gf^2 + \chi f^4) = 0,\\
&h(\varphi' + ck^2 + 3gf^2 + 5\chi f^4 - cp^2c_2) = 0.
\end{aligned}$$

In the above set of equations, the prime stands for the derivative with respect to time $t$. Solving the set of equations, we come to the following results:

$$\begin{aligned}
&f(t) = 0,\\
&p(t) = p \equiv A_{\mathrm{s}}^2,\\
&h(t) = \pm\left(3c\frac{p^2}{\chi}c_6\right)^{1/4} = \pm\left(2c\frac{p^2}{g}c_4\right)^{1/2},\\
&k(t) = \frac{\varepsilon}{\omega}\cos(\omega t) - \lambda t + k_0,\\
&q(t) = -2cp\left[\frac{1}{\omega^2}\,\varepsilon\sin(\omega t) - \frac{1}{2}\lambda t^2 + k_0 t\right] + q_0,\\
&\varphi(t) = c\left(p^2c_2 - k_0^2 - \frac{1}{2\omega^2}\,\varepsilon^2\right)t + c\lambda k_0\,t^2 - \frac{1}{3}c\lambda^2\,t^3
\end{aligned}$$

$$-2\frac{c}{\omega^2}\,\varepsilon(k_0-\lambda t)\sin(\omega t)+2\frac{c}{\omega^3}\,\varepsilon\lambda\cos(\omega t)$$
$$-\frac{c}{4\omega^3}\,\varepsilon^2\sin(2\omega t)+\varphi_0,$$
$$c_6=\frac{4\chi cp^2}{3g^2}\,c_4^2.$$

It should be noted that the latter relation implicitly assumes $gc_4>0$ and $\chi c_6>0$ in view of the above expressions of $h(t)$. Furthermore, the purely cubic case ($\chi=0$) requires $c_6=0$ and the purely quintic case ($g=0$) demands $c_4=0$ to obtain a finite and nonzero value for the amplitude coefficient $h$. Thus, we may let $\chi=\frac{1}{\eta}\,c_6$ with $\eta$ being a positive parameter, and write $g=\pm\sqrt{4cp^2c_4^2/3\eta}$.

Hence the solution of Eq. (1) is given by

$$\psi(x,t)=\pm\left(3c\frac{c_6}{\chi}\right)^{1/4}A_{\rm s}\,u(\zeta)\,{\rm e}^{{\rm i}[k(t)x+\varphi(t)]},\qquad(8)$$

where $u(\zeta)$ is the solution of Eq. (2). The exact solution (8) contains some interesting dynamical properties and features. In the case where equation (8) describes a soliton, the center-of-mass dynamics of the quasi-particle is obtained by taking $\zeta(x,t)=0$. So the center-of-mass position, the velocity, and the acceleration of the soliton are respectively

$$x=2c\left[\frac{1}{\omega^2}\,\varepsilon\sin(\omega t)-\frac{1}{2}\lambda t^2+k_0t\right]-\frac{q_0}{p},$$
$$\dot{x}=2c\left[\frac{1}{\omega}\,\varepsilon\cos(\omega t)-\lambda t+k_0\right],$$
$$\ddot{x}=-2c\,[\lambda+\varepsilon\sin(\omega t)].\qquad(9)$$

Parameters $\omega$, $\lambda$, and $\varepsilon$, as well as $m$ and $\eta$, have some captivating effects on the solution. The center of mass of the macroscopic wave packet behaves like a classical particle, and allows us to manipulate the motion of the soliton by controlling the external trapping potential. In the following we consider some examples to demonstrate the dynamics of kink and antikink solitary waves in the BEC system with different components of the trapping potential.

For a weak laser field ($\varepsilon=0$), the wave packet moves in the $(x,t)$ space along a parabolic trajectory, with acceleration $-2c\lambda$ similar to the one associated with the gravity. When the bias magnetic field is weak ($\lambda=0$), the trajectory is sinusoidal. For the sake of simplicity, we choose $q_0=k_0=0$ in most cases, and set the initial position and speed of the soliton at $x_0=0$ and $\dot{x}_0=0$, respectively.

In order to express the solutions of Eq. (1), we consider the following four cases.

1) When $u(\zeta)$ is given by Eq. (3), the three-body interaction is repulsive, since $\chi>0$. The parameters of quintic and cubic nonlinearities read $\chi=m^2/4\eta$ and $g=\pm\sqrt{4cp^2m^4/3\eta}$, respectively. We obtain through Eq. (8) the families of kink-shaped solitons ($m<0$) and antikink-shaped solitons ($m>0$) expressed as

$$\psi(x,t)=\pm\frac{2A_{\rm s}(3\eta c)^{1/4}\exp\{{\rm i}[k(t)x+\varphi(t)]\}}{\sqrt{2+a\exp[2mp\,x+2mq(t)]}}.\qquad(10)$$

The dynamics of this wave in the absence of the laser field is shown in Figs. 1 and 2. The kink and the antikink solitons travel in the $-x$ direction for $\lambda>0$ and in the $+x$ direction for $\lambda<0$. In both cases, the motion is uniformly accelerated, and the acceleration is proportional to the magnetic field strength $\lambda$. Meanwhile, we have observed that the width of the soliton is inversely proportional to parameter $|m|$. Moreover, for a given value of $m$, the height of the soliton decreases when the quintic parameter increases. So the three-body interaction is responsible for the compression of the soliton.

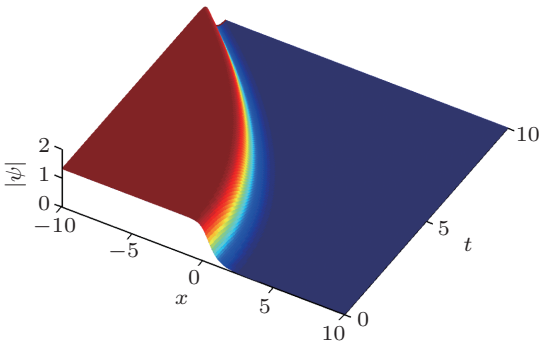



**Fig. 1.** (colour online) Evolution of an antikink soliton given by Eq. (10) under the effect of bias magnetic field. The parameters are $\lambda=0.25$, $\varepsilon=0$, $A_{\rm s}=1$, $a=1$, $m=2$, $c=0.5$, $\eta=0.5$, $\omega=\pi$, $p=1$, $q_0=0$, and $k_0=0$.

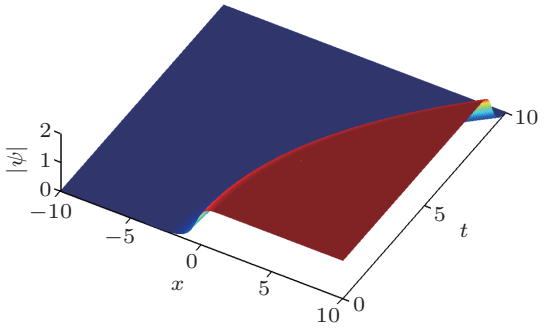



**Fig. 2.** (colour online) Evolution of a kink soliton given by Eq. (10) under the effect of bias magnetic field. The parameters are the same as those in Fig. 1 except for $\lambda=-0.25$ and $m=-2$.

In the absence of the bias magnetic field, the center of mass of the kink soliton oscillates between $x_{\rm min}=-2c\varepsilon/\omega^2$ and $x_{\rm max}=2c\varepsilon/\omega^2$ due

to the temporal periodic modulation of the potential. Similar conclusions have been obtained for a BEC under a quadratic magnetic field obtained through the Darboux transformation method.[36] These oscillations are rather obvious around the line $x = k_0 t$ when the initial speed of the wave packet is nonzero. The dynamics of the oscillating soliton is shown in Figs. 3–5. Let us note in passing that though parameter $\omega$ is the frequency of the wave oscillation, it seriously modifies the oscillation amplitude. Indeed, the wave amplitude increases with the decrease of the frequency.

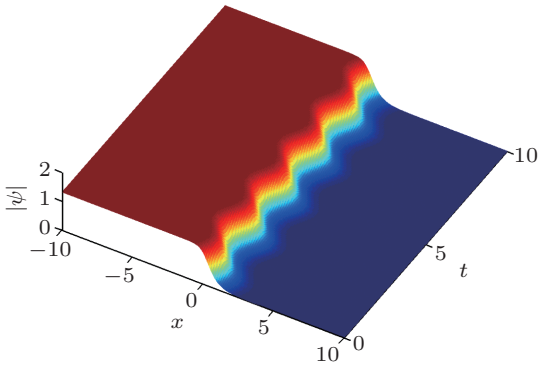



**Fig. 3.** (colour online) Evolution of an antikink soliton given by Eq. (10) with zero initial speed under the effect of laser field. The parameters are the same as those in Fig. 1 except for $\lambda = 0$ and $\varepsilon = 4.0$.

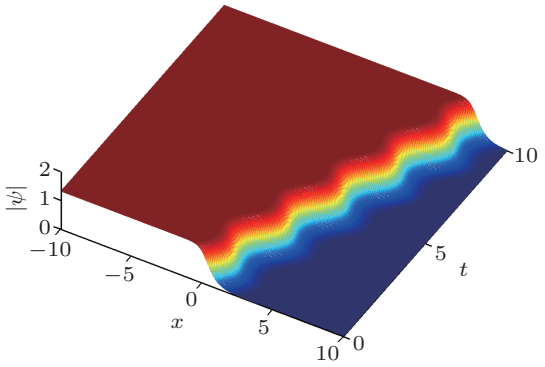



**Fig. 4.** (colour online) Evolution of an antikink soliton given by Eq. (10) with nonzero initial speed under the effect of laser field. The parameters are the same as those in Fig. 3 except for $k_0 = 0.75$.

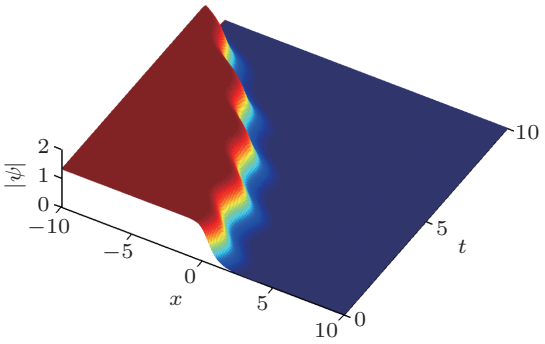



**Fig. 5.** (colour online) Evolution of an antikink soliton given by Eq. (10) with zero initial speed under the combined effect of bias magnetic and laser fields. The parameters are the same as those in Fig. 1 except for $\varepsilon = 4.0$.

2) When $u(\zeta)$ is given by Eq. (4), the three-body interaction is also repulsive. The parameters of quintic and cubic nonlinearities read $\chi = m^2/\eta$ and $g = \pm\sqrt{4cp^2m^4/3\eta}$, respectively. We obtain for Eq. (1) the family of dark solitons

$$\begin{aligned}&\psi(x,t)\\ &= \pm \mathrm{i}(3\eta c)^{1/4}\frac{A_{\mathrm{s}}\tanh(\sqrt{2}mp\,x+\sqrt{2}mq(t)+a)}{\sqrt{2-\tanh(\sqrt{2}mp\,x+\sqrt{2}mq(t)+a)^2}}\\ &\times \exp\{\mathrm{i}[k(t)x+\varphi(t)]\}. \qquad (11)\end{aligned}$$

This dark soliton presents a similar dynamical behavior to the previous kink soliton. It oscillates under the action of the laser field and undergoes the parabolic motion in the $(x,t)$ space because of the linear magnetic field. Figures 6 and 7 display the temporal evolution of the wave.

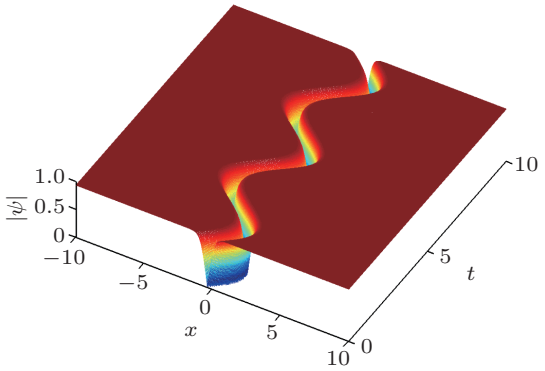



**Fig. 6.** (colour online) Evolution of a dark soliton given by Eq. (11) with zero initial speed under the effect of laser field. The parameters are the same as those in Fig. 3 except for $\omega = \pi/2$.

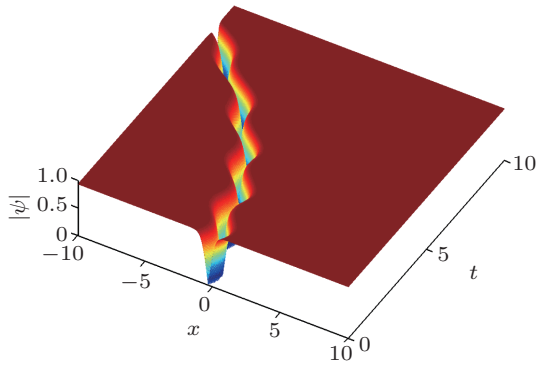



**Fig. 7.** (colour online) Evolution of a dark soliton given by Eq. (11) with zero initial speed under the combined effect of bias magnetic and laser fields. The parameters are the same as those in Fig. 6 except for $\varepsilon = 4.0$.

3) When $u(\zeta)$ is given by Eq. (5), the three-body interaction is attractive, since $\chi < 0$. The parameters of quintic and cubic nonlinearities read $\chi = -m^2/\eta$ and $g = \pm\sqrt{4cp^2m^4/3\eta}$, respectively. We obtain the family of multisolitonic modulated waves

$$\psi(x,t) = \pm(3\eta c)^{1/4} \frac{A_{\rm s}\tan(\sqrt{2}mp\,x + \sqrt{2}mq(t) + a)}{\sqrt{2+\tan(\sqrt{2}mp\,x + \sqrt{2}mq(t) + a)^2}} \times \exp\{{\rm i}[k(t)x + \varphi(t)]\}. \quad (12)$$

The dynamics of this wave is portrayed in Figs. 8 and 9. The laser field creates a pattern made up with wells separating solitonic peaks, and the number of peaks depends on the value of $m$. This pattern is the multisolitonic wave with a modulated envelope.[19] When there is no linear magnetic field, each solitonic peak oscillates around its initial position. When the magnetic field is switched on, the pattern is drifted from its initial position through an accelerated motion. This drift has already been predicted in the previous works through a numerical analysis.[33] Along its trajectory, the density of each solitonic peak of the pattern remains a constant. Meanwhile during the propagation, the distances between neighboring solitons do not change.

4) When $u(\zeta)$ is given by Eq. (6), the three-body interaction is also attractive. The parameters of quintic and cubic nonlinearities read $\chi = -m^2/\eta$ and $g = 0$, respectively. This is the case where the nonlinearity is purely quintic. We obtain the family of bright solitons

$$\psi(x,t) = \pm\delta(3\eta c)^{1/4} A_{\rm s}\sqrt{{\rm sech}(mp\,x + mq(t) + a)} \times \exp\{{\rm i}[k(t)x + \varphi(t)]\}, \quad (13)$$

where $\delta = 1$ or i. The motion of the bright soliton presents the oscillations induced by the laser field. These oscillations are obvious around the parabolic trajectory created by the linear magnetic field. Figures 10 and 11 display the space–time evolution of the wave.

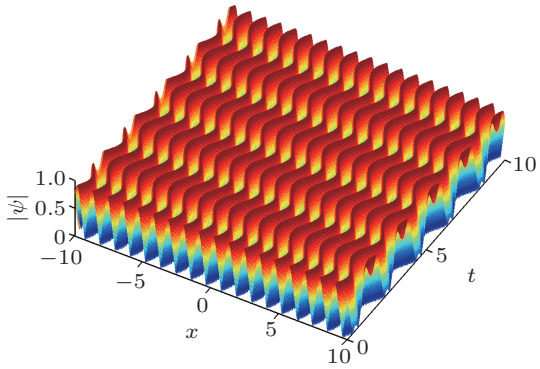



**Fig. 8.** (colour online) Evolution of a multisolitonic modulated wave given by Eq. (12) with zero initial speed under the effect of laser field. The parameters are the same as those in Fig. 3 except for $\varepsilon = 4.0$.

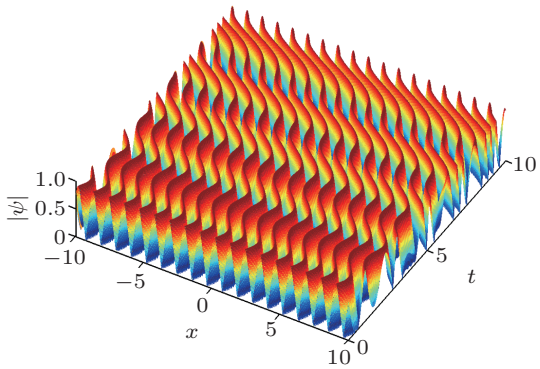



**Fig. 9.** (colour online) Evolution of a multisolitonic wave given by Eq. (12) with zero initial speed under the combined effect of bias magnetic and laser fields. The parameters are the same as those in Fig. 8 except for $\lambda = 0.25$.

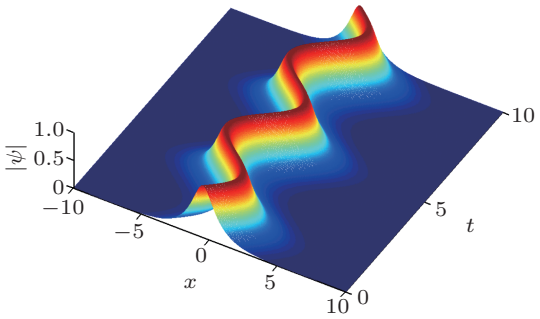



**Fig. 10.** (colour online) Evolution of a bright soliton given by Eq. (13) with zero initial speed under the effect of bias laser field. The parameters are the same as those in Fig. 6.

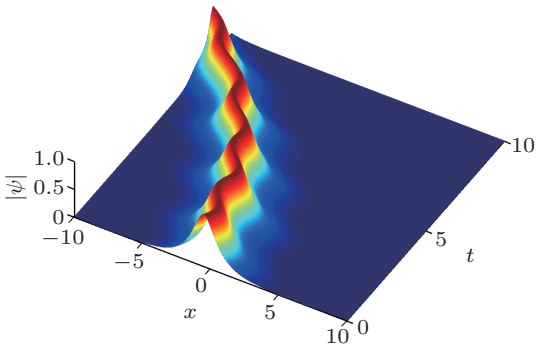



**Fig. 11.** (colour online) Evolution of a bright soliton given by Eq. (13) with zero initial speed under the combined effect of bias magnetic and laser fields. The parameters are the same as those in Fig. 7.

## 3. Conclusion

In summary, we have considered the GP equation with cubic and quintic nonlinearities, which describes

the dynamics of the BEC matter waves in a time-dependent linear potential induced by a bias magnetic field and a time-dependent laser field. With the help of the extended tanh-function method, we have presented the kink and the antikink solitary wave solutions of the BEC system with a three-body interparticle interaction. The other families of matter waves, such as dark solitons, bright solitons, and multisolitonic waves, have also been obtained. The dynamics of these solutions imply that controlling the magnetic and the laser fields may provide a way to generate and manipulate the motion and the shape of different types of solitary waves in the BEC systems.

# References


[1] Theocharis G, Frantzeskakis D J, Kevrekidis P G, Malomed B A and Kivshar Yuri S 2003 *Phys. Rev. Lett.* **90** 120403

[2] Wu L and Zhang J F 2007 *Chin. Phys. Lett.* **24** 1471

[3] Abdullaev F Kh, Abdumalikov A A and Galimzyanov R M 2007 *Phys. Lett.* A **367** 149

[4] Khaykovich L, Schreck F, Ferrari G, Bourdel T, Cubizolles J, Carr L D, Castin Y and Salomon C 2002 *Science* **296** 1290

[5] Shomroni I, Lahoud E, Levy S and Steinhauer J 2009 *Nature Phys.* **5** 193

[6] Matthews M R, Anderson B P, Haljan P C, Hall D S, Wieman C E and Cornell E A 1999 *Phys. Rev. Lett.* **83** 2498

[7] Inouye S, Gupta S, Rosenband T, Chikkatur A P, Gorlitz A G, Gustavson T L, Leanhardt A E, Pritchard D E and Ketterle W 2001 *Phys. Rev. Lett.* **87** 080402

[8] Abo-Shaeer J R, Raman C, Vogels J M and Ketterle W 2001 *Science* **292** 476

[9] Abo-Shaeer J R, Raman C and Ketterle W 2002 *Phys. Rev. Lett.* **88** 070409

[10] Engels P, Coddington I, Haljan P C and Cornell E A 2002 *Phys. Rev. Lett.* **89** 100403

[11] Staliunas K, Longhi S and de Valcárcel G J 2002 *Phys. Rev. Lett.* **89** 210406

[12] Velten H and Wamba E 2012 *Phys. Lett.* B **709** 1

[13] Liu W M, Wu B and Niu Q 2000 *Phys. Rev. Lett.* **84** 2294

[14] Ieda J, Miyakawa T and Wadati M 2004 *Phys. Rev. Lett.* **93** 194102

[15] Li Lu, Li Z D, Malomed B A, Mihalache D and Liu W M 2005 *Phys. Rev.* A **72** 033611

[16] Atre R, Panigrahi P K and Agarwal G S 2006 *Phys. Rev.* E **73** 056611

[17] Liang Z X, Zhang Z D and Liu W M 2005 *Phys. Rev. Lett.* **94** 050402

[18] Matveev V B and Salle M A 1991 *Darboux Transformations and Solitons* (Berlin: Springer-Verlag)

[19] Al Khawaja U 2006 *J. Phys.* A: *Math. Theor.* **39** 9679

[20] Moll K D, Gaeta A L and Fibich G 2003 *Phys. Rev. Lett.* **90** 203902

[21] Kevrekidis P K and Frantzeskakis D J 2003 *Mod. Phys. Lett.* B **18** 173

[22] Perez-García V M, Konotop V V and Brazhnyi V A 2004 *Phys. Rev. Lett.* **92** 220403

[23] Li H M and Wu F M 2004 *Chin. Phys. Lett.* **21** 1425

[24] Liu X Q, Jiang S, Fan W B and Liu W M 2004 *Communications in Nonlinear Science and Numerical Simulation* **9** 361365

[25] Brunner M, Dobnikar J and von Grünberg H 2004 *Phys. Rev. Lett.* **92** 078301

[26] Bulgac A 2002 *Phys. Rev. Lett.* **89** 050402

[27] Pethick C J and Smith H 2002 *Bose-Einstein Condensation in Dilute Alkali Gases* (Cambridge: Cambridge University Press)

[28] Pitaevskii L P and Stringari S 2003 *Bose-Einstein Condensation* (Oxford: Clarendon Press)

[29] Büchler H P, Micheli A and Zoller P 2007 *Nature Phys.* **3** 726

[30] Paredes B, Keilmann T and Cirac J I 2007 *Phys. Rev.* A **75** 053611

[31] Fersino E, Malomed B A, Mussardo G and Trombettoni A 2009 *Eur. Phys. J.* B **68** 417

[32] Roy U, Atre R, Sudheesh C, Kumar C N and Panigrahi P K 2010 *J. Phys.* B: *At. Mol. Opt. Phys.* **43** 025003

[33] Mohamadou A, Wamba E, Doka S Y, Ekogo T B and Kofane T C2011 *Phys. Rev.* A **84** 023602

[34] Wamba E, Mohamadou A and Kofané T C 2008 *Phys. Rev.* E **77** 046216

[35] Huang W H, Mao J W and Qiu W G 2011 *Acta Phys. Pol.* A **119** 294297

[36] Wu L, Zhang J F and Li L 2007 *New J. Phys.* **9** 69